Complexity

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May 31, 2013

Over the last twenty years or so, **complexity** has become a new buzzword in physics. Complexity is concerned with many interacting degrees of freedom, such as particles or so-called agents, which produce **emergent phenomena** in the form of **co-operative behaviour**. Very often, the systems studied under the umbrella of complexity draw on a number of different disciplines, *i.e.* complexity is by definition a **transdisciplinary** or **multidisciplinary** effort.

There is a growing need for a **quantitative understanding** of phenomena involving a large number of interacting participants in many areas of the sciences and humanities, even in those that have traditionally not drawn heavily on mathematical analysis and modelling, such as, say evolution or the social sciences. The primary aims are two-fold, namely to develop a quantitative understanding of a phenomenon on the basis of the interactions of its constituents and, in turn, to build predictive models to forecast and analyse real-world scenarios. The hope is that one validates the other, *i.e.* the detailed understanding of the underlying interactions informs the (predictive, quantitative) model and against the background of the real world, the model refines the understanding of its foundations.

Statistical mechanics is the physics of many entities that interact by the exchange of force, energy or generally something that changes their state. These interactions produce emergent phenomena, features that cannot be derived from a naïve inspection of the microscopic details. The term complexity alludes to the fact that the interaction, as basic as it might be, on the whole produces phenomena that are highly **non-linear**, very often due to some form of (positive or negative) **feedback** *i.e.* effective self-interaction or self-reference that may lead to **self-organisation**.

For example, the spreading of a disease that is communicated through humans, as studied in epidemiology, is crucially affected by infected individuals changing their behaviour. Although there might be a form of **bare** spreading of the disease from individual to individual, if they are in close contact for a long time, the very presence of the disease in an individual may change the probability of that contact. As a result, the **effective** spreading rate and thus the overall disease pattern in a highly structured population is very different from the naïve picture of the disease hoping randomly from person to person.

Phenomena like this have been studied within statistical mechanics for more than one hundred years. The field of **critical phenomena** and phase transitions is most prominently concerned with emergence, whereby local interaction between entities leads to global, long-range features, due to feedback and (effective) self-interaction. These **long-ranged phenomena**, often summarised as **scaling**, display features found in fractals and occur in many systems spontaneously, which led to the development of **self-organised criticality** (SOC) (Bak *et al.* , 1987; Pruessner, 2012) towards the end of the 1980s, seen by many as the answer to the demand for a physics of **fractals** (Kadanoff, 1986) and the beginning of complexity as a subject area in physics.

Some proponents of SOC see it as *the* cornerstone of complexity (*e.g.* Bak & Paczuski, 1993, 1995), as it attempts to explain how emergent, cooperative, long-ranged, non-trivial scaling phenomena are generated from short-ranged, simple interactions governed by simple, local rules. However, complexity does not exhaust itself in studying phenomena that can be readily cast in the language of criticality. It is concerned with every kind of unexpected effect of non-linear interaction and feedback. For example, complexity provides answers to the question how tree lines develop fractal structures, but also how gene regulatory networks control the function of a cell.

Network theory is currently the most important branch of complexity science. In networks, which are conveniently represented in the form of matrices, nodes interact with each other trough edges. As time goes by, often the interaction not only changes the state of a node, but the shape of the network itself. While the understanding of the interaction of nodes can often draw on the established results for interaction on regular lattices (grids), the characterisation of the network connectivity and the possible effect of interaction on that are aspects that are traditionally far less well understood. Fortunately, highly connected networks often behave as if each node interacts with all other nodes, *i.e.* every node sees (almost) the same environment, a setting well-understood and frequently studied as **mean-field theory** in statistical mechanics.

Different areas of science ask complexity different questions. It is the norm that research in complexity is **multidisciplinary**, involving physicists, mathematicians and experts from the respective fields. It is often also **transdisciplinary** as methods are carried over from one area to another. Where complexity research is carried out without the involvement of experts from the target field, it is sometimes criticised for producing answers to questions that have not been asked. To begin with, some questions are in fact difficult to pose. In the theory of networks, for example, determining the local arrangements of nodes and edges (so-called motifs) that constitute closely knit sub-communities requires a definition of the latter which might turn the search for such motifs into a trivial exercise. The key question is therefore much less how to identify important sub-communities and much more how to define them in the first place.

Although complexity is primarily concerned with emergent phenomena, it is still deeply rooted in the **reductionist** tradition, which expects a few, basic, underlying principles to drive a complex system. Stripping such a system down to those basic ingredients leaves a simplified, possibly over-simplified **Mickey Mouse Model**, which may still display the desired characteristics, however, in an uncluttered fashion. Most of these characteristics are **statistical** by nature, such as averages, propensities, or jerky, noisy, yet overall slow drifts. Evolution, for example, is sometimes seen as an exploration of a random, convoluted landscape by a randomly moving particle (representing the state of Nature as a whole), which slowly slides down"valleys" in the pursuit of ever-deepening troughs (Anderson *et al.* , 2004).

Calling such model "Mickey Mouse" may sound unduly critical — this is, however, not intended. Rather, it is a frank admission that these models often fail in many respects, except for the purpose they are designed for. They capture what is thought to be critical in determining the behaviour on the large spatial and temporal scale as it arises (or, rather, emerges) from the smaller, local scale. Complex systems inherently **cross scales**, they are always about something asymptotic, long time and long distance.

As those Mickey Mouse Models help to traverse across different temporal scales, they provide predictive power, ideally in a quantitative manner. This is, ultimately, the scientific acid test, but not one that is easily passed. In fact, after the long, elaborate analysis of many models in complexity, what remains in the conclusion is barely a **narrative** and only very rarely one that is quantitative. What is worse, that narrative might as well have been constructed from the basic ingredients of the model in the first place. In fact, the summarising narrative may be the initial starting point in a different disguise. The outcome is prejudged. For example in (Ispolatov *et al.* , 1998) a wealth distribution model is analysed where the richer person in a trade gains more. Fittingly and unsurprisingly, the overall conclusion is: The rich get richer and the poor get poorer. Although the present example was much rather intended as an entertaining piece of nice Mathematics and certainly not envisaged as a serious attempt to model society, nobody is prevented from interpreting it as such (Hayes, 2002).

An overwhelmingly strong argument in favour of Mickey Mouse Models is **universality**. Universality is expected to be at work (in somewhat broader terms) in complexity just as much as it at work in the asymptotic (scaling) spatio-temporal features of traditional critical systems. However, while it has a strong quantitative origin and meaning in the latter, namely linking symmetry groups of the underlying Hamiltonian, action or Liouvillian to exponents and scaling function, the nature of complex systems and the complex systems intended to model Nature are more complicated and, in fact, richer. Apparently, universality does not apply or only if its notion stretched beyond recognition. This may well have deeply routed technical reasons (Pruessner & Peters, 2006) but as a matter of fact, universality at a technical, quantitative level is rarely found in natural complex systems (but Bak *et al.* , 2002; Peters *et al.* , 2010). What is found is "similar behaviour" among models and natural systems (*e.g.* Reed & McKelvey, 2002), apparent long-ranged (asymptotic) behaviour (Christensen *et al.* , 1996), although sometimes too noisy to allow for a detailed data analysis. Whether this is a coincidence, the effect of some undiscovered mechanism or "universality with noise" is difficult to say. Answering this question is complicated by the bias in research and dissemination that promotes publication of positives rather than negatives of scaling.

Being so vaguely defined that nearly every study of "something complicated" fits under its umbrella, makes complexity prone to abuse. A notorious example is the skillful analysis of hospital waiting times in the NHS (Smethurst & Williams, 2001), the conclusion drawn from it (The British Library Science Technology and Business (STB), 2001; Ball, 2001), and the subsequent case made for a different pay scheme (Papadopoulos *et al.* , 2001). Areas where complexity science is in common use (some may say rife) include biology (in particular micro and cell biology, ecology and evolution), epidemiology, computer science, social science and finance. If complexity is to say that everything goes, it means nothing. If complexity is to say that science has to reach beyond the traditional bounds of disciplines and bring to bear the notions and the techniques developed in one hundred years of statistical mechanics, whenever many degrees of freedom are concerned, then it will be remain as a great success.

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