Scaling in the gradient contact process

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Abstract

The gradient contact process displays a rich variety of scaling features which are relevant for biological processes and very interesting from a more theoretical point of view. We have investigated the finite-size scaling of fluctuations as well as the geometrical features. On the one hand, the aim is to identify biologically relevant observables that would allow us to identify the contact process as the universality class of the underlying microscopic mechanism in the generation of species patterns. Is it possible to infer the universality class of the reproduction mechanism in a biological system by looking at, say, aerial photographs? On the other hand, the gradient contact process can be studied as a form of correlated gradient percolation.

Introduction

Bioecologists would like to understand the structure of species boundaries, such as the timeline shown above. A widely used model in ecology is the contact process. Physicists would like to find the universal behaviour of the contact process in nature. The contact process belongs to the directed percolation universality class, which is thought to be very big, but has never been observed in nature.

The uniform Contact Process

The contact process is a very simple (meta) population model. Sites are either vacant or occupied. Occupied sites become vacant with rate $\nu$ and vacant sites become occupied with rate $\nu c$, where $c$ is the fraction of occupied nearest neighbours. The idea is that each occupied nearest neighbour tries to produce an offspring with rate $\nu$ at a randomly chosen neighbouring site [2].

The activity $\rho$ is the density of occupied sites in the system. Once the lattice is completely empty, there is no way it can become occupied anywhere again, because there is no spontaneous creation. This is the absorbing state.

Finite size scaling arguments can be applied:

$$\Delta x \propto x^{\nu/1+\nu}$$
$$\rho \propto x^{\nu/1+\nu}$$

This scaling has been confirmed numerically. How to probe for this scaling behaviour in static field data, usually single snapshots?

The Gradient Contact Process

Gradient Contact Process: The ordinary contact process has homogeneous $\lambda$ throughout the system. To model a spatially changing environment, $\lambda$ is made space-dependent. Simplest scenario: $\lambda$ changes linearly in position $x$ so that there is a $\lambda_c$ with $\lambda(x) = \lambda_c$.

What are the scaling and the geometrical properties of such a system?

DP scaling features

Determine characteristic scale and apply standard finite size scaling argument.

Characteristic scale:
The characteristic scale at any point in the system is the range to the left and to the right, so that all points within this range have a (bulk) correlation length at least as big as the range.

The scale $\Delta x$ is determined by $\zeta(\Delta x) = \Delta x$, where the correlation length $\xi(x)$ is assumed to correspond to the bulk correlation length $\xi \propto |\lambda - \lambda_c|^{-\nu}$. The parameter $\lambda$ changes linearly in space with slope $\lambda'$ so that $\lambda(x) = \lambda_c + \lambda' x$. As a result, $\Delta x \propto x^{\nu/1+\nu}$. Now standard

The tuning parameter is the ratio $\lambda \equiv \lambda_c/\nu$. As a result, $\Delta x \propto x^{\nu/(1+\nu)}$. Two types of borderlines can be identified, by borrowing the definitions throughout the region of independently occupied sites, unaffected by the particularities of the underlying microscopic process. One should therefore expect the same exponents as in gradient percolation. This scenario is confirmed numerically. Preliminary results in a field study by T. Morschhauser, K. Morschhauser, B. Oborny, and D. Zimmermann, seem to support the conjecture.

Summary

We study the scaling behaviour in the gradient contact process in order to identify reliable observables to be investigated in the field. It is difficult to find features signalling the presence of the contact process as the underlying microscopic dynamics: Block-scaling in absorbing state phase transitions suffers strongly from cross-over phenomena and the scaling features of the apparent borderline are governed by ordinary gradient percolation.

References