HOMOGENIZED TRANSPORT BY A SPATIOTEMPORAL MEAN FLOW WITH SMALL-SCALE PERIODIC FLUCTUATIONS

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Abstract. The transport of a pollutant in the ocean or atmosphere is influenced strongly by both the prevailing large-scale mean flow structure and disordered turbulent motion prevalent on smaller scales. To obtain some insight into the effects of turbulent transport, various authors over the last decade have studied the transport of material in model flows which have a periodic structure. For such flows, one can develop a rigorous homogenization theory to describe their effective transport on large scales. We will present an extension of these homogenization studies to a class of model flows which consist of a superposition of a large-scale mean flow with a small-scale periodic structure (both of which can depend on space and time). Using two-scale convergence and higher order homogenization techniques, we rigorously derive homogenized equations for these models, in which the mean flow and periodic structure are shown to interact nonlinearly. The small-scale structure is responsible for an enhancement in the diffusivity as well as for the presence of an effective drift, both of which are functions of space and time. The effective drift generated in our model provides a concrete and computable instance of the "skew-flux" appearing in turbulence parametrization schemes in atmosphere-ocean science. The remarkable variability of the effective drift and diffusivity is illustrated through a simple example.

1. **Introduction.** A problem of great practical and theoretical interest is that of the transport of a scalar quantity immersed in a fluid flow. Applications include flow in heterogeneous porous media [7, 18, 23, 31], plasma physics [29, 35], fully developed turbulence [19, 25], astrophysics [8] and ocean/atmosphere science [10, 11, 16, 20, 28]. We will for the most part attempt to relate our work to transport processes in atmosphere-ocean science, though the issues are also relevant to the other applied fields.

Accurate modelling of weather or climate requires a proper accounting of the transport of moisture, ozone, and pollutants in the atmosphere, salinity and nutrients in the ocean, and heat and other thermodynamic state variables in both environments [20, 28]. The motion of both the atmosphere and ocean can be thought of as a superposition of some slowly varying mean flow state, determined by regular seasonal or daily forcing and large-scale features of the geometry of the environment, and turbulent fluctuations which depart from this average state [20]. In other words, the mean flow can be identified as the average over an observed data set from a large

number of observations at similar times of the year and/or day. One practical diffi-
culty in the accurate computation of transport in the atmosphere and ocean is that
the turbulent fluctuations typically occur on scales considerably smaller than those
characterizing the mean flow [20]. Modern supercomputers cannot include both the
large scales of primary interest and the small-scale turbulent fluctuations due to the
wide range of scales involved. And even if computers could simulate all relevant
scales, the data which is collected is far too sparse to provide accurate information
about the turbulent fluctuations. Consequently, the atmosphere-ocean scientists
concerned with weather and climate prediction typically strive for a reliable means
to represent the effects of the turbulent fluctuations on the large scales which can
be resolved by data and computational power without actually simulating the tur-
bulent activity on the scales too small to be resolved. The way this is typically
done is to "parametrize" the effects of the small scales through a tractable finite
set of quantities, typically transport coefficients, whose values are related in some
way to the observable variables [16]. This program is far too difficult to carry out
in a precise way in applied problems, so parametrization is typically done through
various phenomenological models [13, 14, 16, 28].

The goal of our work is to study features concerning the parametrization problem
in simplified models where their origins and ramifications can be better understood,
and use this information to guide the development of parametrization schemes in
the more complex situations. To this end, we generalize some previous models for
transport in flows which have strong and *distinct* separation of scales between the
mean flow and turbulent fluctuations [4, 5, 12, 18, 21, 22, 23, 26]. Such situations
do arise in some regions of the stratosphere and ocean; see for example [11].

More precisely, the model flows which we study represent the turbulent fluctua-
tions as a periodic flow with period much smaller than the length scale of the mean
flow. We consider the induced dynamics of a passive scalar field, which represents
an immersed physical density of substances (such as pollutants or nutrients) which
have negligible influence on the flow dynamics. One of our main contributions is
to extend some previously developed results for the case of zero or constant mean
flow [22, 26] to the case where the mean flow is permitted to have a fairly general
spatio-temporal structure. Such a model has been recently considered by other
authors to illustrate phenomena such as resonance-enhanced diffusion [9, 24] and
steady flow through porous media with boundary layer effects [7], but our approach
here will focus on other issues.

Namely, after introducing the model in Section 2, we present in Section 3 a rig-
orous approximation theorem to characterize the large-scale dynamics of a passive
scalar field through an effective advection-diffusion equation where both the drift
and diffusion term are renormalized from their original bare values. The drift correc-
tion is a manifestation of a "skew-flux" phenomenon discussed in the atmosphere-
ocean science literature [13, 14, 28]. We briefly present some asymptotic results and
numerical examples which characterize the magnitude and variability of the the ef-
fective drift and diffusion terms in the large-scale equations. We close in Section 4
with a brief discussion of our methods of derivation, which involve a combination of
multiple scale expansions, higher order homogenization techniques, and the method
of two-scale convergence.

2. **Scaling and Nondimensional Mixing Parameters.** We model the veloc-
ity field as a superposition of a large-scale mean flow with a small-scale periodic

fluctuating field. The small-scale component is intended as a crude model for the unresolved turbulence which we adopt to keep the mathematical technicalities to a minimum. To save space, we write the velocity field immediately in a form nondimensionalized with respect to the space and time scales of the large-scale mean flow:

$$\mathbf{u} = \mathbf{V}\left(\mathbf{x}, t\right) + a\mathbf{v}\left(\frac{\mathbf{x}}{\delta}, \frac{t}{\eta}\right) \tag{1}$$

with the first term representing the large-scale mean flow and the second term the fluctuations with period 1 in each argument. The nondimensional parameters a, δ, and η represent the ratios of, respectively, the small-scale velocity magnitude, length scale, and time scale to their large-scale counterparts. We consider situations in which there is strong separation between the space and time scales of the mean flow and fluctuations $\delta, \eta \ll 1$, but allow for both large and small values of the velocity amplitude ratio a. Such restrictions are still relevant to some atmosphere-ocean applications [11], and can be thought of more broadly as a simplified but nontrivial representation of resolved and unresolved scales in a more general context.

The evolution of a passive scalar field $T(\mathbf{x}, t)$ in such a flow is given by the (nondimensionalized) advection-diffusion equation:

$$\begin{aligned}
\mathrm{St}_g\,\frac{\partial T(\mathbf{x}, t)}{\partial t} + \mathbf{u}(\mathbf{x}, t) \cdot \boldsymbol{\nabla} T(\mathbf{x}, t) &= \mathrm{Pe}_g^{\ -1}\Delta T(\mathbf{x}, t) \\
T(\mathbf{x}, t = 0) &= T_{\mathrm{in}}(\mathbf{x}),
\end{aligned} \tag{2}$$

set on Euclidean space $\mathbb{R}^d$ with appropriate decay conditions at infinity. Here Pe_g is the global (large-scale) Péclet number representing the inverse of the nondimensionalized molecular diffusivity and St_g is the global (large-scale) Strouhal number representing the ratio of sweeping and fluctuation times of the mean flow.

3. **Effective Large-Scale Transport Equations.** The equation (2) is stiff in the sense that it involves variability on two widely separated space and time scales. We wish to think of the periodically fluctuating component as the unresolved scales in an application whose effect on the large scales we wish to capture in some effective way. Using homogenization theory [3, 21] we can approximate rigorously the solution to (2). We present here a theorem for a special but relevant distinguished limit which illustrates the main ideas. The time scale separation between the velocity components will be chosen to be comparable to the spatial scale separation ($\eta = \delta$) (as is true for the oceanic data in [11]), the mean flow and fluctuations will be taken to have comparable magnitude ($a = 1$), and the global Péclet number will be expressed as $\mathrm{Pe}_g = \delta^{-1}\,\mathrm{Pe}_\ell$ with the *local* Péclet number independent of δ. The local Péclet number is a nondimensional measure of the molecular diffusivity relative to the space and time scales of the periodic fluctuations. The assumption that Pe_ℓ is independent of δ corresponds to a limit process in which the ratio of the scales of the velocity fluctuations to the scales of molecular diffusion is held fixed (at some possibly large value) while the separation of scales between the velocity fluctuations and the mean flow is made very large. (The analogously defined local Strouhal number coincides for our particular distinguished limit with its global value St_g, which is $\mathrm{ord}\,(1)$).

Theorem 1. *Let $T^{(\delta)}(\mathbf{x}, t)$ be the solution of the advection-diffusion equation (2) with*

$$\eta = \delta, \qquad a = 1, \qquad \text{and } \mathrm{Pe}_g = \delta^{-1} \mathrm{Pe}_\ell, \tag{3}$$

with Pe_ℓ independent of δ:

$$\mathrm{St}_g \frac{\partial T^{(\delta)}(\mathbf{x}, t)}{\partial t} + \left(\mathbf{V}(\mathbf{x}, t) + \mathbf{v}\left(\frac{\mathbf{x}}{\delta}, \frac{t}{\delta}\right) \right) \cdot \boldsymbol{\nabla} T^{(\delta)}(\mathbf{x}, t) = \delta \, \mathrm{Pe}_\ell^{-1} \Delta T^{(\delta)}(\mathbf{x}, t),$$

$$T^{(\delta)}(\mathbf{x}, t = 0) = T_{\mathrm{in}}(\mathbf{x}). \tag{4}$$

The velocity field components $\mathbf{V}$ and $\mathbf{v}$ are smooth and bounded, and $\mathbf{v}$ has period 1 in each of its arguments. The initial data T_{in} is also assumed to be smooth and bounded with $T_{\mathrm{in}} \in L^2$.

Let $\Phi(\mathbf{x})$ be a smooth, integrable large-scale filter function, independent of δ, and consider a coarse-graining of the solution of (4):

$$T^\Phi(\mathbf{x}, t) \equiv \int_{\mathbb{R}^d} \Phi(\mathbf{x} - \mathbf{x}', t) T^{(\delta)}(\mathbf{x}', t) \, \mathrm{d}\mathbf{x}'.$$

Then, $T^\Phi(\mathbf{x}, t)$ can be approximated by the solution $\bar{T}(\mathbf{x}, t)$ to the following effective advection-diffusion equation:

$$\mathrm{St}_g \frac{\partial \bar{T}(\mathbf{x}, t)}{\partial t} + \mathbf{V}(\mathbf{x}, t) \cdot \boldsymbol{\nabla} \bar{T}(\mathbf{x}, t) = \mathrm{Pe}_g^{-1} \boldsymbol{\nabla} \cdot (\mathsf{K}^*(\mathbf{x}, t) \boldsymbol{\nabla} \bar{T}(\mathbf{x}, t)), \tag{5}$$

$$\bar{T}(\mathbf{x}, t = 0) = \int_{\mathbb{R}^d} \Phi(\mathbf{x} - \mathbf{x}', t) T_{\mathrm{in}}(\mathbf{x}') \, \mathrm{d}\mathbf{x}',$$

where the effective diffusion tensor *is defined as:*

$$\mathsf{K}^*(\mathbf{x}, t) \equiv \mathsf{I} - \mathrm{Pe}_\ell \int_0^1 \int_{[0,1]^d} \mathbf{v}(\mathbf{y}, \tau) \otimes \boldsymbol{\chi}(\mathbf{x}, t, \mathbf{y}, \tau) \, \mathrm{d}\mathbf{y} \, \mathrm{d}\tau \tag{6}$$

and I is the identity tensor. The "corrector function" $\boldsymbol{\chi}(\mathbf{x}, t, \mathbf{y}, \tau)$ has period 1 in its $\mathbf{y}$ and τ arguments and satisfies the cell problem:

$$\mathrm{St}_g \frac{\partial \boldsymbol{\chi}}{\partial \tau} + (\mathbf{V}(\mathbf{x}, t) + \mathbf{v}(\mathbf{y}, t)) \cdot \boldsymbol{\nabla}_\mathbf{y} \boldsymbol{\chi} - \frac{1}{\mathrm{Pe}_\ell} \Delta_\mathbf{y} \boldsymbol{\chi} = -\mathbf{v}(\mathbf{y}, \tau), \tag{7}$$

where the spatial differential operators are with respect to the auxiliary variable $\mathbf{y}$. The large-scale variables $\mathbf{x}$ and t appear as parameters in the cell problem.

The quality of the approximation can be quantified as follows: For every $t_1 > 0$ (independent of δ) there exist constants $\{C_j = C_j(\mathrm{Pe}_\ell, \mathrm{St}_g, t_1)\}$, independent of δ, such that

$$||T^\Phi - \bar{T}||_{L^\infty((0,t_1); L^2(\mathbb{R}^d))} \leq C_1 \delta^2 + C_2 \delta e^{-C_3 t/\delta} \tag{8}$$

$$||\boldsymbol{\nabla} T^\Phi - \boldsymbol{\nabla} \bar{T}||_{L^2((0,t_1) \times \mathbb{R}^d)} \leq C_4 \delta^{\frac{3}{2}} + C_5 \delta e^{-C_3 t/\delta} \tag{9}$$

This theorem can be readily extended to allow for large-scale modulations in the amplitude of the periodic fluctuations of the velocity field [32].

We observe first that the effective diffusion tensor K^* is a function of the large-scale space and time variables. The reason for this is that the mean flow appears in the cell problem (7). In other words, the effective diffusion at some point in space and time depends on the local value of the mean flow. The large-scale variation of the mean flow therefore implies the large-scale variation of the effective diffusivity. The spatial dependence of K^* in addition to its general lack of

symmetry [12, 17, 29] moreover induces an effective drift. To see this, we define the symmetric and antisymmetric parts of the effective diffusion tensor as $\mathsf{S}^*(\mathbf{x}, t) \equiv \frac{1}{2}\left(\mathsf{K}^*(\mathbf{x}, t) + (\mathsf{K}^*(\mathbf{x}, t))^\dagger\right)$ and $\mathsf{A}^*(\mathbf{x}, t) \equiv \frac{1}{2}\left(\mathsf{K}^*(\mathbf{x}, t) - (\mathsf{K}^*(\mathbf{x}, t))^\dagger\right)$. We can then rewrite the effective equation (5) in the form:

$$\mathrm{St}_g\,\frac{\partial T(\mathbf{x}, t)}{\partial t} + \left(\mathbf{V}(\mathbf{x}, t) + \mathrm{Pe}_g{}^{-1}\mathbf{U}^*(\mathbf{x}, t)\right) \cdot \boldsymbol{\nabla} T(\mathbf{x}, t) \tag{10}$$
$$= \frac{1}{\mathrm{Pe}_g}\boldsymbol{\nabla} \cdot \left(\mathsf{S}^*(\mathbf{x}, t) \cdot \boldsymbol{\nabla} T(\mathbf{x}, t)\right),$$
$$T(\mathbf{x}, t = 0) = T_{\mathrm{in}}(\mathbf{x}).$$

The advection term now involves an effective drift correction proportional to

$$\mathbf{U}^* \equiv -\boldsymbol{\nabla} \cdot \mathsf{A}^*$$

which acts in addition to the mean flow. The effective diffusion induced by the small-scale velocity fluctuations is now represented by the symmetric matrix $\mathsf{S}^*(\mathbf{x}, t)$, which is everywhere enhanced over the bare molecular diffusion value ($\mathsf{S}^* - \mathsf{I}$ is positive definite) [2, 5, 12, 18, 21, 26]. Atmosphere-ocean scientists have previously recognized that the effects of small unresolved velocity scales will generally give rise to these features [13, 28, 34]. The effective drift has been variously called a skew flux [14, 28], an eddy-induced transport velocity [13], or a bolus velocity [8]. Our mathematical model provides a context in which these effects can be rigorously established, computed, and analyzed.

In particular, we can assess what type of flow conditions give rise to large or small values of the effective drift and diffusivity. For large values of Pe_ℓ, which is the physically relevant regime, the effective drift and diffusivity will have rather dramatic spatio-temporal variability. We provide a simple illustration for a steady, two-dimensional case in which the mean flow is given by a steady strain:

$$\mathbf{V}(\mathbf{x}, t) = \begin{bmatrix} -\gamma x_1 \\ \gamma x_2 \end{bmatrix} \tag{11}$$

and the velocity fluctuations are modeled as a perturbed cellular flow:

$$\mathbf{v}(\mathbf{y}) \;\; = \;\; \begin{bmatrix} \partial\psi(\mathbf{y})/\partial y_2 \\ -\partial\psi(\mathbf{y})/\partial y_1 \end{bmatrix},$$
$$\psi(\mathbf{y}) \;\; = \;\; \sin 2\pi y_1 \sin 2\pi y_2 + (\cos 2\pi y_1)^2. \tag{12}$$

The streamlines of both velocity field components are plotted in Figure 1. Our model for the fluctuating velocity field is one of the simplest which will yield a nonzero effective drift. We plot in Figure 2 a representation of the symmetric part of the effective diffusion and the effective drift vector field for our example model. The effective diffusivity exhibits a remarkable variability, which can be partially understood from analytical results [15, 22, 12] describing how the symmetric part of the effective diffusivity of a steady periodic flow depends on a constant mean flow. Loosely speaking, when the local Péclet number Pe_ℓ is relatively large, then S^* will be $O(\mathrm{Pe}_\ell{}^2)$ large ("maximally enhanced") when the mean flow is aligned along certain directions with rational slope that resonate with the periodic fluctuations. On the other hand, when the mean flow is aligned with an irrational slope, the effective diffusivity remains bounded in the large Pe_ℓ limit and is said to be "minimally enhanced." These asymptotic theorems produce observable differences at large but finite local Péclet number, as documented in [27]. In our present model where the mean flow varies smoothly in space and $\mathrm{Pe}_\ell = 32$ is only moderately large, we also

observe a significant variability in the magnitude and principal directions of effective diffusivity. From the asymptotic studies [15, 22, 12], these variations can be expected to be more pronounced and exhibit finer scale fluctuations at larger values of Pe_ℓ. The effective drift coefficient $\mathbf{U}^*$, by contrast, can be shown generically to have magnitude $\mathrm{ord}(\mathrm{Pe}_\ell)$, regardless of the mean flow [17, 32]. Nonetheless, we still do observe an interesting geometric structure for $\mathbf{U}^*$ in Figure 2. Somewhat more realistic examples will be studied in [32] to ascertain the extent to which certain observed features are model-specific or generic.
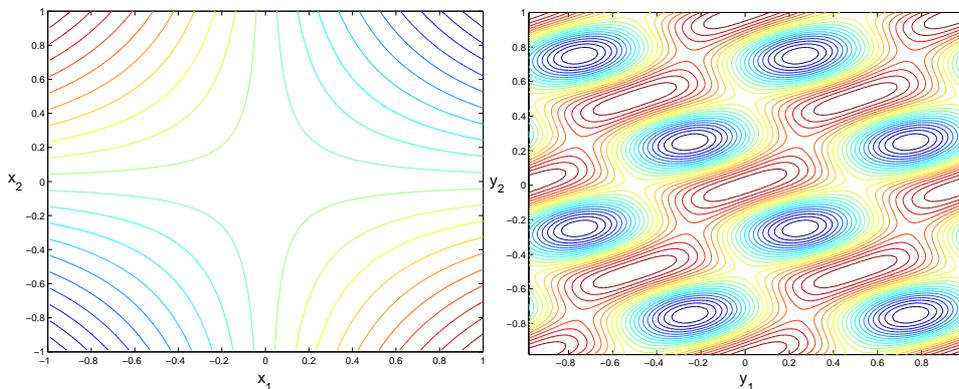

FIGURE 1. Left panel: streamlines of the mean strain flow (11) with $\gamma = 10$. Right panel: streamlines of modified cellular flow representing the fluctuations (12).
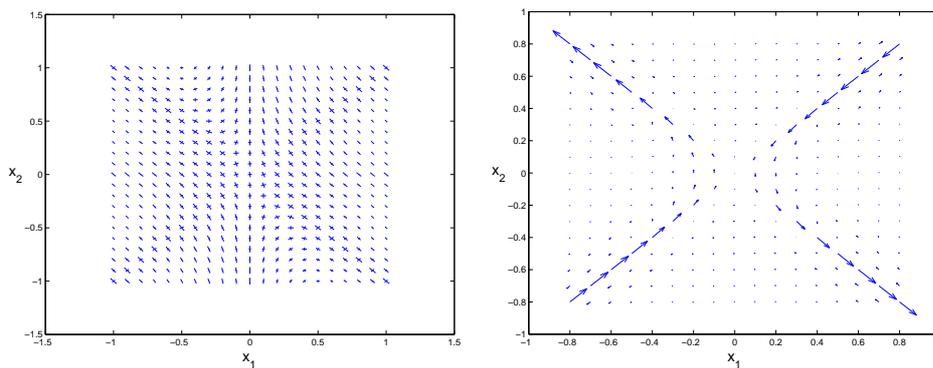

FIGURE 2. Effective transport coefficients in mean strain flow ($\gamma = 10$) with modified cellular periodic fluctuations at $\mathrm{Pe}_\ell = 32$. Left panel: Symmetric part S^* of effective diffusivity. At each grid point, the two line segments denote the directions of the principal axes, and their length is proportional to the log of the corresponding eigenvalue of S^*. Right panel: Drift correction vector $\mathbf{U}^*$.

A final remark concerning the magnitude and importance of the diffusivity enhancement and effective drift is in order. Both corrections are $\operatorname{ord}(\operatorname{Pe}_g{}^{-1})$, which is $\operatorname{ord}(\delta)$ due to the assumption (3). Therefore, the effective dynamics of the passive scalar field on the large scale are dominated by simple mean flow advection, and the small-scale velocity field induces a formally small correction to the large-scale transport. The $\operatorname{ord}(\delta)$ correction, however, represents qualitatively distinct effects which can build significantly over time. Diffusion is naturally a fundamentally different process than laminar advection, and is the only means by which the passive scalar field can be mixed into a larger area. Moreover, the effective diffusion as well as the effective drift can induce transport across the streamlines of the mean flow, and will play particularly important roles in regions where the mean flow is small.

4. **Techniques of derivation.** For the special distinguished limit (3) considered in this paper, the homogenization theorem in Section 3 can be proved using multiple-scale techniques in a fairly standard way [3, 21, 26, 32]. The main novelty is the need to pursue a higher order expansion in order to identify the $\operatorname{ord}(\delta)$ correction, and the requisite introduction of an initial $\operatorname{ord}(\delta)$ transient term [7]. Full details will be provided in [32].

For other choices of distinguished limits, the multiple scale technique becomes either extremely cumbersome (due to a multitude of fractional powers) or breaks down all together due to the generation of fundamentally unsolvable equations [33]. Effective equations can be rigorously and more efficiently derived instead through the more technical theory of two-scale convergence [1, 30] coupled with higher order homogenization. A detailed discussion will be provided in [33]; here we crudely summarize the findings. For mean flows which have amplitude balanced with the fluctuations ($a = 1$), the effective equation takes the advection-diffusion form (5), but the cell problem can degenerate in ways previously studied [3, 6]. For weak mean flows ($a \gg 1$), the mean flow drops out of the cell problem, so the effective diffusivity is simply constant and no effective drift emerges. Finally, for strong mean flows ($a \ll 1$), a more complicated, coupled system of effective equations emerge. The properties of these exotic effective equations will be analyzed in future work.

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