EE2 Mathematics

Example Sheet 1: Functions of Multiple Variables

1) Consider $f(p, q) = 2p(q^2 + 2q - 1)$. Find the components of $\nabla f$, i.e. $(\frac{\partial f}{\partial p}, \frac{\partial f}{\partial q})$, at the points i) (-1,0), ii) (1,0), iii) (-1,1) and iv) (1,1). Produce a sketch showing $\nabla f$, the gradient vector, at each of these points.

Pick your answers from:
- a) (0,-4)
- b) (4,8)
- c) (2,-4)
- d) (0,4)
- e) (-2,-4)
- f) (4,-8)
- g) (4,4)
- h) (-2,4)

2) I am an indifferent experimentalist and can be slightly inaccurate in my measurements of time intervals and distances. Recall that a pendulum has period $P$ and length $l$ and these are related by $P = 2\pi\sqrt{\frac{l}{g}}$. What is the percentage error in my estimate of $g$ if
i) I measure $P$ perfectly but make a 0.2% error in estimating $l$.
ii) I measure $l$ perfectly but make a 0.3% error in estimating $P$.

It might help you to rearrange the above expression in $P$.

Pick your answers from:
- a) 0.1
- b) 0.2
- c) 0.4
- d) 0.3
- e) 0.6
- f) 0.8
- g) 0.5

3) Consider $f(x, y) = \exp(-2xy)$. Find: i) $\frac{\partial f}{\partial y}|_x$ and ii) $\frac{\partial f}{\partial x}|_y$

Check that $\frac{\partial}{\partial y}|_x \frac{\partial f}{\partial y}|_x = \frac{\partial}{\partial x}|_y \frac{\partial f}{\partial y}|_x$. Points $(x, y)$ can be re-expressed in polar coordinates $(r, \theta)$. Use a change of variables (via the chain rule) to obtain expressions for: iii) $\frac{\partial f}{\partial r}|_\theta$ and iv) $\frac{\partial f}{\partial \theta}|_r$.

Check that you have obtained the correct answers by first re-expressing $f(x, y)$ as $f(r, \theta)$.

Pick your answers from:
- a) $-2x \exp(-2xy)$
- b) $-2y \exp(-2xy)$
- c) $-2y \exp(-2xy)$
- d) $-2x \exp(-2xy)$
- e) $-2x \exp(-2xy)$
- f) $2(y^2 - x^2) \exp(-2xy)$
- g) $-xy \exp(-2xy)$
- h) $-4xy \exp(-2xy)$

4) Consider the equation $xyz + x^3 + y^4 + z^6 = 0$. What is the value of the product below?

$$\frac{\partial x}{\partial y}|_z \cdot \frac{\partial y}{\partial z}|_x \cdot \frac{\partial z}{\partial x}|_y$$ (1)

Pick your answer from:
- a) 1
- b) 0
- c) -1
- d) $\infty$

[harder] Show that this holds in general for $f(x, y, z) = 0$. It might help to consider $x = x(y, z)$, $y = y(x, z)$ and the total differentials $dx$ and $dy$.

5) Expressions for $f(x, y)$ can be re-expressed in polar coordinates as $f(r, \theta)$. i) Does the following expression have the correct dimensions? ii) And is it correct?

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2}$$ (2)

Pick your answers from:
a) yes! b) no.

6) For:
- \((x^2 + y^2 + 1)^{-1}\) find i) all stationary points and ii) their characters.
- \(\sin x \sin y\) with \((0 < x < \pi, 0 < y < \pi)\) find iii) all stationary points and iv) their characters.
- \(x^4 + y^4\) find v) all stationary points and vi) their characters. [tricky to do properly]
Calculating the Hessian in each case might be useful.

Pick your answers from:
a) (1, 1) b) (1, 0) c) (0, 1) d) (0, 0) e) \((\pi/4, \pi/2)\) f) \((\pi/2, \pi/2)\) g) max h) min i) saddle