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Regularity theory of Fourier integral operators with complex phases and singularities of affine fibrations. (English)
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The main subject of the present monograph in the regularity theory of Fourier integral operators with real and complex phases and related questions of the singularity theory of affine fibrations. It is known that the geometry and singularities of the canonical relation of an operator are reflected in the boundedness properties of operators in L^p-spaces. Therefore, a part of the book is devoted to the general singularity theory of affine fibrations. Conditions for the continuity of Fourier integral operators are related to the singularities of affine fibrations in (subsets of) $\mathbb{C}^n$, defined by the kernels of matrix valued functions. Singularities of such fibrations are analyzed in the general case. Fourier integral operators lead to fibrations, given by the kernels of the Hessian of a phase function of the operator.
One of the aspects of the current work is that one deals with operators with complex phase functions. The theory of such operators is well developed but their regularity has not been much studied. In a way, the use of complex phases provides a more natural approach to Fourier integral operators. In complex valued terms the geometric obstructions of the global theory with the real phase can be avoided and it is a remarquable fact that every Fourier integral operator with a real phase can be globally parametrized by a single complex phase. The sharp orders for Fourier integral operators with real phase to be bounded in L^p, are known for operators satisfying the so-called smooth factorization condition. In the monograph this is extended to the complex phases.
Results are applied to derive L^p estimastes for solutions of the Cauchy problem for hyperbolic partial differential operators. The use of the complex phase allows to treat several new examples, such as non-hyperbolic Cauchy problems for pseudo-differential equations and the oblique derivative problem.
The background information on Fourier integral operators with real and complex phases as well as the singularity theory of affine fibrations and relevant methods of complex analytic geometry is provided in the book.
Contents: 1) Fourier integral operators. 2) Affine fibrations. 3) Affine fibrations of gradient type. 4) Further estimates for analytic Fourier integral operators. 5) Applications.

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Keywords : regularity theory of Fourier integral operators with real and complex phases; singularity theory of affine fibrations; L^p estimastes for solutions of the Cauchy problem for hyperbolic partial differential operators; oblique derivative problem
Classification :
  ∗35S30 Fourier integral operators
  35-02 Research monographs (partial differential equations)