Embedding Computer Models for Stellar Evolution into a Coherent Statistical Analysis

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Outline

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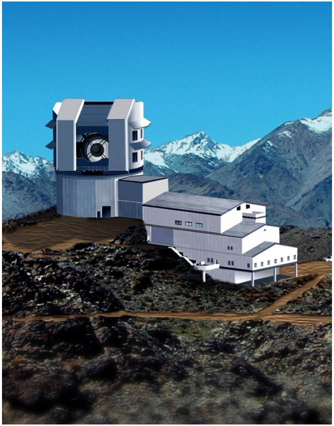
Complex Data and Sophisticated Models



- The volume of astronomical data is growing... astronomically!
- A great leap forward: *Large Synoptic Survey Telescope* (1.28 petabytes per year).
- *Not just massive:* rich & deep.
- LSST: Trigonometric Parallax, Proper Motion, and Photometric data in 5 bands.
- Rich data enables us to fit complex models.

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Stellar Formation

Stars form when the dense parts of a molecular cloud collapse into a ball of plasma.

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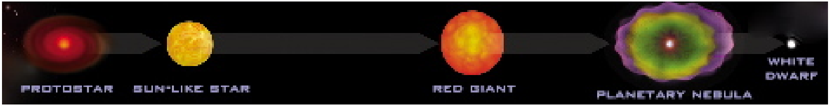
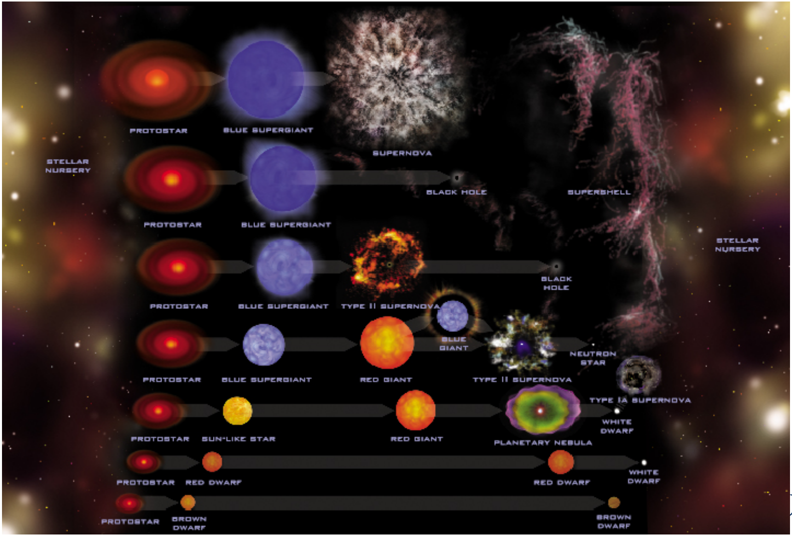
Evolution of a Sun-like Star



- Eventually the core of the *protostar* ignites with the fusion of Hydrogen into Helium.
- This reaction can last for millions or billions of years, depending on the initial stellar mass.
- When the Hydrogen in the core is depleted, the star may fuse Helium into heavier elements
- At the same time the star goes through dramatic physical changes, growing and cooling into a *red giant* star.
- Soon the star undergoes mass loss forming a *planetary nebula*.
- Eventually only the core is left, a *white dwarf star*.

Stellar Evolution Depends on Initial Mass



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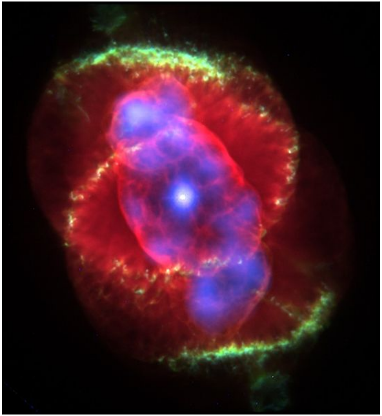
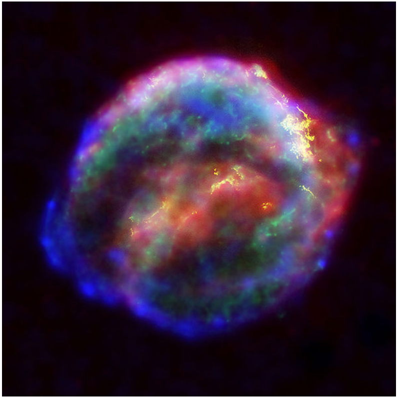
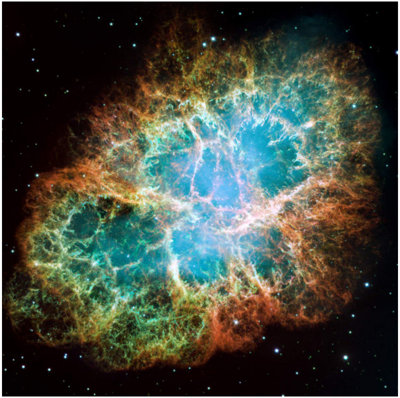
Planetary Nebula



Planetary Nebulae are the illuminated, expanding atmospheres of red giants as they lose the bulk of their mass to become white dwarfs.

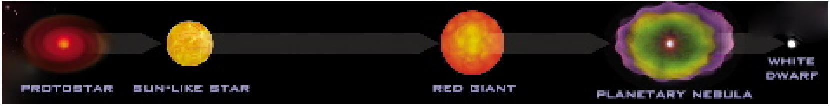
Supernovae





Supernovae are dramatically exploding Giants and result in *neutron stars* or *black holes*.

Stellar Characteristics



Six Unknown Parameters Affect a Star's Appearance as it Ages

1. More *massive* stars are denser, hotter, bluer, and burn their fuel much more quickly.

- Composition also effects the color spectrum
    2. *"Metals"* absorb more blue light.
    3. Excess *Helium* at the core reduces the efficiency of the nuclear reaction.

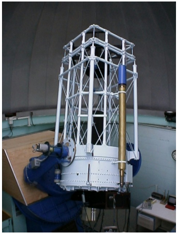
4. The spectrum of the star changes as the star *ages*.
5. Some light from a star is *absorbed* by interstellar material.
6. More *distant* stars are fainter.

Data Collection

Photometric Magnitudes

- To fit the parameters, we study light emitted by each star.
- Using filters, we measure the luminosity of a star's electro-magnetic radiation in several wide wavelength bands.
- An inverted log transform of luminosities gives magnitudes.
- We have 2–3 magnitudes for several hundred stars.

GOAL: Use data to learn about the six stellar parameters.

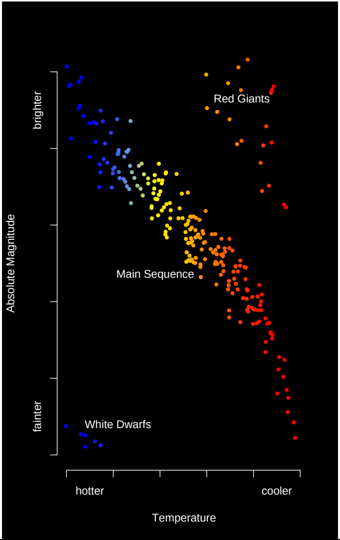


Stellar Clusters



- Stellar Clusters are physical groups of stars formed at the same time out of the same material.
- Cluster stars have the same *metallicity, helium abundance, age, distance,* and *absorption*.
- We call these five common parameters *cluster parameters*.
- Only the stars' *initial masses* vary.
- This significantly simplifies statistical analysis.

Classifying Stars Using HR Diagrams



Hertzsprung-Russell Diagrams

- Plot Temperature vs. *Absolute Magnitude*[a].
- Identifies stars at different stages of their lives.
- Evolution of an HR diagram.
- Must measure Temperature and Absolute Magnitude.

[a] Magnitude at a fixed distance (10 parsec).

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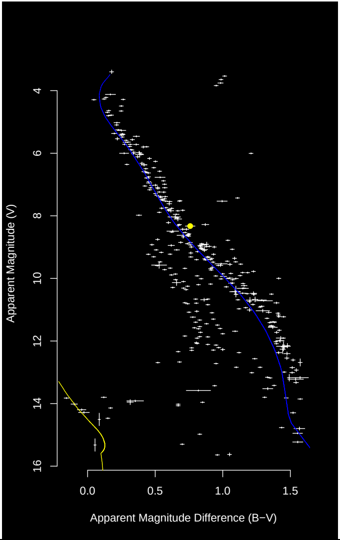
Color Magnitude Diagrams



Color-Magnitude Diagram

- With a star cluster, we can use *Apparent* magnitude.
- Magnitude difference (color) is highly correlated with temp.
- The stars below the main sequence are non-cluster stars in the same field of view, called *field stars*.

Computer-Based Stellar Evolution Models

Computer Models Predict Magnitudes From Stellar Parameters

- Must iteratively solve set of coupled differential equations.
- This creates a static physical model of a star, which is how a star of a particular mass and *radial abundance profile* would appear in terms of its luminosity and color.
- Stars are evolved by updating the mass and abundance profile to account for the newly produced elements.
- Finally interstellar absorption and distance can be used to convert absolute magnitudes into apparent magnitudes.

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Computer-Based Stellar Evolution Models

A Comprehensive Stellar Evolution Model

- There are separate implementations for Main Sequence / Red Giant and White Dwarf evolution.
- There are competing implementations for Main Sequence.
- An empirical model is used to link the models via an initial mass / white dwarf mass relationship.
- Evaluating the full model can take seconds to more than an hour, depending on the evolutionary state of the star.

Embed into a likelihood fit rather than use a "chi-by-eye" fit.

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Basic Likelihood Function

The Stellar Evolution Model as Part of a Complex Likelihood

- The model predicts observed magnitudes as a function of mass, M_i, and cluster parameters, $\mathbf{\Theta}$:

$$\boldsymbol{G}(M_i, \mathbf{\Theta})$$

- We assume independent Gaussian errors with known variances:

$$L_0(\boldsymbol{M}, \mathbf{\Theta} | \boldsymbol{X}) = \prod_{i=1}^{N} \left(\prod_{j=1}^{n} \left[\frac{1}{\sqrt{2\pi\sigma_{ij}^2}} \exp\left(-\frac{(x_{ij} - G_j(M_{i1}, \mathbf{\Theta}))^2}{2\sigma_{ij}^2} \right) \right] \right).$$

- We use the computer model as a component of a principled statistical analysis.

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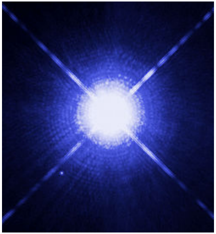
Binary Star Systems

- Between 1/3 and 1/2 of stars are actually binary star systems.
- Most are unresolved.
- The luminosities of the component stars sum.



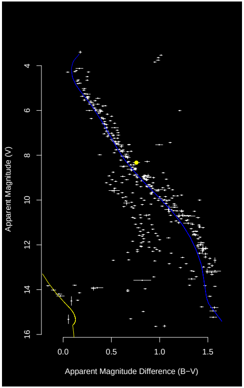
- Resulting offset on the CMD is informative for the masses.
- The expected observed magnitudes for binaries are

$$-2.5 \log_{10} \left[10^{-\boldsymbol{G}(M_{i1},\boldsymbol{\Theta})/2.5} + 10^{-\boldsymbol{G}(M_{i2},\boldsymbol{\Theta})/2.5} \right].$$

- The “secondary masses” of single stars are zero.

Field Stars

Field Stars appear in the field of view but are not part of cluster.



- Their magnitudes do not follow the pattern of the CMD.
- More distant stars are dimmer and below main sequence.
- We use a mixture model.
- Field star magnitudes are assumed uniform over the range of the data.

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Combining Computer Models

Separate Models for Main Sequence & White Dwarf Evolution

- Main Sequence models depend on the initial mass, M.
- White Dwarf models depend on the current white dwarf mass, $f(M)$.
- The predicted model magnitudes are

$$\boldsymbol{G}(M, \boldsymbol{\Theta}, i) = \begin{cases} G^{\mathrm{MS}}(M, \boldsymbol{\Theta}) & \text{if star } i \text{ is from main sequence} \\ G^{\mathrm{WD}}(f(M), \boldsymbol{\Theta}) & \text{if star } i \text{ is a white dwarf} \end{cases}$$

- For a narrow range of M, $f(M)$ is approximately linear.
- Goals: allow uncertainty in f and fit it over wide range of M.

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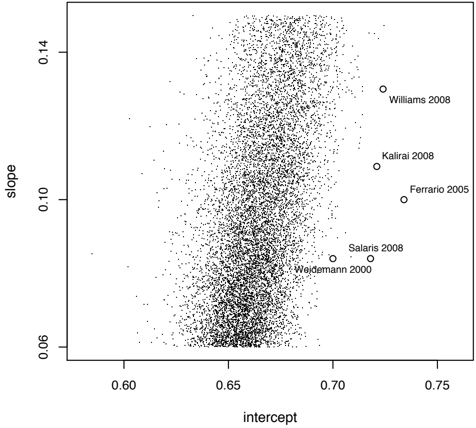
Traditional Method for Fitting IFMR

Initial-Final Mass Relation:

- *Main-sequence age:* Fit the color-magnitude diagram.
- *White Dwarf Mass:* Spectroscopy to determine temperature and surface gravity.
- *White Dwarf Age:* How long white dwarf has been cooling.
- *Progenitor's Life on Main Sequence:* Subtract white dwarf age from main-sequence age
- *Derive white dwarf's progenitor mass*.

We aim to use a coherent statistical model to fit the IFMR.

Comparison with Other Estimates



- Other estimates based on a wider range of M.
- This lead to bias.
- We use informative prior that truncates the slope's posterior.
- Bayesian analysis: coherent accounting of uncertainty.

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Likelihood Function

The resulting Likelihood function is

$$L(\boldsymbol{M}, \Theta, \boldsymbol{Z} | \boldsymbol{X}) =$$

$$\prod_{i=1}^{N} \prod_{j=1}^{n} \Bigg[\frac{Z_i}{\sqrt{2\pi\sigma_{ij}^2}} \exp\left(-\frac{1}{2\sigma_{ij}^2} \left\{ x_{ij} + 2.5 \log_{10} \left[10^{\frac{-G_j(M_{i1}, \boldsymbol{\Theta}, i)}{2.5}} + 10^{\frac{-G_j(M_{i2}, \boldsymbol{\Theta}, i)}{2.5}} \right] \right\}^2 \right)$$

$$+ (1 - Z_i) p_{\text{field}}(\boldsymbol{X}_i) \Bigg],$$

where Z_i is an indicator for cluster membership for star i.

Prior Distributions

We use both informative and non-informative prior distributions:

- An informative truncated Gaussian is used on log mass, representing the population distribution of stellar masses.
- The ratio of the smaller and larger mass is uniform.
- For well studied clusters there are informative star-by-star priors on cluster membership.
- A mildly informative population-based prior is used for age.
- The remaining cluster parameters must be considered on a case-by-case basis.

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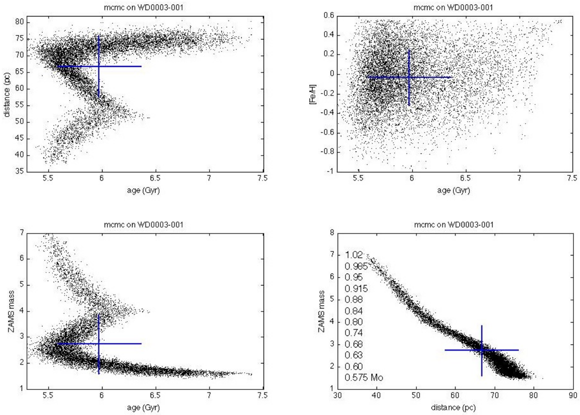
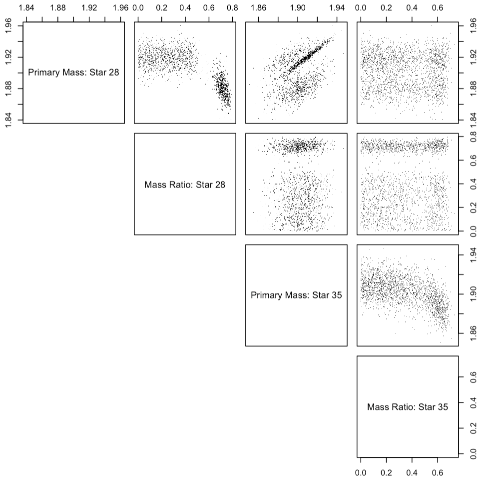
Complex Posterior Distributions



As white dwarf precursor mass varies, values of other parameters change non-linearly to make mass possible.

Complex Posterior Distributions



The classification of certain stars as field or cluster stars can cause multiple modes in the distributions of other parameters.

Basic MCMC Strategy

Metropolis within Gibbs Sampling

- $3N + 5$ parameters, none with closed form update.
- Strong posterior correlations among the parameters.

Evaluation of Computer Stellar Evolution Model is *Very* Costly.

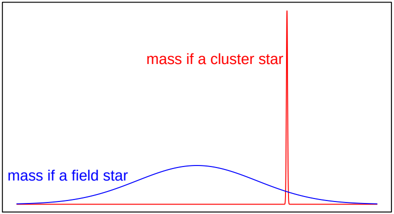
- Instead we use a tabulated form to avoid online evaluation.
- Evaluation points are not evenly spaced, but chosen to capture the complexity of the underlying function.
- Tables provided by developers of computer models.

Correlation Reduction with alternative Prior Dist'n

Field/Cluster Indicator is Highly Correlated with Masses

- Data are uninformative for the masses of field stars.
- Data are highly informative for cluster star masses.
- Cannot easily jump from field to cluster star designation.

Solution: Replace prior for masses given field star membership by approximation of the posterior given cluster star membership.



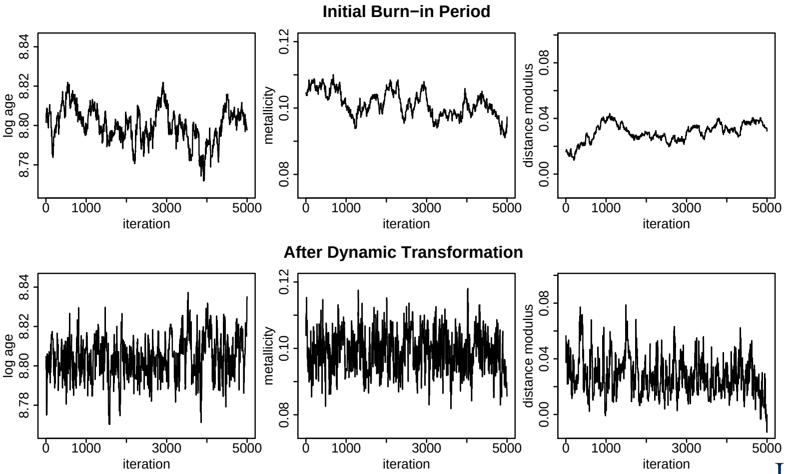
Does not effect statistical inference & enables efficient mixing.

Correlation Reduction via Dynamic Transformations

Strong Linear and Non-Linear Correlations Among Parameters

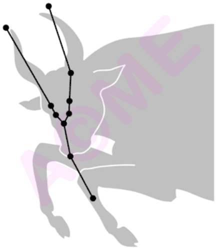
- Static and/or dynamic (power) transformations remove non-linear relationships.
- A series of preliminary runs is used to evaluate and remove linear correlations.
- We tune a linear transformation to the correlations of the posterior distribution on the fly.
- Results in a dramatic improvement in mixing.

Correlation Reduction via Dynamic Transformations



Outline

The Hyades



The Hyades

- Nearest cluster to the solar system.
- We adjust magnitudes for differential distances.
- Visible to the unaided eye as nose of Taurus the Bull.

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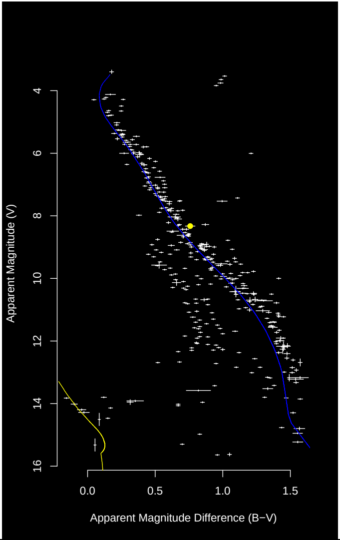
Prior Information for the Hyades



Perryman et al. (1998)

- Distance: 151 light years (stellar parallax)
- Age: 625 ± 50 million years (main sequence turnoff)

Weideman et al, 1992

- Age based on White dwarfs: 300 million years

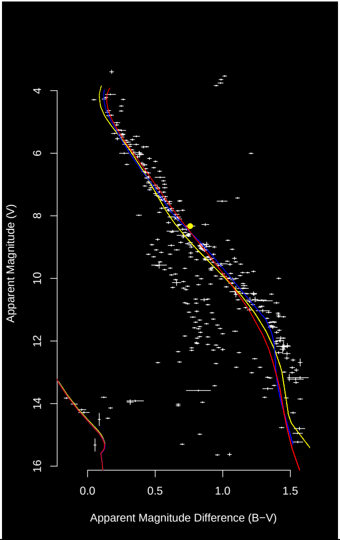
The two ages should agree better.

Scientific Goal

Compare Main Sequence Turn Off Estimate with Estimate Based Primarily on White Dwarf Magnitudes.

- We remove Red Giants and stars near turn off from data.
- Side goal: Evaluate underlying computer/physical models.
- Compare existing external measures with those produced under our fit.
- Most sophisticated empirical check of computer models.

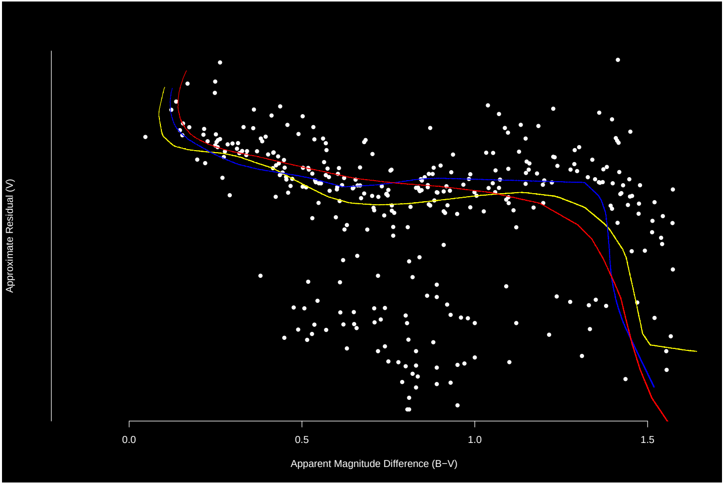
Comparing Three Computer Stellar Evolution Models



3 models with previous best values

- Agree for white dwarfs.
- All three models have trouble with the faintest stars.
- We compare fits based on varying data depths.

Approximate Main Sequence Residuals



(Does not adjust for Binary or Field stars.)

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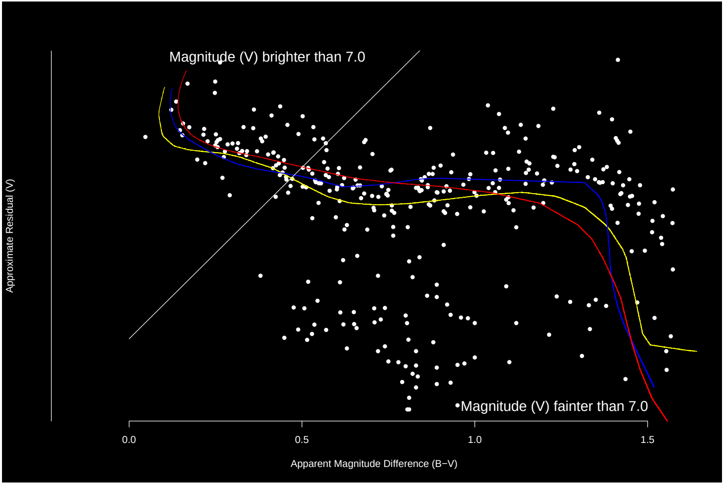

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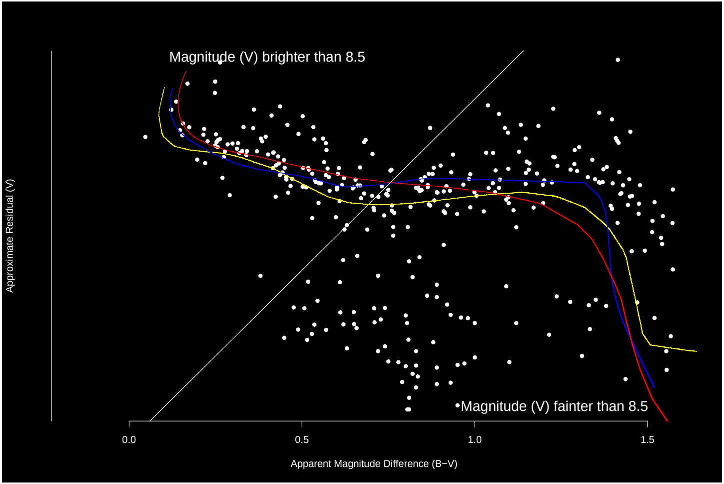

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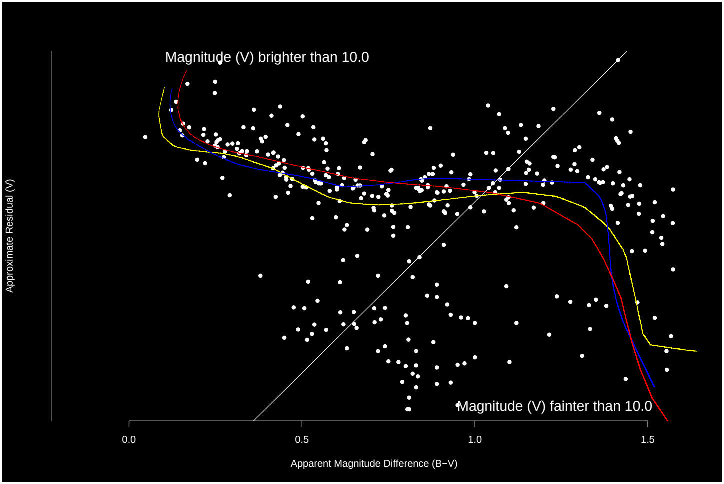
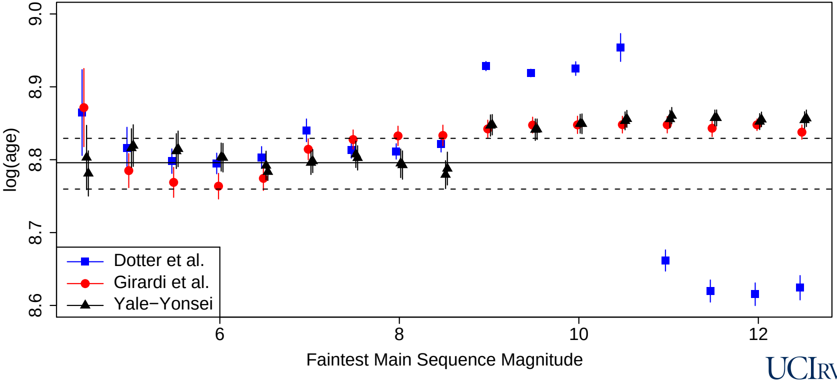


(Does not adjust for Binary or Field stars.)

Fitting the Cluster Age

Comparing posterior intervals of each model with MSTO age.
All fits used a flat prior distribution for age.



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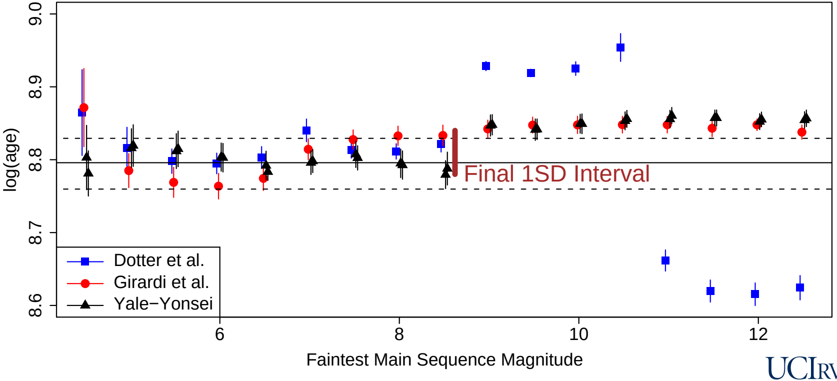
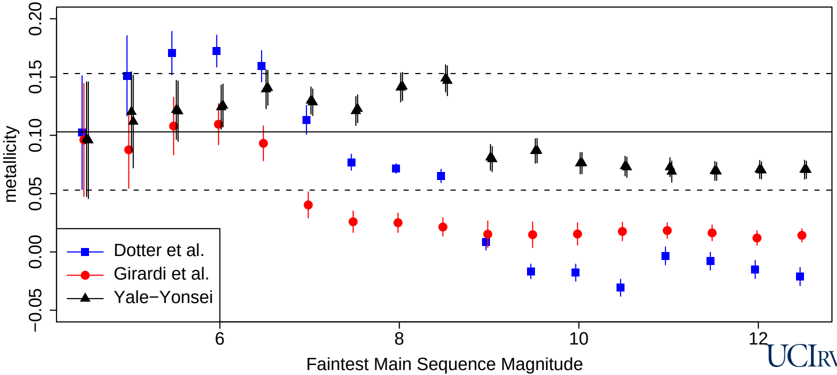
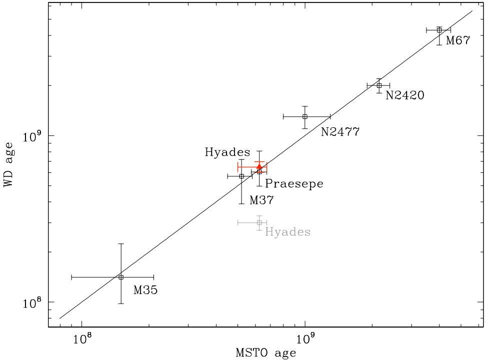


Fitting the Cluster Metallicity

Comparing posterior intervals of each model with prior interval based on best prior information.



Comparing Fitted Age with Best MSTO Age



We demonstrate the ability of the bright white dwarf technique to derive ages (Jeffery et al. 2007) consistent with main sequence turn-off ages.

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Reliable Mass Estimates as Model Diagnostics

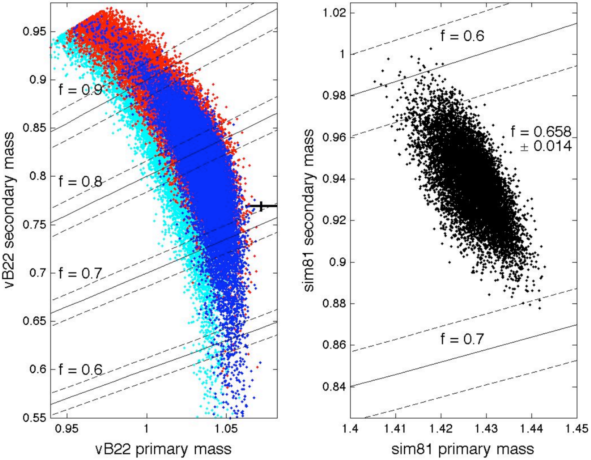
Double-Line Eclipsing Binaries

- Orbital plane is along the line of sight: very rare.
- We can obtain direct accurate measure of the two component masses.

Double-Line Spectroscopic Binaries

- Spectrum exhibits a changing mixture of Dopper-shifted spectral lines as the stars orbit.
- We can obtain direct accurate measure of the **ratio** of the two component masses.

The Binary Diagnostic



- Masses of eclipsing binary in Hyades.
- Simulated ratios with 2% errors.
- Simulated 1Gyr star with BVI photometry.

From Model Checking to Model Improvement

The Power of the Binary Diagnostic

- Direct check of a quantity that resides deep in our statistical model and is highly model dependent.
- Mass is strongly correlated with cluster age—the parameter of most interest.

Goal: A Coherent Analysis

- Use discrepancies to tune computer models[1].
- Include direct binary masses in the likelihood.
- Potential to improve accuracy of other fitted parameters.

[1] I.e., through parameters such as core convective overshoot, convective mixing lenght, and color-T_{eff} relations.

Summary

What we have done:

- First principled statistical fit of stellar evolution model.
- Likelihood-based estimates & errors of cluster parameters.
- Greatly improved agreement of age estimates of Hyades.
- We have tested method on many clusters: NGC 2477, NGC 2360, NGC 2420, NGC 2660, NGC 3960, NGC 188.
- With some constraints, we can use a single white dwarf.

What we still need to do:

- Use available binaries to check models and improve fit.
- Combine data from multiple clusters of different ages to improve estimate of IFMR.
- Evaluate on less studied, more distant, and larger clusters.
- Improve MCMC sampler (multiple modes, parallel temporing, and the variance parameter)

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For Further Reading I

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