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Statistical Techniques and Qualitative Adjustments for LDA

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Summary

The main focus in this presentation is a commentary on the current and emerging statistical procedures for analysing operational risk losses as part of the Loss Distribution Approach to advanced measurement. The Basel Capital Accord, Basel II, proposes this as a requirement for regulatory capital calculations for operational risk. In particular the limitations of the ability of existing statistical methodology to do this are highlighted.

Key words Operational Risk, Statistical analysis of loss data, Basel Capital Accord, Basel II, Loss Distribution Approach (LDA), Advanced Measurement Approach (AMA), Qualitative Adjustment, Operational Risk Research Forum.

Contents

1. Where subjective intervention might arise in LDA

My interpretation of "qualitative adjustment" is the subjective intervention in the data analysis process. The following table lists opportunities for this.

- Loss data capture
  - Categorising, dating and valuing the loss
    - and the potential loss (or near miss), the excess over realised loss
  - Setting minimum thresholds for loss data capture
- Loss data modelling
  - Frequency and severity
- Model fitting
  - Model expected losses, unexpected losses and stress losses
  - Assign boundaries between these
- Data augmentation
  - External data
  - Generating simulated data from fitted models

We note that this list does not include subjective methodologies collectively described as scenario analysis.

2. Statistical features in emerging best practice

A conference on "*Leading Edge Issues in OR Measurement*" organised by the Basel Committee's Risk Management Group took place in May this year at the New York Federal Reserve Bank (RMG, 2003). Among the contributors was the Industry Technical Working Group (ITWG) of major international banks. The ITWG presented a draft discussion paper on LDA subtitled "*Ideas, issues and emerging practices*" (ITWG, 2003a). This paper is essential reading for anyone seeking guidance on LDA. Below, I have highlighted passages relevant to the statistical methodology, and other passages just for comment. Some remarks are meant to be provocative, and in this post-presentation revision, I have included some responses to it made in the discussion at ORRF.

(a) The [Basel] Accord permits an unprecedented amount of flexibility in the approach used to assess capital requirements.

My comment: This could lead to problems of supervision and audit (see §4.3 below).

(b) There is a huge interest in developing a risk sensitive and transparent quantitative approach.

(c) The ITWG shares a common view that loss data should really be the *foundation* of an LDA-based AMA.

Note: Regarding this statement of the obvious, it was pointed out at the Forum that the phrase "LDA-based" was introduced late in the drafting.

(d) The fundamental premise underlying LDA is that operational losses are a reflection of underlying operational risk exposure.

(e) LDA uses standard actuarial techniques to model operational losses through frequency and severity estimation to produce an objective estimate of both expected and unexpected losses.

My comment: It is not clear how "objective" should be interpreted when the subjectivity of data capture, and of loss distribution modelling under true model uncertainty, are involved.

(f) The ITWG define an operational loss to be the amount charged to the P & L statement net of recoveries in accordance with GAAP, in the resolution of the OR event. Near misses, opportunity costs and contingent liabilities are excluded. Similarly, time spent by staff rectifying a problem.

My comment: In (j) below, we read of robust estimation of capital to protect against one in a thousand year events. Near misses involving potentially catastrophic losses are liable to occur rather more frequently than this, so to reject them in the name of objectivity seems perverse.

(g) Once loss data have been collected, sorted and scrubbed, LDA involves modelling the loss severity and loss frequency separately, and then combining these to form a total loss distribution.

My comment: My response is to ask why we should be modelling these separately when all the information is embodied in the severity distribution regularly updated within a rolling time period (Cruz *et al.* , 1998). It was pointed out at the Forum that changes in the bank's structure,

perhaps due to acquisition, would require fresh starts in data collection leading to the unavailability of loss data model results during the first several years after every such change.

(h) To ensure that the selected distribution is a reasonable fit, standard statistical techniques are used.

My comment: If we are interested mainly in the tail, standard techniques still need to be developed and validated. (See §4.2 below.)

(i) The biggest challenge is selecting the distribution that best fits the *tail* of the observed data, due to the inherent scarcity of low frequency, high impact operational loss events. One of the key objectives is to ensure that the tail estimate is *relatively* robust and stable, because this is where the estimate for required capital is derived.

(j) If the distribution reflects a time horizon of one year, and the 99.9 percentile, a firm will be able to determine the amount of capital required to protect itself from a total annual loss so high that it would only be expected to occur once in a 1000 years.

My comment: Regardless of the problems firms will have in making this determination of capital, how will carrying out the calculation help risk management protect the firm from such rare events?

(k) Regulatory capital will be determined using the above methodology. The Basel CP3 document of April 2003 (Basel Committee, 2003) suggests a one year time horizon and a percentile equivalent to that used for credit and market risk assessment. Capital is required for "unexpected" losses only. A firm would simply read the total loss amount at the appropriate percentile, and then subtract the mean or expected losses to determine the capital required.

My comment: This seems to imply a move away from a 99.9 percentile to a 95 percentile. We might well hope to develop statistical methodologies based on VaR calculations giving robust and stable tail estimates at this level.

My comment: "The mean or expected losses" is rather too loosely phrased. At the Forum it was pointed out that this was using "mean" and "expected" in their statistical sense. This measure is thus the difference between a percentile and an average, which could cause problems relating to its statistical properties. I would recommend using the median in place of the mean giving a direct difference of percentiles.

(l) Under the threshold losses: Should these be collected?

My comment: We do not have in place standard statistical methodology for modelling and testing when data are censored below by a threshold beyond fitting the generalised Pareto distribution, which incorporates a threshold.

(m) In order to test the impact of the threshold on capital estimation, the bank should ensure that the fitted distribution passes a goodness of fit test.

My comment: Goodness of fit tests are inherently conservative, reluctant to reject. In practice there may be no goodness of fit test that has sufficient power to reject the fit when the fit is poor.

(n) In the case of a large number of small losses, the ITWG proposes that the properties of the Maximum Likelihood Estimator (MLE) may be used to give assurance that the resulting capital

calculations are credible. In the situation of few large losses, the MLE becomes less reliable as a basis for assessing the credibility of capital estimates, and bootstrapping sampling techniques may be employed. Bootstrapping is well suited to situations where the number of data points is small and the underlying distribution is non-normal.

My comment: The properties of MLE are those of large sample normality, which may not apply for tail estimation, regardless of sample size. Resulting calculations based on bootstrap data would still lack credibility when the number of actual data points is small.

(o) The choice of threshold is tied to a cost-benefit analysis of collection and to the level that local management finds useful. Therefore, it is not possible to endorse a single data collection threshold (ie that proposed by CP3 of 10,000 euro).

My comment: An alternative method of threshold specification is is given in §3 below.

(p) Relevancy (of external data): Use of expert judgement on the individual external data point to determine if that point, from the perspective of a particular bank, is an outlier, and remove those outliers.

My comment: Stress losses are by definition outliers. Outliers are the fundamental reason for regulatory capital calculations and supervision. It is dangerous to exclude them.

Note: An interesting discussion on external data banks followed at the Forum about the strict requirements that are imposed before a loss is accepted by the data bank. Each data point carries its own scenario and the data bank may hold as many as 15,000 or 20,000 of these points. How the external data borrowed from the data bank is used in relation to losses that have arisen within the firm could influence how forecasts might be perceived.

(q) Scenario analysis is defined as the forecast of operational losses and events that cause them, based on the knowledge of business experts.

My comment: How will the knowledge of business experts be audited?

(r) A potential loss event could arise under three types of scenarios:
- Expected Loss (optimistic scenario)
- Unexpected Serious Case Loss (pessimistic scenario)
- Unexpected Worst Case Loss (catastrophic scenario)

My comment: This appears to mimic the Expected Loss, Unexpected Loss, Stress Loss of the actuarial approach.

3. Standardising thresholds and severity quantiles

The problem that managing OR for lots of different business lines and loss types requires thresholds for data capture and for capital-at-risk for each, can be mitigated by mapping each fitted severity distributions onto, for example, the standard Gumbel. If thresholds are set for the standard Gumbel, they can be mapped back to give monetary values. This is done in an environmental context relating different measures of air pollution in a paper (Heffernan & Tawn, 2004) read to the Royal Statistical Society on 15 October 2003. (See also Appendix A2 below.)

4. Footnotes

4.1. The banks in ITWG are shown to be whole-heartedly into the application of LDA for economic capital computations. The regulators demand the use of loss data for OR management, and these banks are well on the way to achieving this. As to the LDA computation of regulatory capital, so much subjectivity and qualitative adjustment is taking place within the Advanced Measurement Approach that, with statistical methodologies inadequate at the present time, I cannot see an objective quantification for regulatory capital being achievable in the near future. Preparing to meet a once in 50 years event should not be made into a significant worry for OR management, nevermind a once in 1000 years event. The Italian press is chastised about scare-mongering regarding Mount Vesuvius being nearly due for its once in every 2000 years eruption. ITWG also stands for Italian Tourist Web Guide.

4.2. The standard statistical methodologies referred to by the ITWG seem to be those of undergraduate statistics courses where emphasis is on the centre (mean, median) of the population rather than the tails. I am not proposing we try to employ methodologies understandable only by the superstars of mathematical finance, merely that investment be made in research into how standard statistical methodologies might be developed for example for fitting censored data and for testing the fit.

4.3. A legal viewpoint

Jonathan Walsh, head of the financial institutions group of the law firm Norton Rose, wrote in *The Lawyer* on 29 September (WALSH, 2003)

> Complexity makes for bad regulation. There is immense scope within Basel II for misinterpretation [. . .]. If adopted on the date set for its implementation, Basel II will be difficult to comply with, almost impossible to police consistently and result in a divisive approach among institutions. Legal opinions covering the compliance by banks with the new rules are likely to become more heavily qualified and significantly less certain. [. . .], business divisions of banks and the lawyers who advise them will have their work cut out for the forseeable future.

A. Appendices

A1. Frequency and severity modelling

At the RMG conference (see §2 above), Anthony Peccia of the Bank of Montreal on behalf of ITWG gave a presentation (ITWG, 2003b) showing how seven of the banks were each developing their LDA-based AMA. It was clear that there is no agreed best model for frequency and severity. There is much that could be usefully said about the choice of models, to include also those studied in the statistics literature and the mathematical finance literature. This is not the best place for the technical summary that it requires.

A2. Mapping onto Gumbel

Suppose that we have fitted to the loss data a generalised Pareto distribution with threshold parameter u, scale parameter β and shape parameter ξ. Then the formula for the distribution of a random variable X from the fitted model gives

$$P(X > x+u \,\Big|\, X > u) \;=\; \begin{cases} \left\{1+\frac{\xi}{\beta}\,x\right\}_+^{-1/\xi} & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

where $\{z\}_+ = z$ if $z > 0$, and $= 0$ if $z \leq 0$.

For the loss data $x_1, x_2, \ldots, x_n$, the sample distribution function

$$\widetilde{F}(x) \;=\; \frac{1}{n}\sum_i 1(x_i \leq x)$$

is the proportion of data values not exceeding value x, and gives weight $1/n$ to each data value.

The semi-parametric distribution for the fitted model allowing for the threshold u is

$$\widehat{F}(x) \;=\; \begin{cases} 1 \;+\; \{1-\widetilde{F}(u)\}\left\{1+\frac{\xi}{\beta}\,(x-u)\right\}_+^{-1/\xi} & (x \geq u) \\ \widetilde{F}(x) & (x < u) \end{cases}$$

Then $y_i = -\ln\{-\ln \widehat{F}(x_i)\}$ are observations from the standard Gumbel distribution.

The random variable $Y = -\ln\{-\ln \widehat{F}(X)\} = T(X)$ is such that
$P(Y \leq y) = \exp(-e^{-y})$ and $P(Y > y) \sim e^{-y}$, an exponential upper tail, as $y \to \infty$.

A3. The bootstrap for augmenting loss data

We fit a distribution to the data, and then obtain simulated values from the fitted distribution which are added to the data. From time to time a distribution is fitted to the combined data. Simulated values are obtained from this augmented data set, and so on.

The fitted distribution may be the sample distribution function $\widetilde{F}(x)$ (see §A2 above). Alternatively we may fit a distribution from a family of distributions such as the log-normal, Gumbel or generalised Pareto.

References

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HEFFERNAN J E & TAWN J A (2004) "A conditional approach for multivariate extreme values" *J Roy Statist Soc B* 66, 1–34.
{A pre-publication preprint may be downloaded at http://www.rss.org.uk/publications/preprints.html . Some details are given in Appendix A2 above.}

ITWG (2003a) "An LDA-based AMA for the Measurement of OR: Ideas, issues and emerging practices"
{See RMG (2003) for pdf format (25 pages).}

ITWG (2003b) "The LDA-based Advanced Measurement Approach for Operational Risk – Current and In Progress Practice, RMG Conference, May 29, 2003"
{See RMG (2003) for MS Powerpoint (39 slides).}

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{The conference proceedings including ITWG (2003a, 2003b) can be downloaded at http://www.newyorkfed.org/newsevents/events/ banking/2003/con052903.html}

WALSH, J (2003) "Complex questions" *The Lawyer* (Issue 38, 29 Sept) Vol. 17, page 27.
{The full article can be found at: http://www.thelawyer.com . Search the archive for "Basel II".}

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