Noncommutative Analysis and Partial Differential Equations

11-15 April 2016

Pure Analysis and PDE Group
http://wwwf.imperial.ac.uk/~ruzh/IC-conference-noncommutative.htm
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We kindly acknowledge the support of EPSRC, The Taught Course Centre (TCC) and the Department of Mathematics, Imperial College.
1 Abstracts

Group von Neumann algebras and the $L^p - L^q$ boundedness of pseudo-differential operators

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(Joint work with Michael Ruzhansky) In this talk, we present a sufficient condition for the $L^p - L^q$ boundedness of pseudo-differential operators on locally compact groups. Our approach is based on the operator algebras techniques. The result depends on a version of the Hausdorff-Young-Paley inequality that we establish for this purpose. In particular, we establish the sufficient conditions for the $L^p - L^q$ boundedness of Fourier multipliers on general locally compact groups. The obtained result also implies the corresponding Hörmander's Fourier multiplier theorem on $\mathbb{R}^n$ and the corresponding known results for Fourier multipliers on compact Lie groups. If time permits, a possible extension to quantum groups will be discussed.

References


1 Abstracts

**The Schrödinger equation for the fractional Laplacian on hyperbolic spaces**

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(Work in progress with Yannick Sire (Marseille)). We are studying the linear and the nonlinear Schrödinger equation for the fractional Laplacian on real hyperbolic spaces. In this talk we will discuss mainly dispersive estimates and Strichartz inequalities.

**Generalised Spherical Functions and the Levy-Khintchine Formula on Groups and Symmetric Spaces**

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In 1964 Ramesh Gangolli published a Levy-Khintchine type formula which characterised $K$ bi-invariant infinitely divisible probability measures on a symmetric space $G/K$. His main tool was Harish-Chandra’s spherical functions which he used to construct a generalisation of the Fourier transform of a measure. In this talk I will introduce generalised spherical functions (or Eisenstein integrals), and extensions of these which are constructed using representation theory, to obtain such a characterisation for arbitrary infinitely divisible probability measures on a non-compact symmetric space.  
Based on joint work with Tony Dooley (Bath, Sydney)

**Two classes of Lie groups contractions**

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(Joint work with Rauan Akylzhanov) In this talk we will present a generalization of the Lie group contractions of İnönü and Wigner that contracts a generic Lie group to either a nilpotent Lie group or a semi-direct product of Lie groups. We will then sketch an implementation of these contractions in the unitary dual of a Lie group directly by applying the Borel-Weil theorem to the original group. For this purpose, we assume that it is compact and semi-simple.
References

The Fourier transform on the Heisenberg group: a distribution point of view

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In this talk, we want to construct a theory of Fourier transform which can be extended to the tempered distributions on the Heisenberg group. This implies a precise description of the range of the Schwartz space by the Fourier transform.

Banach space representations of nilpotent Lie groups, smooth vectors and pseudo-differential operators

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We discuss uniformly bounded Banach space representations of nilpotent Lie groups. We then give some results on contragredient representations of Lie groups, with a view toward applications to the abstract characterization of some spaces of pseudo-differential operators. In particular, we give an abstract approach to the description of the norm closure of the pseudo-differential operators of order zero.

Potential Theory results for a class of PDOs admitting a global fundamental solution

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We shall present some potential-theoretic results for a class of hypoelliptic PDOs L admitting a positive and global fundamental solution \( G(x,y) \) on \( N \)-dimensional space: characterizations of \( L \)-harmonic and \( L \)-subharmonic functions are available by means of suitable mean-value operators on the level sets of \( G \); by the latter, one can also prove in a simple way the Strong Maximum Principle for \( L \) and, in some selected cases, Harnack/Liouville
results as well. Due to the relevance of such a global fundamental solution, we present a class of homogeneous (i.e., endowed with dilations) PDOs for which the existence of $G(x,y)$ can be proved via a (global) lifting procedure.

**On the algebraic decay of travelling waves**

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Travelling wave-type solutions of nonlinear evolution equations can be often derived as solutions of certain semilinear elliptic equations of the form $p(D)u = F(u)$, where $p(D)$ is a Fourier multiplier and $F(u)$ is a locally bounded nonlinear term. Under suitable assumptions on the symbol $p(\xi)$ of the linear part, it is possible to derive decay estimates for the solutions. In particular, we are interested to the case when the symbol $p(\xi)$ is a sum of homogeneous functions and at least one of them is only finitely smooth at $\xi = 0$. This case occurs in several physical models in fluid dynamics and plasma physics. We derive sharp algebraic decay estimates for the solutions of the equation and we state a precise relation between the smoothness of $p(\xi)$ at $\xi = 0$ and the decay rate at infinity of the solutions. Similar estimates can be proved for the derivatives of the solution when $F(u)$ is smooth. The content of the talk is based on recent results obtained in collaboration with F. Nicola, T. Gramchev and L. Rodino.

**References**


**Non-Commutative Harmonic Analysis in generalized Clifford Analysis**

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The Dirac operator in standard Clifford Analysis describes Fermions as well as in general SU(2)-symmetries. But there is a high interest in physics in the study of SU(n)-
symmetries. This is nowadays mainly modeled by using supersymmetry. But there is another approach to this problem, based on fractional derivatives and higher order decompositions of the Laplacian (see e.g. Herrmann 2005). In this talk we describe fractional Clifford analysis with respect to ternary Clifford algebras. These algebras allow to construct a Dirac operator whose third power is the Laplacian. We begin with the basic tools of a fractional function theory in higher dimensions established by fractional Weyl relations. This enable us to construct a Fischer decomposition and give a full characterization of the spaces of fractional homogeneous monogenic polynomials w.r.t. the fractional Dirac operator. We end this presentation with a description of the group symmetries.

Eigenfunction expansions of ultradifferentiable functions and ultradistributions

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(Joint work with Michael Ruzhansky) In this talk we give a global characterisation of classes of ultradifferentiable functions and corresponding ultradistributions on a compact manifold X. The characterisation is given in terms of the eigenfunction expansion of an elliptic operator on X. This extends the result for analytic functions on compact manifolds by Seeley in 1969, and the characterisation of Gevrey functions and Gevrey ultradistributions on compact Lie groups and homogeneous spaces by the authors (2014).

Schatten-von Neumann properties on compact manifolds

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In this talk we present some recent results on the study of Schatten-von Neumann properties for operators on compact manifolds. A notion of full matrix-symbol on manifolds has been recently introduced in [6] based on a discrete Fourier analysis developed in [3]. We will compare the point of view of full matrix-symbol with the one of kernel. The special case of compact Lie groups is treated separately as an application of the setting introduced in [7]. We will also discuss the case of operators on \( L^p \) spaces by using the notion of nuclear operator in the sense of Grothendieck and deduce Grothendieck-Lidskii trace formulas in terms of the matrix-symbol ([1], [2], [4], [5]). (Joint work with Michael Ruzhansky.)

References

[1] J. Delgado The trace of nuclear operators on \( L^p(\mu) \) for \( \sigma \)-finite Borel measure on
Abstracts


Analysis of the minimal representation for pseudo-orthogonal groups

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The minimal representation of the pseudo-orthogonal group $O(p,q)$ can be realized on a Hilbert space of homogeneous functions on the isotropic cone. This is the Kobayashi-Ørsted model. It can also be realized on a Hilbert space of holomorphic functions on a complex manifold. This is the Brylinski-Kostant model. We will describe a transformation which maps one model to the other. It can be seen as an analogue of the Segal-Bargmann transform.

Wigner measures and effective mass theorems

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The dynamics of an electron in a crystal in the presence of impurities is described by a wave function that solves a semi-classical Schrödinger equation where the semi-classical parameter is the ratio between the mean spacing of the lattice and the characteristic length scale of variation of the external potential. Effective Mass Theory consists in showing that, under suitable assumptions on the initial data, the wave function can be approximated in the semi-classical limit thanks to a solution of a simpler Schrödinger equation, the effective
mass equation, which is independent of the semi-classical parameter. It is classical to use in this context Floquet-Bloch decomposition which relies on the spectral theory of periodic Schrödinger operators developed by Bloch in the context of solid state physics. Our goal in this talk is to describe how Wigner measure approach, conjugated with Floquet-Bloch decomposition, can be used to derive effective mass equations. We shall mainly consider two different situations depending on the geometric structure of the set of critical points of Bloch bands: when it consists of isolated points or when it is a submanifold of codimension larger than 1. These results are joint work with Victor Chabu and Fabricio Macia.

**Pseudo-differential operators on Lie groups**

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In this talk, I will present some recent developments in the theory of pseudo-differential operators on Lie groups.

**A survey on weakly hyperbolic equations and systems**

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This talk is a survey on some recent work, in collaboration with Michael Ruzhansky (Imperial College London) on weakly hyperbolic equations and systems. The expression weakly refers to the presence of multiple roots/eigenvalues. We will discuss well-posedness for the corresponding Cauchy problem in suitable function spaces and how to deal with low regular coefficients.
1 Abstracts

**Semi-classical analysis of magnetic Schrödinger operators**

**Bernard Helffer**
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After Fournais, Helffer, Kordyukov, Morame, Raymond, Sjöstrand, Vu Ngoc ...

Our main object of interest is the Laplacian with magnetic field on a riemannian manifold, but in this talk we will mainly consider, except for specific toy models, a magnetic field

\[ \beta = \text{curl} A \]

on a regular domain \( \Omega \subset \mathbb{R}^d \) \((d = 2 \text{ or } d = 3) \) associated with a magnetic potential \( A \) (vector field on \( \Omega \)). Starting from the closed quadratic form \( Q_h \)

\[ W^{1,2}_0(\Omega) \ni u \mapsto Q_h(u) := \int_{\Omega} |(-ih\nabla + A)u(x)|^2 \, dx, \tag{1.1} \]

we consider the magnetic Laplacian \( \mathcal{H}^D(A, h, \Omega) \) to be the self-adjoint operator associated to \( Q_h \).

Motivated by various questions in geometry and mathematical physics we consider the following connected problems in the asymptotic \( h \to +0 \).

**Pb 1** Determine the structure of the bottom of the spectrum of the magnetic Laplacian: gaps, typically between the first and second eigenvalue.

**Pb 2** Find an effective Hamiltonian which through standard semi-classical analysis can explain the complete spectral picture including tunneling.

**Wavelet and Gabor frames on the three-sphere**

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There is a wide range of applications for function systems on the three sphere, among them X-Ray diffraction tomography. In this application one needs to reconstruct a function on the three-sphere which is well-localized. To approximate such a function we discuss the construction of Wavelet and Gabor frames on the three-sphere. In both cases we use the representation of the corresponding group (Lorentz group in the case of the wavelet transform and Euclidean group in the case of the Gabor transform). While this provides us with the continuous transforms in reality we need a discrete version. To this end we will use coorbit space theory to construct wavelet and Gabor frames. Here we will show the difference in the construction of both cases. To illustrate the applicability of our frames we present an algorithm for the inversion of the spherical X-Ray transform.
Covariant and contravariant calculus of operators on homogeneous spaces

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It is common to study operators by means of certain symbolic calculus, that is, expressing properties of operators in terms of certain functions called their symbols. Symbols may be scalar- or operator-valued functions. The former are usually easier to study, but even the later may provide definite advantages: they resolve a single operator into a bundle of simpler ones.

There are various techniques to obtain symbols out of operators and correspondingly decode properties of operators out of their symbols: Simonenko’s localisation, Berezin’s calculus, Toeplitz operators, PDO calculus, etc. Furthermore, the bilateral connections between functions and operators are usually associated to relations between classical and quantum observables. Thus, there is a physical remake for every mathematical theory in terms of certain (de-)quantisation.

In many (if not all) cases we may find a group, which (at least locally) acts by a non-trivial representation. For example, for operators on a manifold we may bring the action of the Heisenberg group by spatial and phase shifts to every local chart. Thus PDOs, generated according to Roger Howe by the Heisenberg group, become a universal tool for symbolic calculus.

Yet, the universal tool may not be the best adopted to a particular setup. If our operators are related to some other group of symmetries we may prefer to use it instead of the Heisenberg group. Many symbolic methods mentioned above are generated by the affine group, SL(2,R) or certain semi-direct product just in the same way as PDO are generated by the Heisenberg group. We discuss some fundamental results and open problems in this approach.

Onset of instability for a class of non-linear PDE systems

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We consider some quasi-linear systems of PDE and we are looking for ways of detecting some Hadamard instability. The first example that comes up is the elliptic case where the matrix defining the system has a non-real eigenvalue. We explore here the case where the characteristic roots at initial time are real-valued, but leave the real axis for positive time.

We prove several instability results and provide examples such as Burgers systems, Van der Waals gas dynamics, and Klein-Gordon-Zakharov systems.
1 Abstracts

A global quantization for type I locally compact groups.

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(Joint work with M. Ruzhansky) Let $G$ be a unimodular type I second countable locally compact group and $G'$ its unitary dual. We introduce and study a global pseudo-differential calculus for operator-valued symbols defined on $G \times G'$, and its relations to suitably defined Wigner transforms and Weyl systems. We also unveil its connections with crossed products $C^*$-algebras associated to certain C-dynamical systems, and apply it to the spectral analysis of covariant families of operators. Applications are given to nilpotent Lie groups, in which case we relate quantizations with operator-valued and scalar-valued symbols.

$L^p$ functional calculus for the Kohn Laplacian on complex spheres

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The Kohn Laplacian $\Box_b$ associated to the tangential Cauchy–Riemann complex on a strictly pseudoconvex CR manifold $M$ is a classic example of “non-elliptic Laplacian”. In the case $M$ is the unit sphere in $\mathbb{C}^n$, a great amount of information on the spectral theory of $\Box_b$ can be obtained via representation theory. Following this approach we prove a sharp $L^p$ multiplier theorem of Mihlin–Hörmander type for $\Box_b$. In particular, we require on the multiplier a smoothness condition of order $s > (2n - 1)/2$, i.e., half the topological dimension of $M$. It is still an open question whether the same result holds for an arbitrary compact strictly pseudoconvex CR manifold $M$.

This is partly joint work with V. Casarino (Padova), M. Cowling (Sydney) and A. Sikora (Sydney).
Necessary and sufficient conditions for $L^p$ spectral multipliers on 2-step groups

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Let $G$ be a 2-step stratified group of topological dimension $d$ and homogeneous dimension $Q$. Let $L$ be a homogeneous sub-Laplacian on $G$. By a theorem due to Christ and to Mauceri and Meda, an operator of the form $F(L)$ is of weak type $(1,1)$ and bounded on $L^p(G)$ for all $p \in (1, \infty)$ whenever the multiplier $F$ satisfies a scale-invariant smoothness condition of order $s > Q/2$. It is known that, for several 2-step groups and sublaplacians, the threshold $Q/2$ in the smoothness condition is not sharp and in many cases it is possible to push it down to $d/2$. Here we show that, for all 2-step groups and sublaplacians, the sharp threshold is strictly less than $Q/2$, but not less than $d/2$.

This is joint work with Alessio Martini.

Pseudo-differential calculus on Motion Groups

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We describe a pseudo-differential calculus on the semi-direct product between $V$ and $K$, where $V$ is a finite-dimensional vector space and $K$ is any compact connected Lie group acting linearly on $V$.

On certain a priori estimates for systems

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In this talk I will be concerned in the main (known) positivity estimates for systems: the Sharp-Garding inequality and the Fefferman-Phong inequality. After recalling them, I will discuss their validity in the case of systems and speculate about their nature and generalizations. I will finally discuss the existence of the positive and negative part of certain symmetric 2x2 systems.
1 Abstracts

Parseval localized frames for subelliptic function spaces on compact manifolds

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We consider a compact homogeneous manifold \( M = G/H \), where \( G \) is a compact Lie group and \( H \) is a closed subgroup. Let \( Y_1, ..., Y_m, m \leq n = \dim M \), be elements of the Lie algebra \( g \) of \( G \) that algebraically generate entire \( g \). Let \( X_1, ..., X_m \) be the natural images of the vector fields \( Y_1, ..., Y_m \) on the manifold \( M \). We consider the so-called sub-elliptic Laplace operator \( \mathcal{L} = X_1^2 + ... + X_m^2 \) in the natural space \( L^2(M) \) and introduce sub-elliptic Sobolev and Besov spaces associated with \( \mathcal{L} \).

The main objective is to construct a frame \( \{ \phi_\nu \}_{\nu \in J} \) in \( L^2(M) \) which has the following properties

1. it is Parseval, i.e.
   \[
   \| f \|^2 = \sum_\nu |\langle f, \phi_\nu \rangle|^2, \quad f \in L^2(M);
   \]

2. each \( \phi_\nu \) has strong localization on \( M \);

3. each \( \phi_\nu \) is bandlimited in the sense of the spectral resolution of \( \mathcal{L} \).

Such frame is used to describe relevant Sobolev and Besov sub-elliptic spaces in terms of coefficients \( \{ \langle f, \phi_\nu \rangle \}_{\nu \in J} \).

Uncertainty inequalities on Lie groups

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In this talk I present several recent developments concerning the Heisenberg-Pauli-Weyl inequality and related inequalities involving sublaplacians.
Decay and regularity of global solutions of elliptic partial differential equations

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We first survey some results obtained by Gramchev with Cappiello, Rodino, Nicola, Pilipovic and others, concerning decay and regularity of omoclinic solutions of partial differential equations, i.e. global solutions in the Euclidean spaces, tending to zero at infinity. Basic examples are given by the harmonic oscillator of Quantum Mechanics and, in the non-linear setting, by the profile equation for the KdV waves. We then fix attention on the explicit expression of the Weyl symbol of the inverse of the harmonic oscillator (results of Cappiello, Rodino, Toft and Melin) and present some open problems concerning the analytic version of the Shubin operators in Euclidean spaces.

Orthonormal Bases for Square-Integrable Representations of Nilpotent Lie Groups

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Let $G$ be a connected, simply connected nilpotent group and $\pi$ be a square-integrable unitary irreducible representation modulo its center on $L^2(\mathbb{R}^d)$. We prove that under reasonably weak conditions on $G$ and $\pi$ there exist a uniform subgroup $\Gamma$ of $G$ and some compact set $F \subseteq \mathbb{R}^d$ such that

$$\{ \mu(F)^{-1/2} \pi(\gamma)1_F \mid \gamma \in \Gamma/Z(G) \}$$

forms an orthonormal basis of $L^2(\mathbb{R}^d)$. This construction generalizes the well-known example of Gabor orthonormal bases in time-frequency analysis.
1 Abstracts

Index Theory for Hyperbolic Operators

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I will discuss the Atiyah-Singer index theorem, give some example and then explain the
Atiyah-Patodi-Singer index theorem for Dirac operators on manifolds with boundary. I will
then show that choosing appropriate function spaces and boundary conditions hyperbolic
operators such as the Lorentzian Dirac operator on a globally hyperbolic space-time may
also be Fredholm. In this case an appropriate analog of the APS index formula holds can
be shown to hold. (Joint work with C. Baer).

Hardy and Rellich type inequalities, identities, and sharp
remainders on homogeneous groups

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In this talk we give sharp remainder terms of $L^p$ and weighted Hardy and Rellich in-
equalities on one of most general subclasses of nilpotent Lie groups, namely the class of
homogeneous groups. As consequences, we obtain analogues of the generalised classical
Hardy and Rellich inequalities and the uncertainty principle on homogeneous groups. We
also present higher order inequalities of Hardy-Rellich type, all with sharp constants. A
number of identities are derived including weighted and higher order types. This talk is
based on joint works with Michael Ruzhansky.
Sub-Laplacian Gevrey Spaces on the Heisenberg Group and Applications

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In our work in progress, we define Gevrey Spaces with respect to the Sub-Laplacian on manifolds. In this talk, we focus on the case of the Heisenberg Group. Keeping in mind the Euclidean case, we show a global characterisation of these spaces on the Fourier transform side. This turns to be fundamental in applications. Indeed we prove the Gevrey well-posedness of the Cauchy problem for the time-dependent wave equation for the sub-laplacian on the Heisenberg group.

References

Hardy’s inequality for fractional powers of the sublaplacian on the Heisenberg group

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In this talk we discuss Hardy inequalities for the conformally invariant fractional powers of the sublaplacian on the Heisenberg group $H^n$. We have two versions of such inequalities depending on whether the weights involved are non-homogeneous or homogeneous. In the first case, the constant arising in the Hardy inequality turns out to be optimal. In order to get our results, we use ground state representations. The key ingredients to obtain the latter are some explicit integral representations for the fractional powers of the sublaplacian and a generalized result by M. Cowling and U. Haagerup. The approach to prove the integral representations is via the language of semigroups. As a consequence of the Hardy inequalities we also obtain versions of Heisenberg uncertainty inequality for the fractional sublaplacian. The talk is based on a joint work with Luz Roncal.
Abstracts

Schatten-von Neumann properties of Weyl operators of Hörmander type

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Let $t \in \mathbb{R}$ be fixed and consider the pseudo-differential operators $\text{Op}_t(a)$ with symbol $a$ which is defined by the formula:

$$\text{Op}_t(a)f(x) \equiv (2\pi)^{-n} \int_{\mathbb{R}^n \times \mathbb{R}^n} a((1-t)x+ty,\xi)f(y)e^{i\langle x-y,\xi \rangle} \, dyd\xi$$

A fundamental result for such operators reads: Assume that $0 \leq \delta < \rho \leq 1$ and $r \in \mathbb{R}$. Then each $\text{Op}_t(a)$ with $a \in S_{\rho,\delta}^r(\mathbb{R}^{2n})$ is $L^2$-continuous, if and only if $S_{\rho,\delta}^r \subseteq L^\infty$ (i.e. $r \leq 0$). Here recall that $S_{\rho,\delta}^r(\mathbb{R}^{2n})$ consists of all $a \in C^\infty(\mathbb{R}^{2n})$ such that

$$|\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| \leq C_{\alpha,\beta}(1+|\xi|)^{-\rho|\beta|+\delta|\alpha|}.$$

A somewhat weak property here is that no conclusion concerning $L^2$-continuity can be done for a particular operator $\text{Op}_t(a)$, when $a \in S_{\rho,\delta}^r$ and $r > 0$.

In [1] the theory at this point was performed more complete. For example, if $a \in S_{\rho,\delta}^r$, then it follows from [1] that $\text{Op}_t(a)$ is $L^2$-continuous, if and only if $a \in L^\infty$.

The theory, which contains the latter result as a special case, is formulated by means of Hörmander-Weyl calculus, where the symbol classes $S(m,g)$ are parameterized with appropriate weight functions $m$ and Riemannian metrics $g$. The continuity investigations are also performed in a broader context, which involve Schatten-von Neumann properties for such operators. Then it is proved in [1] that the following general result holds true: Assume that $p \in [1,\infty]$, and that the $g$-Planck’s constant $h_g$ satisfies $h_g^N m \in L^p$, for some $N \geq 0$. Then $\text{Op}_t(a)$ is a Schatten-von Neumann operator of order $p$, if and only if $a \in L^p$.

The previous result was partially recently extended in such way that the Schatten parameter $p$ was allowed to be smaller than 1. More precisely, in [2] it is proved that if $p \in (0,\infty]$, $h_g^N m \in L^p$, for some $N \geq 0$ and $a \in L^p$, then $\text{Op}_t(a)$ is a Schatten-von Neumann operator of order $p$.

An important example concerns globally defined pseudo-differential operators with symbols in the SG class $\text{SG}_{\rho,\delta}^{(\omega)}(\mathbb{R}^{2n})$, which consists of all $a \in C^\infty(\mathbb{R}^{2n})$ such that

$$|\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| \leq C_{\alpha,\beta}\omega(x,\xi)(1+|x|)^{-\delta|\alpha|}(1+|\xi|)^{-\rho|\beta|},$$

where $\omega$ is bounded by a polynomial and $\rho, \delta > 0$. In this case we have that $\text{Op}_t(a)$ is Schatten-$p$ operator, if and only if $a \in L^p$, when $p \geq 1$.

In the talk we explain these results and present some ideas of their proofs.
References


Pseudo-differential operators generated by boundary value problems

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(Joint work with Michael Ruzhansky) We consider the development of pseudo–differential operators generated by boundary value problems. In particular, we derive an explicit formula for the quantization of pseudo–differential operators induced by the derivative operator on a segment. Starts an interesting direction of discrete analysis based on elliptic boundary value problems, continuing, in a sense, the analysis on the torus started by M. Ruzhansky and V. Turunen [1], [2], in which case one may think of a problem having periodic boundary conditions. Some researches on the development of the Fourier analysis based on a non self–adjoint boundary value problem are given in [3]–[5].

References


1 Abstracts

**Theory and applications of time-frequency analysis**

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When and how often something happens in a signal? By properly quantizing these questions, we obtain the Born–Jordan time-frequency transform, defining a sharp phase-space energy density. We study properties of different time-frequency transforms, and also present computed examples from acoustic signal processing, quantum mechanics and medical sciences.

**Operators on compact groups**

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In this talk we will present some recent results on the global symbolic calculus of operators on compact Lie groups. Operators on compact Lie groups are described in terms of global symbols, which are matrix-valued functions on the non-commutative phase space \( G \times \hat{G} \), where \( G \) denotes the compact Lie group under consideration and \( \hat{G} \) the set of equivalence classes of irreducible unitary representations of the group \( G \). Symbol classes for operators are characterised in terms of differential-difference conditions.

We will focus on the particular example of the 3-sphere \( S^3 \), understood as set of unit quaternions, and show how the differential-difference calculus arises in a natural way when considering spaces of homogeneous polynomial as the representation spaces. We will give explicit formulas and recurrence relations and explain central parts of the associated calculus. Applications involve differential operators, the Szegő projector and projections associated to the 2-sphere as homogeneous space of \( S^3 \).
2 Posters

On a nonlocal initial boundary value problem for the time-fractional diffusion equation

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In this work we discuss methods for constructing trace formulae for the heat-volume potentials of the time-fractional diffusion equation to piecewise smooth lateral surfaces of cylindrical domains and use these conditions to construct as well as to study a nonlocal initial boundary value problem for the time-fractional diffusion equations. We also obtain similar results for higher powers of the time-fractional diffusion equations.
3 Participants

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Hajer Bahouri (Université Paris-Est-Créteil, France)
Ingrid Beltiţă (Institute of Mathematics of the Romanian Academy, Romania)
Andrea Bonfiglioli (University of Bologna, Italy)
Marco Cappiello (University of Torino, Italy)
Paula Cerejeiras (University of Aveiro, Portugal)
Aparajita Dasgupta (École Polytechnique Fédérale de Lausanne, Switzerland)
Julio Delgado (Imperial College London, UK)
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Nicola Garofalo (University of Padova, Italy)
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Remi Lodh (Springer UK)
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Luigi Rodino (University of Torino, Italy)
David Rottensteiner (University of Vienna, Austria)
Alexander Strohmaier (University of Loughborough, UK)
Durvudkhan Suragan (Imperial College London, UK)
Chiara Taranto (Imperial College London, UK)
Joachim Toft (Linnaeus University, Sweden)
3 Participants

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Niyaz Tokmagambetov (Kazakh National University, Kazakhstan)
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Jens Wirth (Stuttgart University, Germany)