Bayesian Inference for the White Dwarf Initial-Final Mass Relation

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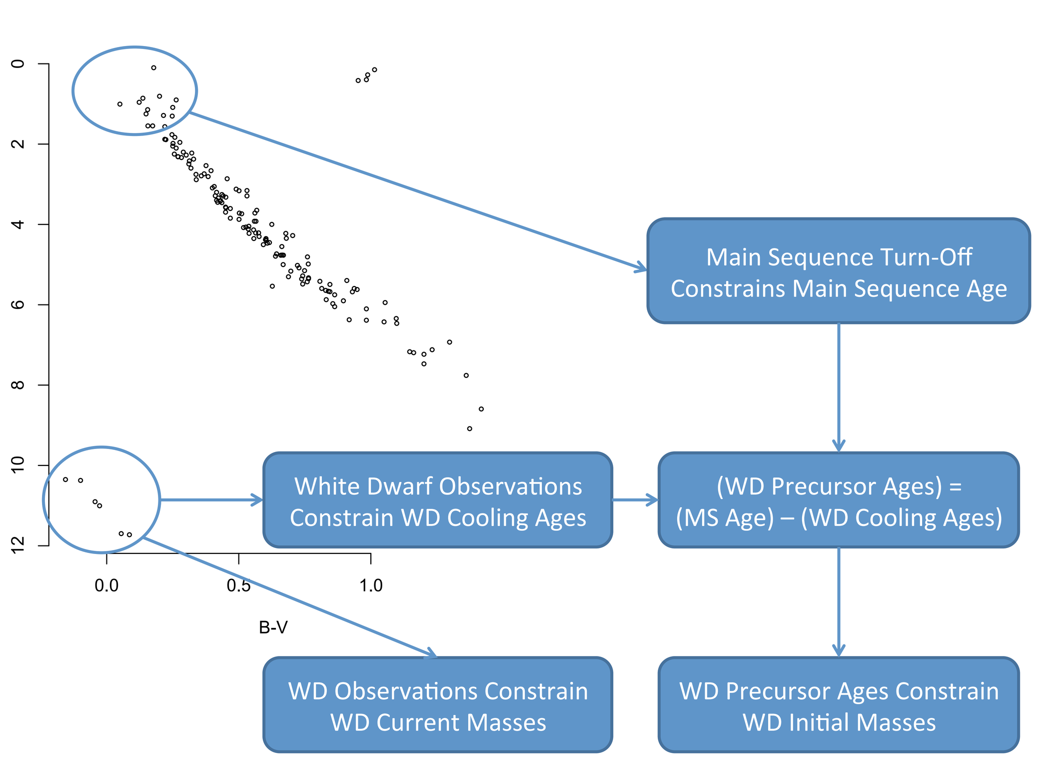
Summary

- Stars lose mass as they age, and understanding mass loss is important for understanding stellar evolution.
- The **initial-final mass relation** (IFMR) is the relationship between a white dwarf's initial mass on the main sequence and its final mass.
- We have developed a new method for fitting the IFMR based on a Bayesian analysis of photometric observations, combining deterministic models of stellar evolution in an internally coherent way. No mass data are used.
- Our method yields precise inferences (with uncertainties) for a parameterized linear IFMR. Our method can also return posterior distributions of white dwarf initial and final masses.

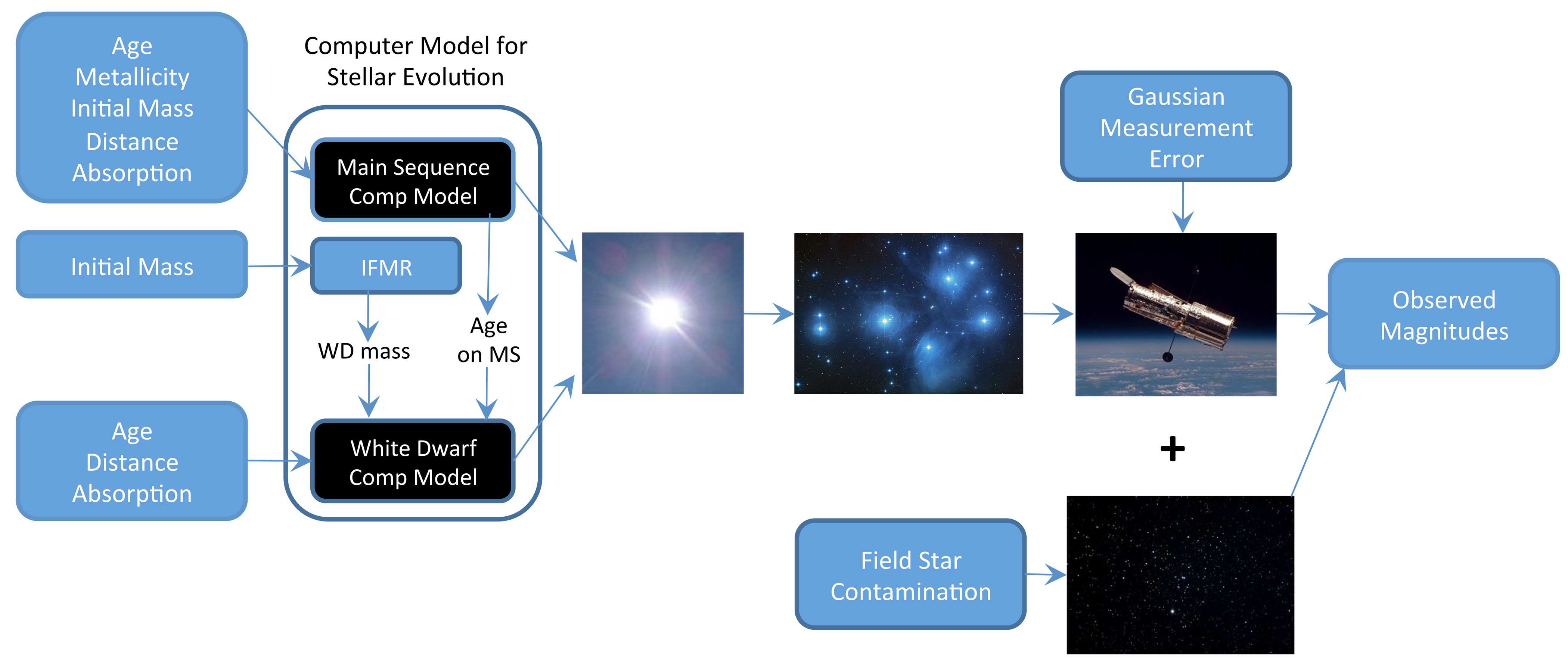
Background: Color-Magnitude Diagrams

- Observe stars' luminosities through different filters
- For the star clusters we study, several parameters are common to all stars:
  - Chemical composition (metallicity)
  - Age
  - Distance
  - Absorption
- Initial masses vary star to star
- **Color-magnitude diagrams** show the temperature (horizontal axis) and brightness (vertical axis) of stars in different evolutionary states
- For single-age clusters, these different evolutionary states are determined by stars' initial masses

Fitting the IFMR



Statistical Model



Cluster Star Likelihood

- Gaussian errors:
$$\boldsymbol{Y}_i \mid M_i, \boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\Sigma}_i \overset{indep}{\sim} N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$$
- $\boldsymbol{Y}_i$ = vector of observations of magnitudes through different filters
- $M_i = (M_{i1}, M_{i2})$ = the primary and secondary mass of star i
- $\boldsymbol{\theta}$ = vector of cluster parameters, including age, metallicity, distance, and absorption
- Observational uncertainties $\boldsymbol{\Sigma}_i$ are assumed known
- Means $\boldsymbol{\mu}_i$ are functions of unknown parameters and depend on deterministic stellar evolution models $\boldsymbol{G}_{\text{ms}}$ and $\boldsymbol{G}_{\text{wd}}$
- If star i is a main sequence star, we model it as a binary system:
$$\mu_{ij} = -2.5 \log_{10}\left(10^{-G_{\text{ms},j}(M_{i1},\boldsymbol{\theta})/2.5} + 10^{-G_{\text{ms},j}(M_{i2},\boldsymbol{\theta})/2.5}\right)$$
  Single star systems will have small M_{i2}, with negligible effect on modeled luminosity
- If star i is a white dwarf, then $\boldsymbol{\mu}_i$ depends on the IFMR:
$$\mu_{ij} = G_{\text{wd},j}(M_i, \boldsymbol{\theta}, f, \boldsymbol{\alpha})$$
- f is the IFMR, which we parameterize as a linear relationship:
$$f(M_i, \boldsymbol{\alpha}) = \alpha_0 + \alpha_1(M_{i1} - M^*)$$
  where M^* is a fixed value for centering the white dwarf initial masses.
- $M_{i2} = 0$ for all white dwarfs (we do not model binary systems involving white dwarfs)

Mixture Model for Field Stars

- Field stars appear in observational field of view, but are not part of the cluster.
- For simplicity, field stars are assumed uniformly distributed in magnitude space.
- Mixture model
$$Z_i \sim \text{Bernoulli}(\pi_i)$$
$$(M_{i1}, R_i) \mid Z_i \sim p(M_{i1}, R_i \mid Z_i)$$
$$\boldsymbol{Y}_i \mid M_{i1}, R_i, Z_i \sim \begin{cases} p_1(\boldsymbol{Y}_i \mid M_{i1}, R_i, \boldsymbol{\theta}, \boldsymbol{\alpha}) & \text{if } Z_i = 1 \\ p_0(\boldsymbol{Y}_i) = \text{constant} & \text{if } Z_i = 0 \end{cases}$$
  - $Z_i = 1$ if star i is a cluster member, $Z_i = 0$ otherwise
  - π_i = prior probability of cluster membership for star i
  - $R_i = M_{i2}/M_{i1}$ = ratio of secondary to primary mass
  - p_1 = cluster star likelihood
  - p_0 = field star likelihood

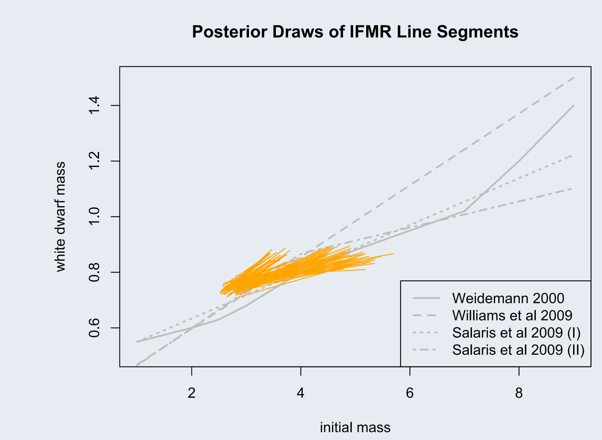
Prior Distributions

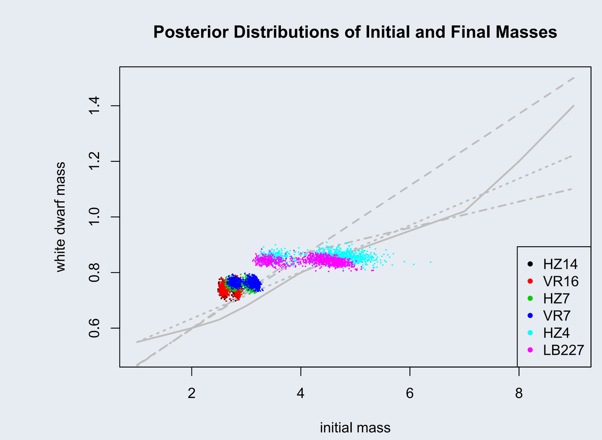
- Primary mass:
$$\log_{10}(M_{i1}) \sim N(-1.02, 0.677^2),\ \ 0.1M_\odot < M_{i1} < 8.0M_\odot$$
  based on Miller and Scalo's initial mass function
- Cluster membership prior probabilities come from external information when available
- Uniform on R_i, $\log_{10}$(age), and $\boldsymbol{\alpha}$, with appropriate boundaries
- Gaussian prior distributions on metallicity and distance
- Truncated Gaussian prior distribution on absorption (absorption is positive)

Model Fitting

- Unknown parameters: $\boldsymbol{M}_1, \boldsymbol{R}, \boldsymbol{Z}, \boldsymbol{\theta}, \boldsymbol{\alpha}$
- MCMC (Metropolis algorithm) on lower-dimensional marginal distribution $p(\boldsymbol{\theta}, \boldsymbol{\alpha} \mid \boldsymbol{Y})$, where it is more reliable
- Numerical integration to marginalize over $(\boldsymbol{M}_1, \boldsymbol{R})$
- Because of conditional independence, $2N$-dimensional integral factors into N 2-dimensional integrals that can be evaluated in parallel within each MCMC iteration

Results: Hyades





- Analyzed Hyades data after adjusting for different distances to individual cluster members.
- Inferences agree with IFMRs from the literature, without using white dwarf mass data.
- Bimodality due to two possible age solutions, at approximately 525 Myr and 665 Myr.

Acknowledgements

We wish to thank Kurtis Williams for helpful discussions. This material is based upon work supported by the National Aeronautics and Space Administration under Grant No. 10-ADAP10-0076 issued through the Astrophysics Data Analysis Program, and by NSF grants DMS 09-07522 and 09-07185.

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