The Leverhulme Trust

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Research Fellowship

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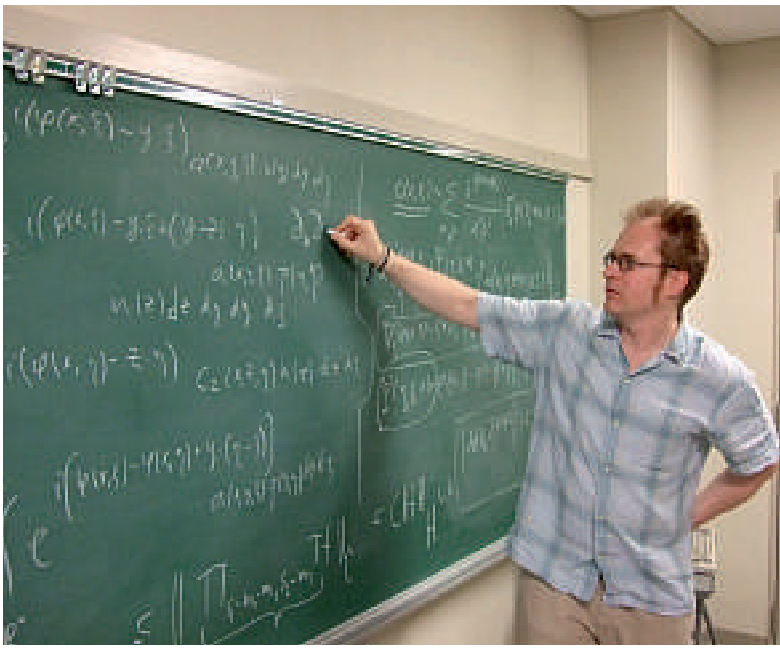
Global smoothing properties of Schrödinger equations

There are many equations of mathematical physics that are of great importance for understanding various physical and chemical processes. Among many, one can mention wave equations describing propagation of waves (like light, sounds, and many other wave-like signals), Schrödinger equations describing different phenomena from quantum physics, fibre optics, internet and television signal transmission, KdV equations for shallow water waves (including tsunamis), the list goes on. Partial differential equations rarely allow one to determine the solutions explicitely, but in reality one does not need to know the solution precisely to be able to say something about its important properties. Consequently, mathematicians try to develop efficient techniques for deducing properties of solutions that are important for particular applications. For example, the so-called smoothing properties exhibited by all these equations are extremely important for telling whether the process will be controllable over a long period of time, or whether it will blow up or collapse quickly. Clearly, this knowlegde is of crucial importance for physical, chemical, and other processes under consideration.

The standard approach to these problems is based on the areas of mathematics called the spectral theory and the harmonic analysis. However, at the moment these methods break down almost completely when considering processes that take place in varying media or under changing conditions, posing a huge problem for applications. The Leverhulme Research Fellowship allowed me to work on a completely new approach to these problems. The new method is based on the application of another area of mathematics, so-called microlocal analysis, and its geometric techniques, to analyse precise properties of solutions to equations of different types, concentrating above all on the extremely important class of Schrödinger equations. The main idea is to reduce many problems to so-called normal forms, which are simpler models of the same equations with solutions exhibiting similar properties. The huge obstacle is that in the theory of nonlinear partial differential equations one needs to carry out these constructions globally in time and space. These global reductions incorporate the important analytic and geometric information about the original equation and have to be thoroughly analysed.

To successfully carry out this program, one needs two main ingredients. First, one needs the precise analysis of the transformation operators. Second, one needs to find model cases of equations with similar properties of solutions. The funding of the Leverhulme Trust has enabled me to visit internationally renowned mathematical centres of excellence in both Italy and Japan where I was able to discuss these ingredients with leading mathematicians working in this area.

The Leverhulme Research Fellowship provided for an opportunity to establish contacts, to carry out scientific discussions and collaboration with these mathematicians. This played a very important role in the major development of the whole area.



Dr Ruzhansky working on Fourier integral operators which is a new approach he has devised to analyse Schrödinger equations.