Tipping Points Toolbox for Climatic Records

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In collaboration with Tim Lenton (Exeter University)

Training and career: bifurcations

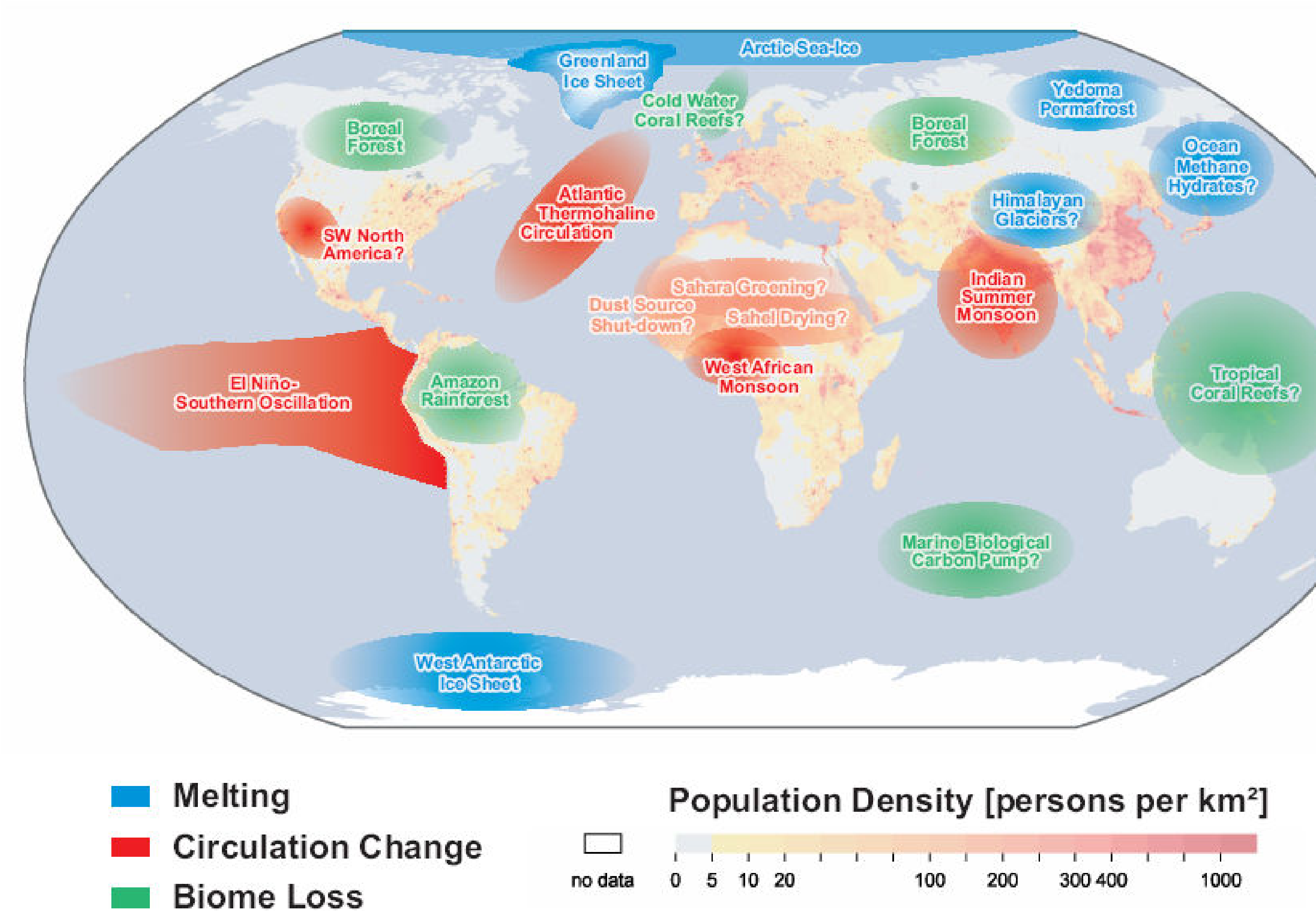
- MSc in maths, Russia: functional analysis and theory of bifurcations
- PhD in physics, Israel: times series analysis of hydrological records and earthquakes catalogs
- Postdoc in the UK (2006-present) funded by NERC and AXA: bifurcation analysis of time series
- 2012:heading for the next bifurcation

  [early warning]…

Approach in research

- Consider a general stochastic model
- Find an idea
- Develop codes
- Test on artificial data with known properties
- Apply to paleo and historic records with known transitions
- Apply to model data to find future transitions

Tipping elements in the climate system

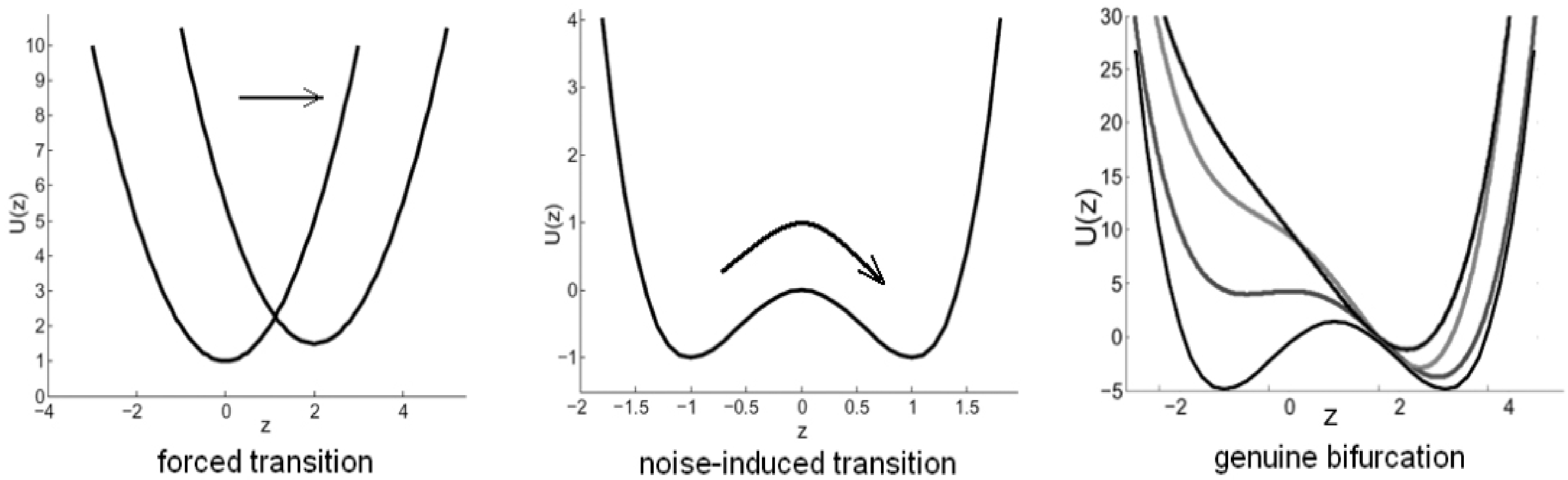


Lenton et al. PNAS, 2008

What we are studying

transitions and bifurcations in time series

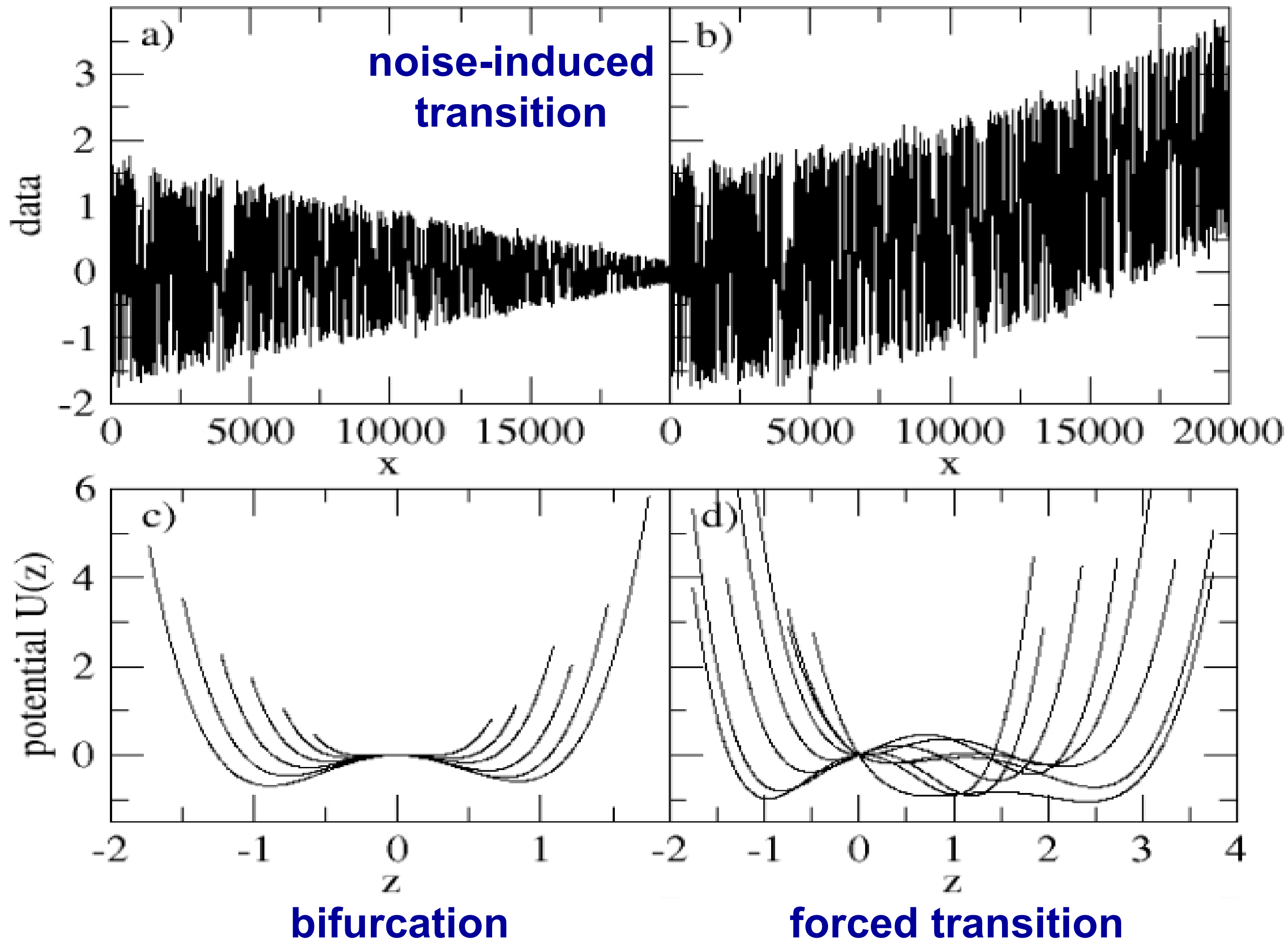
These may be classified in terms of the system potential, which defines the states of a climatic variable



Livina et al, Climate Dynamics 2011 (started in 2008)

Tipping elements of the Earth system may approach tipping points that may be transitions as well as bifurcations

Examples of artificial series



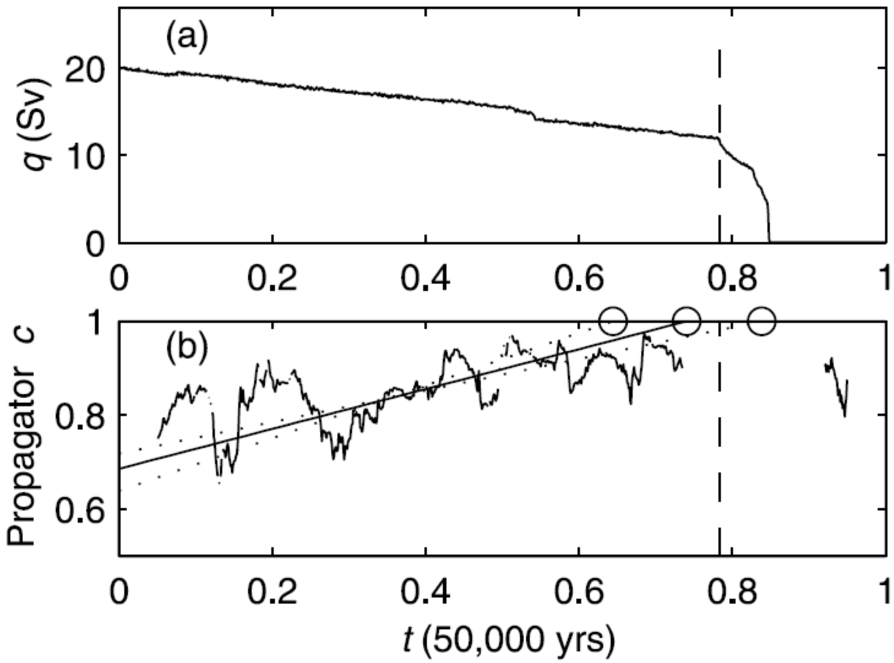
How we are studying

- Degenerate fingerpinting (monitoring autocorrelations)
- Potential analysis (monitoring structure of the system potential)

Degenerate fingerprinting: studying critical behaviour

Degenerate fingerprinting

Held & Kleinen, GRL 2004



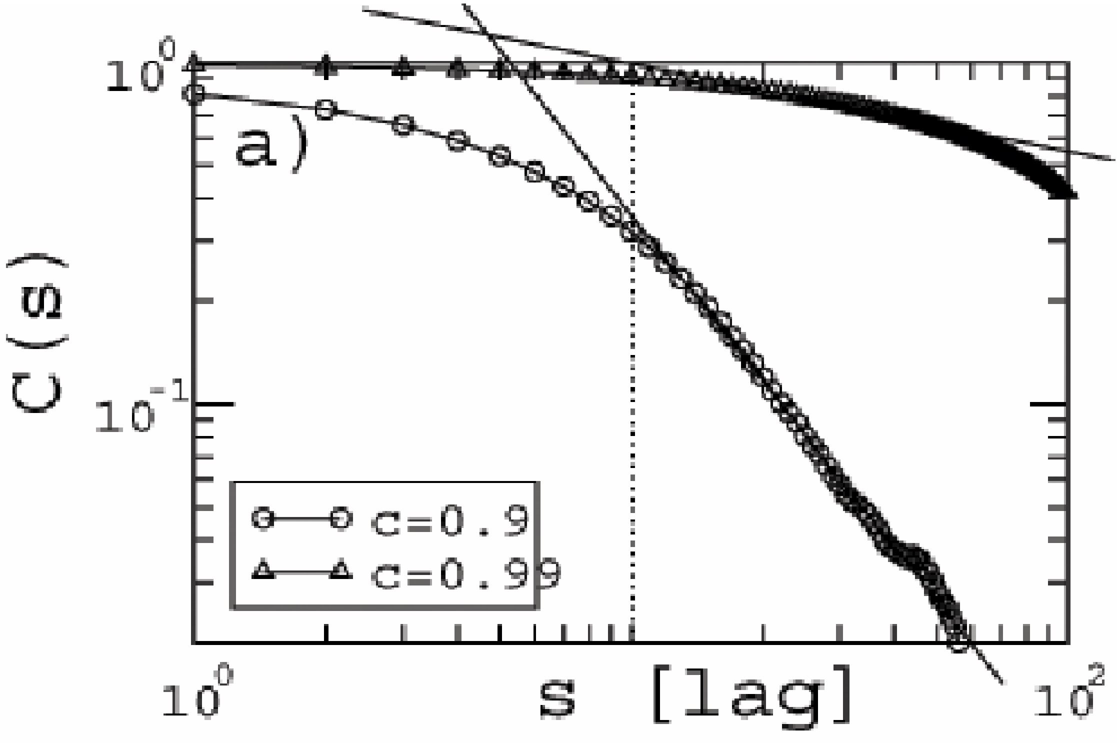
aggregation with $\Delta t \approx 1/\kappa$

North Atlantic stream function from CLIMBER2 model and propagator used to detect bifurcation; THC collapse due to linear increase of CO_2 and statistically perturbed increased fresh water forcing

Series is approximated by an AR(1) process, and exponential decay of the auto-correlation function (ACF) is estimated. Thus ACF-propagator *c* is defined; its gradual trend towards value 1 indicates critical behaviour.

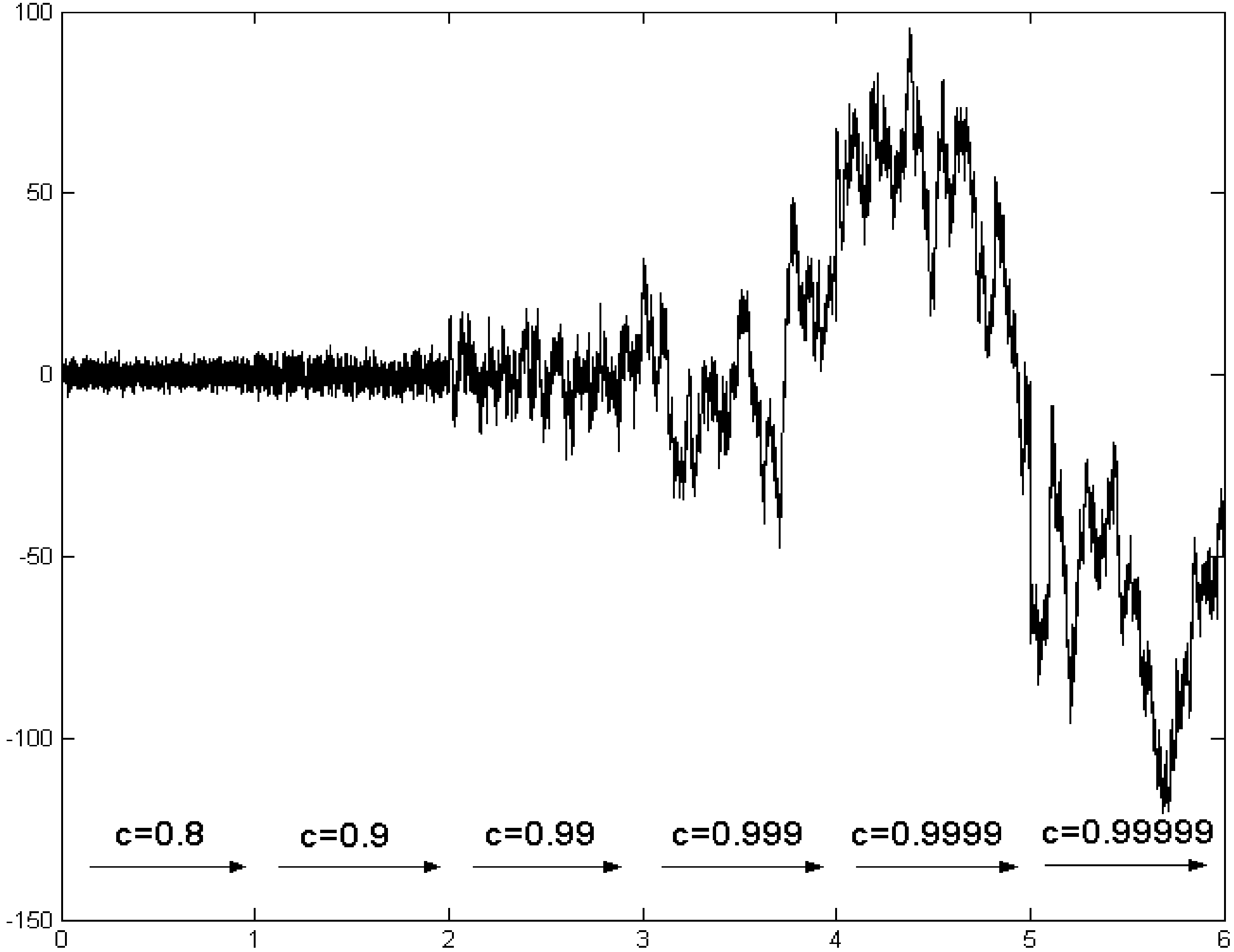
$$y_{n+1} = c y_n + \sigma \eta_n,$$
$$c = \exp(-\kappa \Delta t),\ \kappa \text{ is decay rate}$$
$$(\kappa = 0 \text{ when } c = 1)$$

Autocorrelation function and lag-1 ACF of AR(1) data at critical *c* values



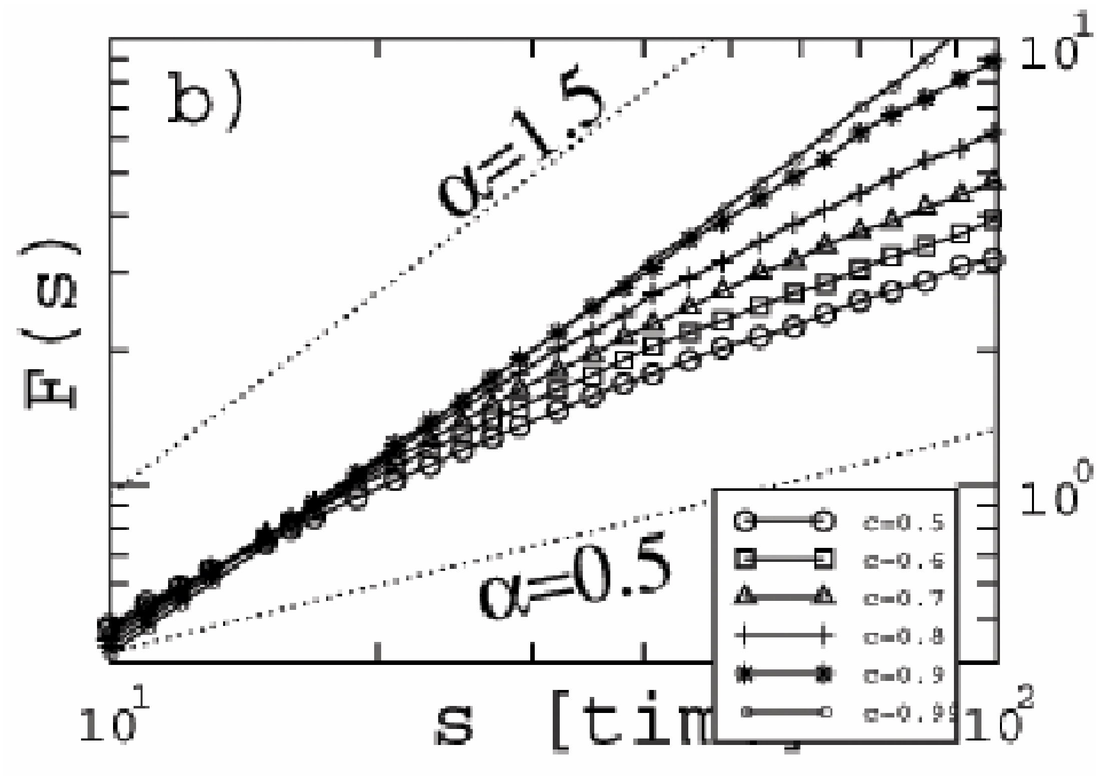
Lag-1 autocorrelations can be affected by trends and high noise level in the series

AR(1) data at critical values of parameter *c*



Increasing c – increasing nonstationarities – increasing short-term memory

DFA of AR(1) data at various *c* values

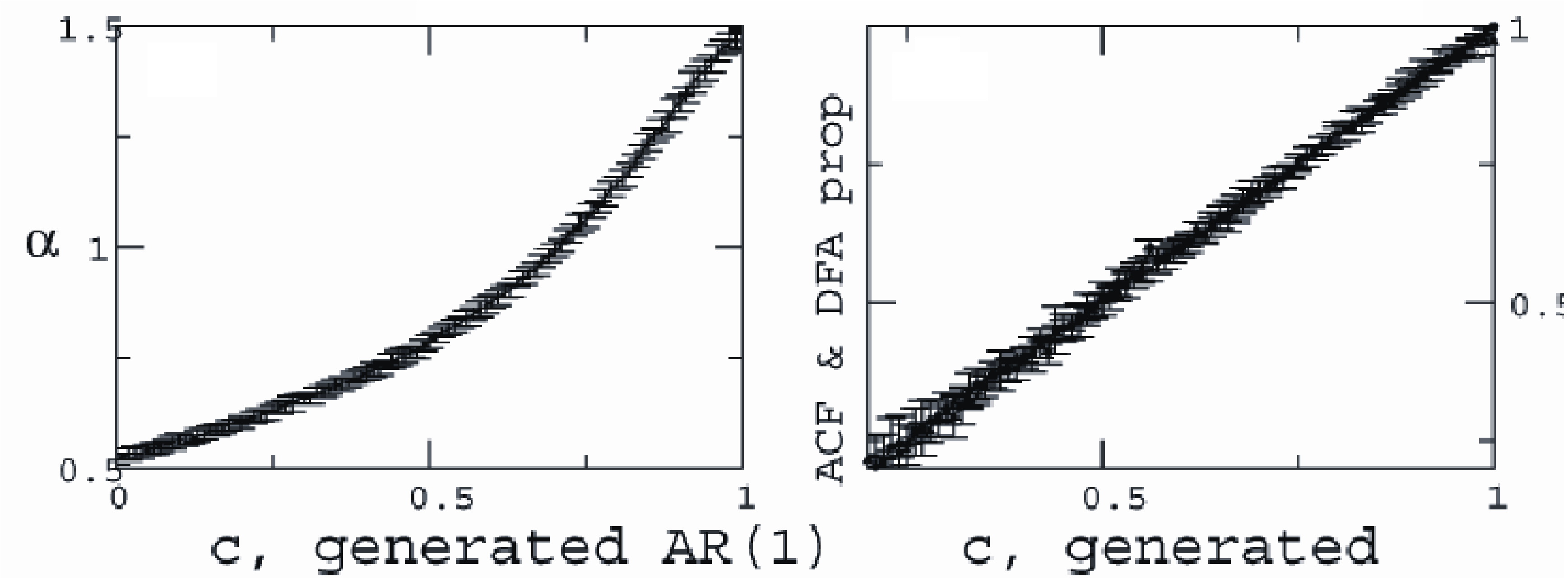


$$\text{if } c \to 1 \text{ then } y' = \sigma\eta$$

AR(1) process is not long-term correlated (Bogachev et al, 2009)
The short-memory effects are observed for 10-100 time units

Modified degenerate fingerprinting

Livina and Lenton, GRL 2007

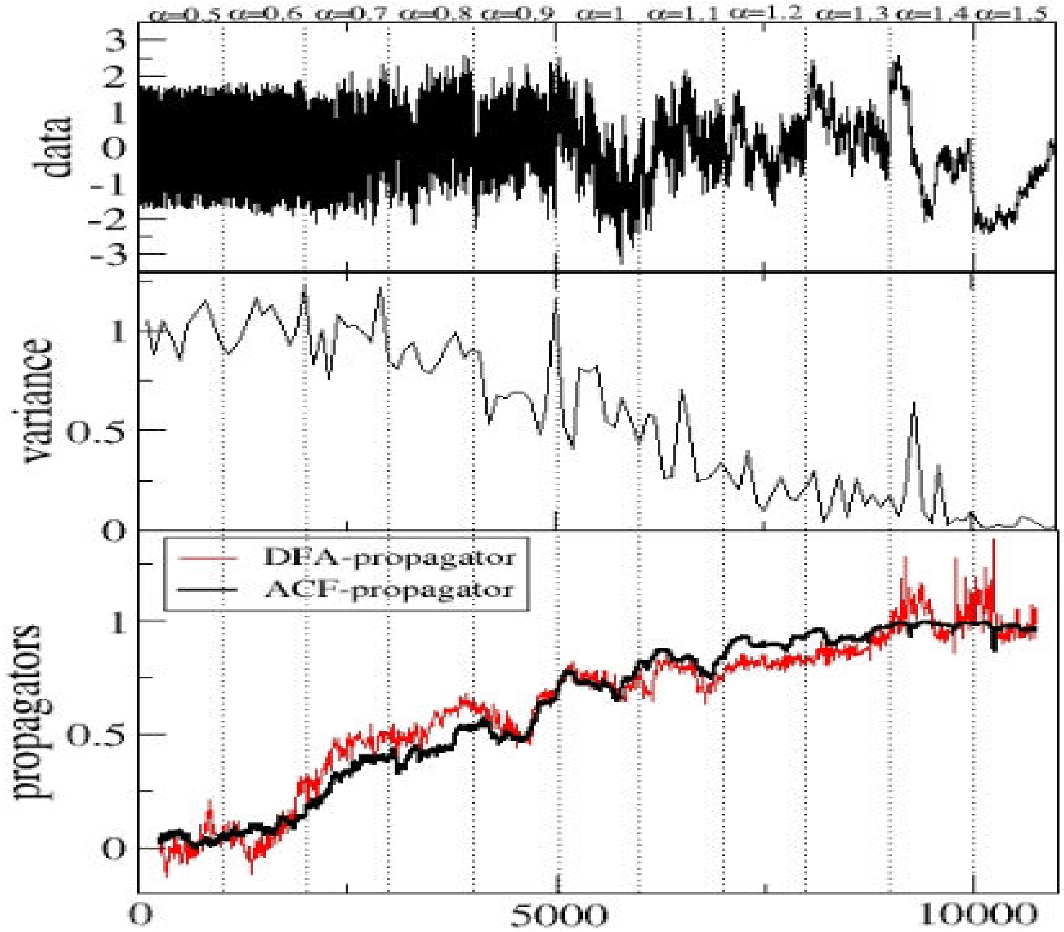


Errorbars obtained for 20 samples of length 20,000 at each *c* value

We calculated DFA exponent and calibrated it towards the ACF propagator. The resulting estimator is called DFA-propagator. Its trend to critical value 1 indicates approaching a tipping point (similar to ACF)

Artificial data with increasing memory

Livina, Ditlevsen, Lenton, *Physica A*, 2011



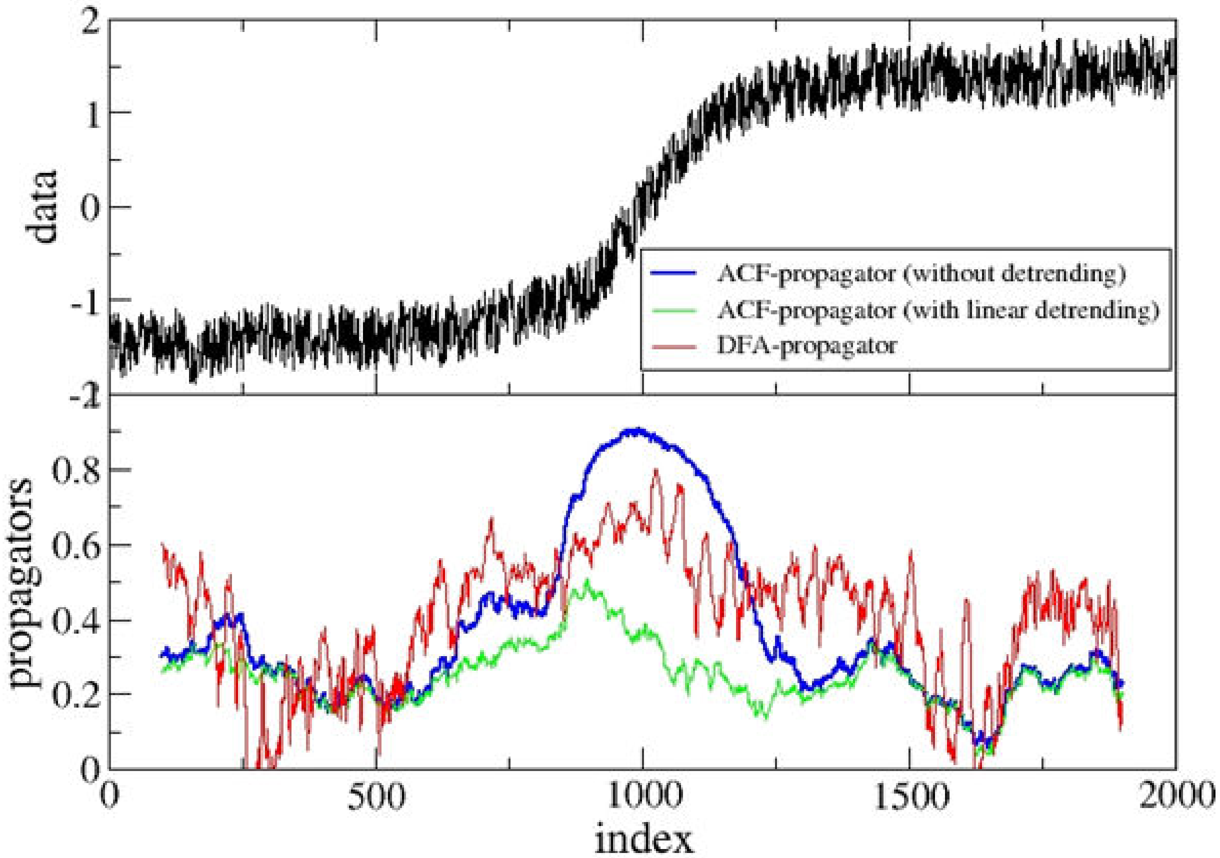
+ recent work in KNMI

When ACF-propagator reaches critical value 1, DFA-propagator is still capable to reflect the variability in the variance

Example of transition: sigmoid function

added red noise, fluctuation exponent 0.7

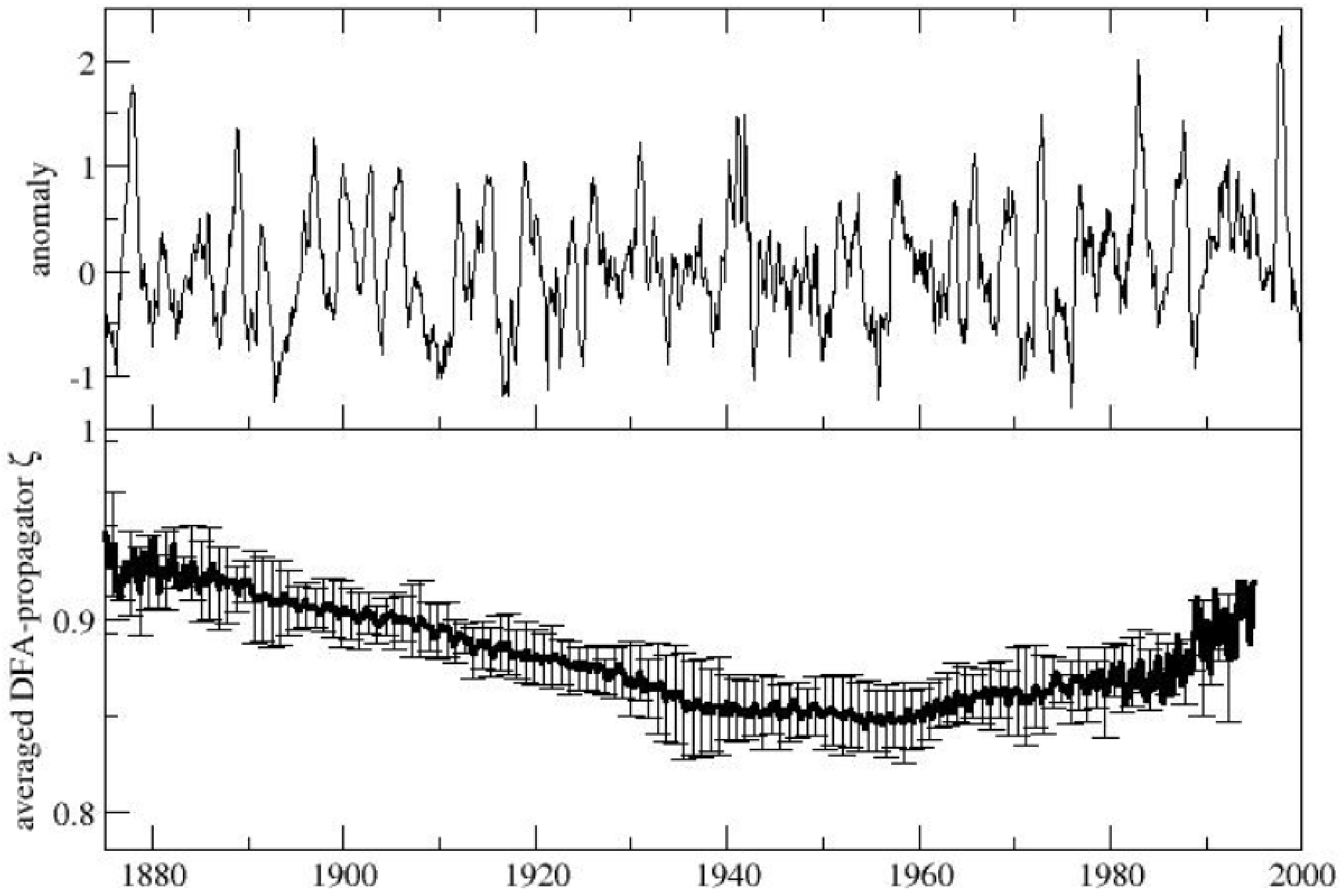
Livina, Ditlevsen, Lenton, Physica A, 2011



ACF-propagator without detrending is sensitive to transitions

Testing robustness: variable window size

Kaplan SST anomaly, monthly data, wl=300:50:1000



Window size vary between 5% and 50% of the data length

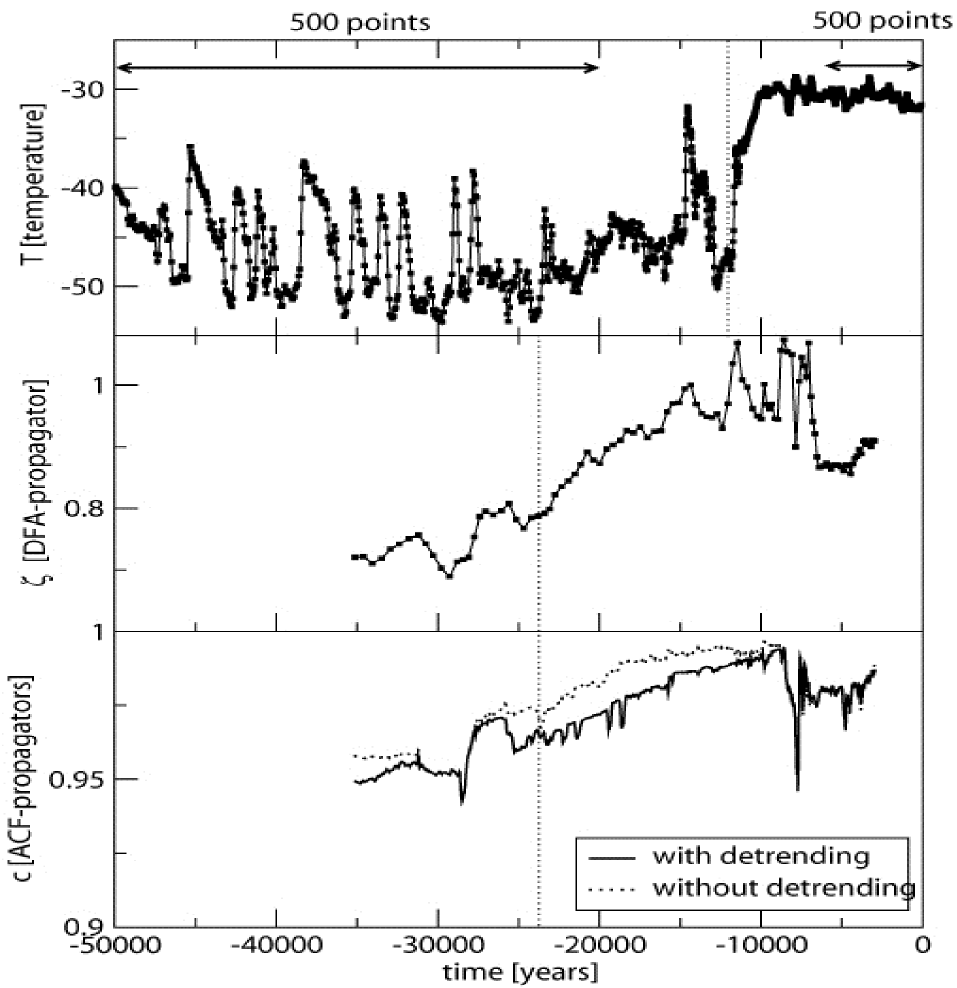
GISP paleotemperature

Livina and Lenton, GRL 2007

Greenland ice-core regional temperature record

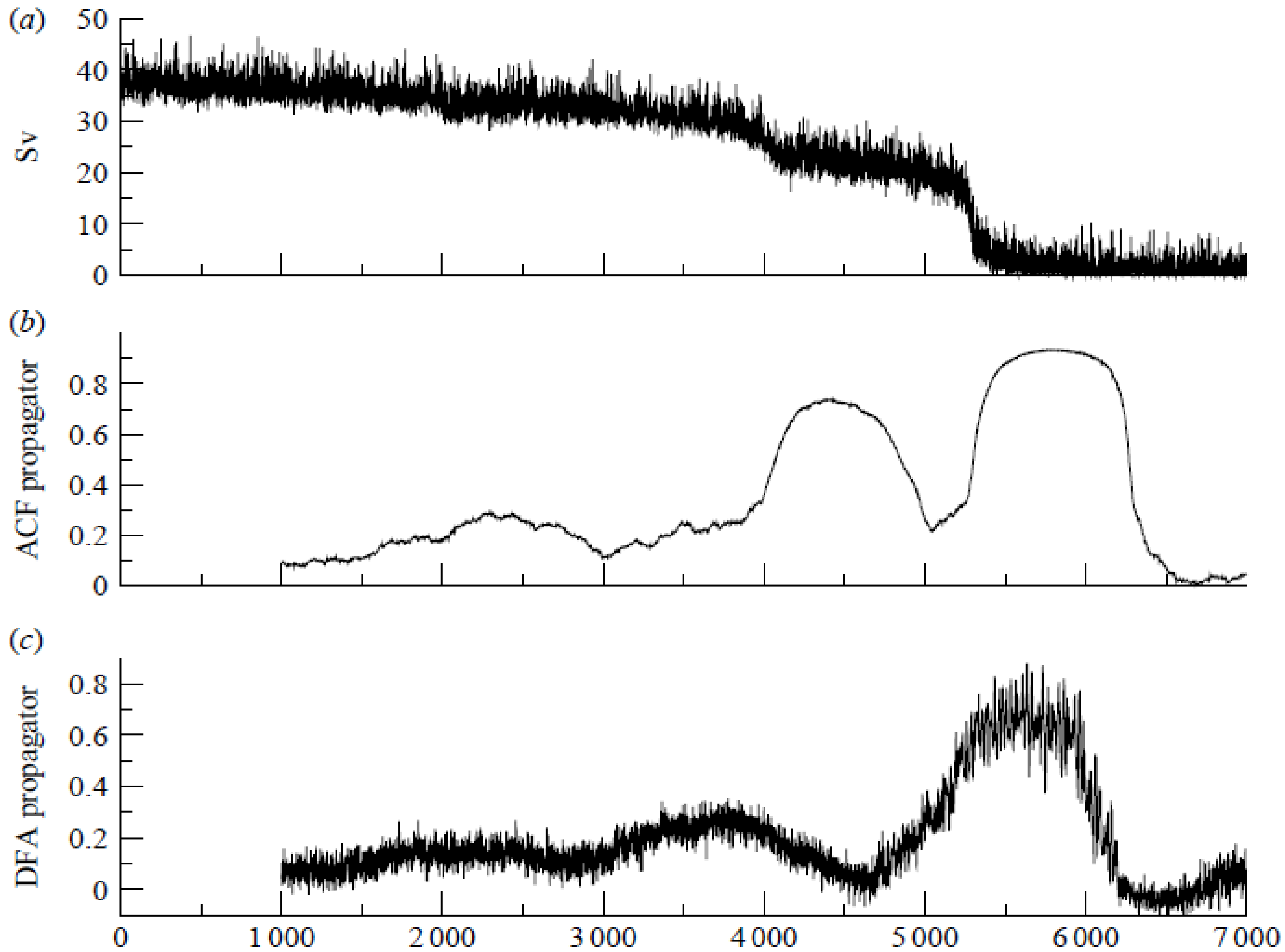
Early warning indicator from detrended fluctuation analysis

Early warning indicator from autocorrelation function



GENIE-2 streamfunction

Lenton et al, Phil. Trans A 2009



Branch of THC hysteresis loop with collapse of dynamics

Preprocessing geophysical data for degenerated fingerprinting

- Held & Kleinen used aggregation of data to reduce the effect of weather noise
- Dakos et al used residuals after applying Gaussian filter
- Alternatively, it is possible to use wavelet denoising for the same.
- When there are gaps or poor temporal resolution, we cannot interpolate data, because that would introduce spurious correlations in the data, which would affect estimation of lag-1 autocorrelations
- Many datasets are studied in “raw” format

Potential analysis: studying system bifurcations

Simple stochastic model for climatic variables (temperature etc.)

$$\dot{z}(t) = -U'(z) + \sigma\eta$$

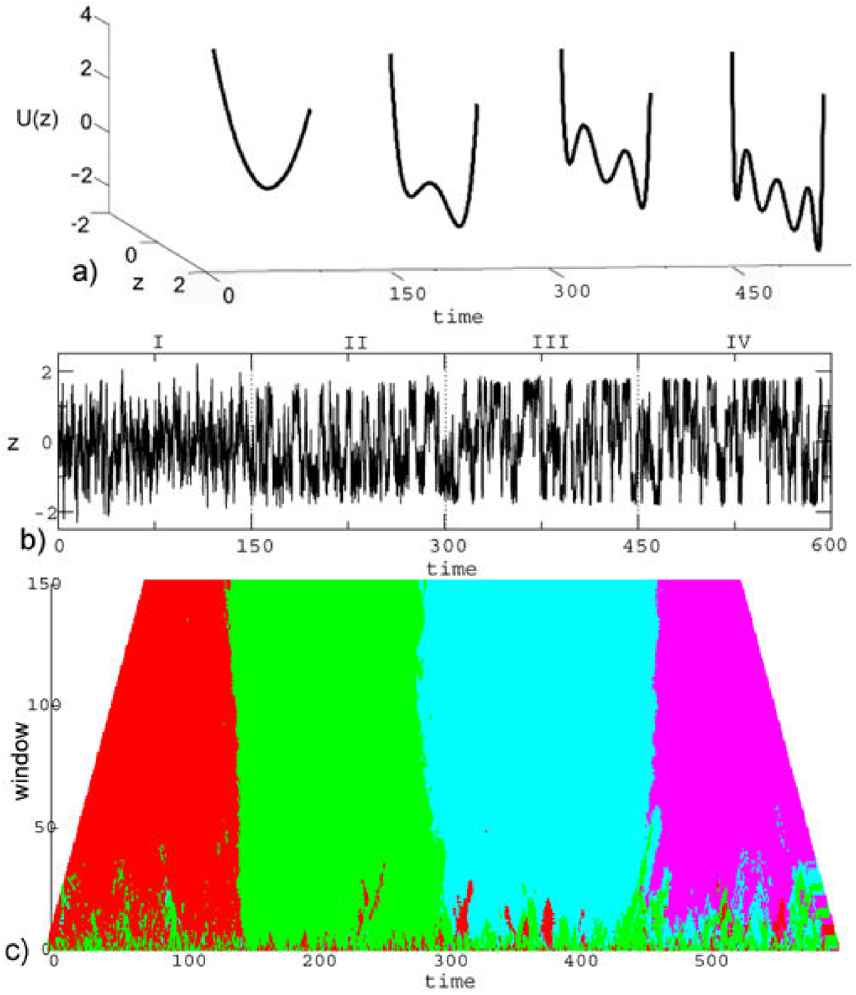
$$U(z) = a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z$$

- Estimation of the number of states – polynomial degree of U(z)
- Estimation of noise level
- Derivation of potential coefficients using Unscented Kalman Filter (UKF)

double-well potential

Kwasniok & Lohmann, Phys Rev E, 2009
Livina, Kwasniok, Lenton, Climate of the Past, 2010

Artificial data with four potentials



Potential contour plot at different time scales

We generate artificial data using Euler scheme

$$x_{t+\Delta t} \approx x_t - \left.\frac{dU}{dx}\right|_t \cdot \Delta t + (W_{t+\Delta t} - W_t)$$

W is a Wiener process

Potentials:

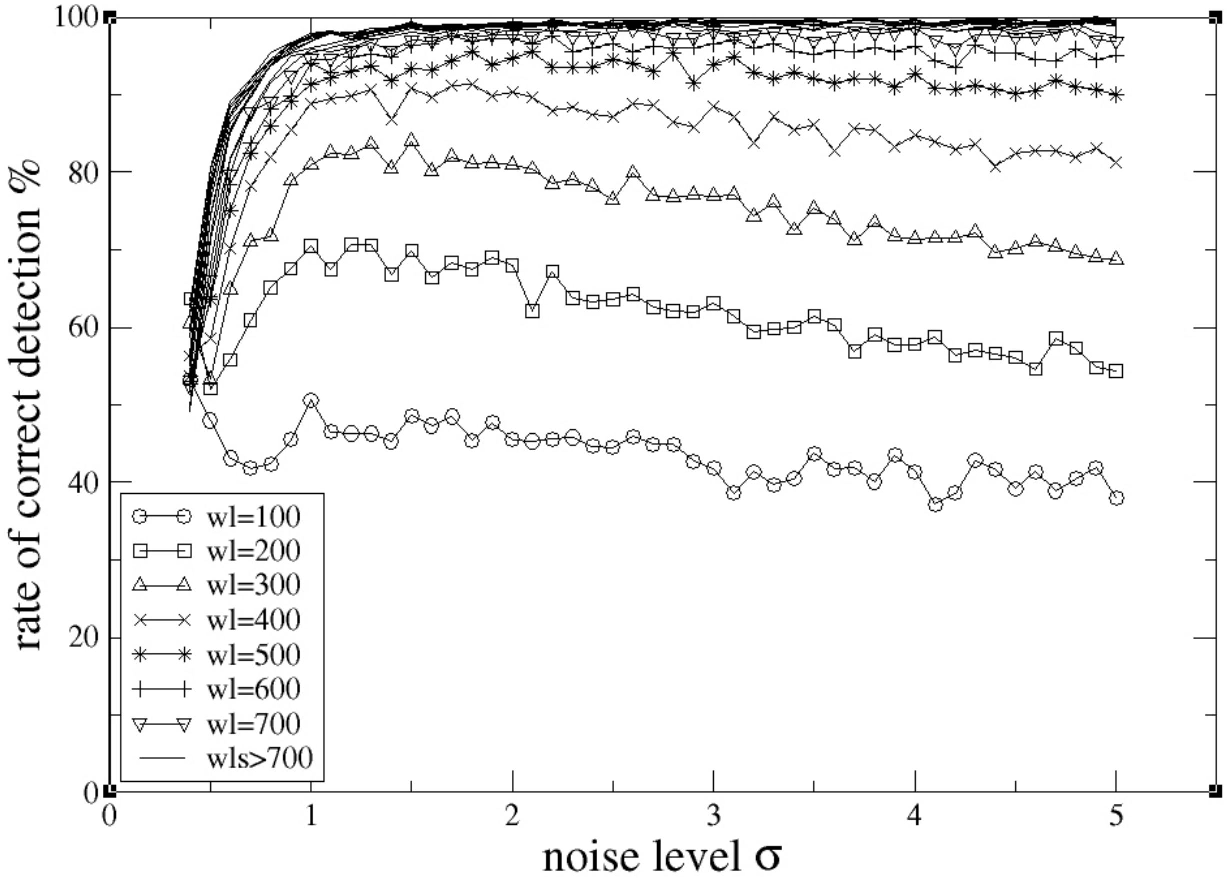
$$U(z) = z^2$$

$$U(z) = z^4 - 2z^2$$

$$U(z) = z^6 - 4.5z^4 + 5z^2$$

$$U(z) = z^8 - 6.5z^6 + 13z^4 - 8z^2$$

Rate of correct detection



Detection of two wells in artificial double-well potential data
(depth of wells = 1, consider 1000 samples per each value of noise level)

Livina et al, *Climate Dynamics*, 2011

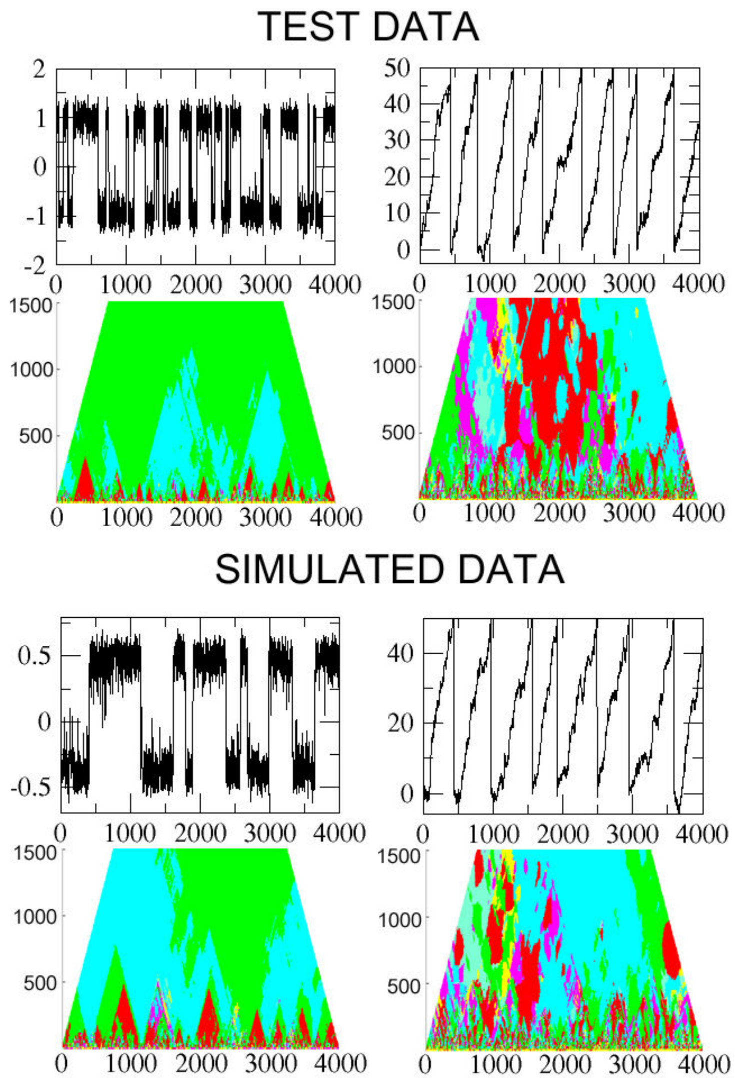
“Blind test” experiment

Livina, Ditlevsen, Lenton, *Physica A*, 2011

- 9 samples of data generated from different (unknown) models provided (Ditlevsen’s visit in 2009)

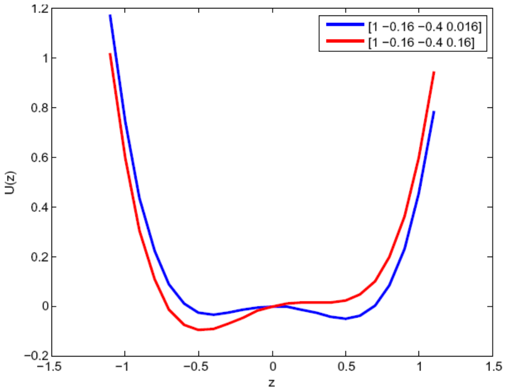
- Potential analysis used to try and deduce underlying models, then simulate data equivalent to the test samples

- Method correctly reconstructs generating equation where there is potential behaviour, and recognises sample with non-potential behaviour

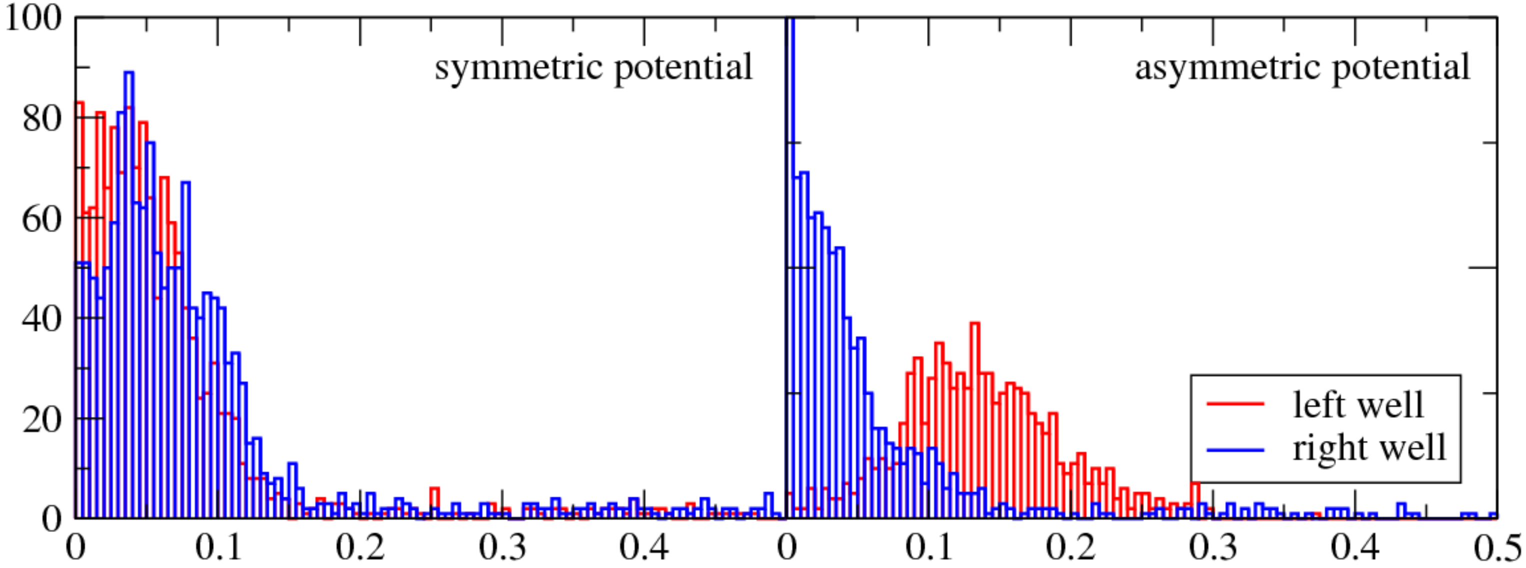


Artificial data with symmetric and asymmetric potential: comparing detected depths of wells

sets of 1000K points, wl=5K, σ=0.4



We generated two samples of artificial data and in sliding windows estimated the potentials coefficients, from which we derived the depths of the potential wells. Analytically calculated depths were: for symmetric potential 0.0339 and 0.0488, for asymmetric potential 0.1124 and 0.0007



Preprocessing geophysical data for potential analysis

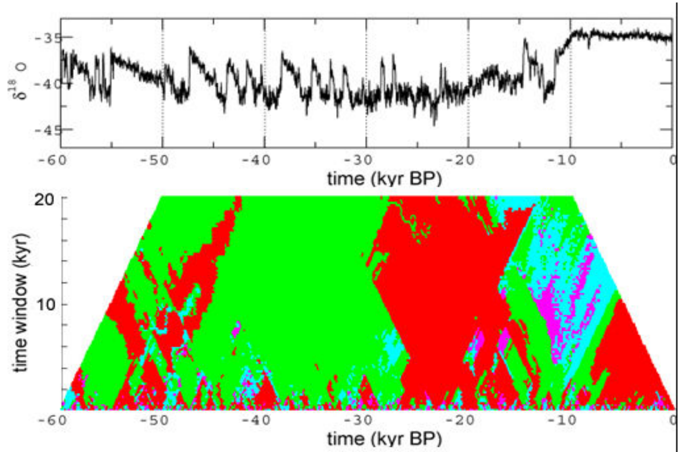
- When there are gaps or poor temporal resolution, we can interpolate data, because we deal with probability distribution. [To some extent: too high resolution together with small sliding window may give meaningless results].
- If there are nonstationarities in the data, it is helpful to use wavelet denoising for the estimation of the noise level and also detrending/filtering if there is an obvious trend
- Still, many datasets are studied in “raw” format

GRIP & NGRIP temperature proxy data

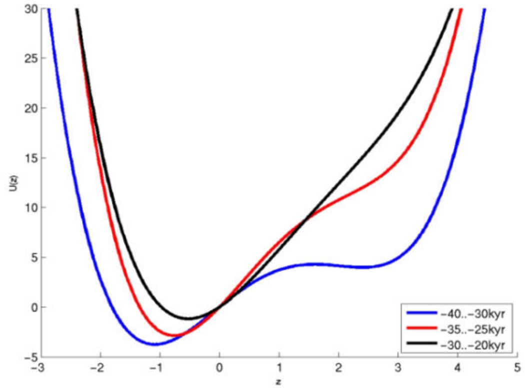
$\delta^{18}O$ data: bifurcation at 25-28 kyr BP

(Livina et al, Climate of the past, 2010)

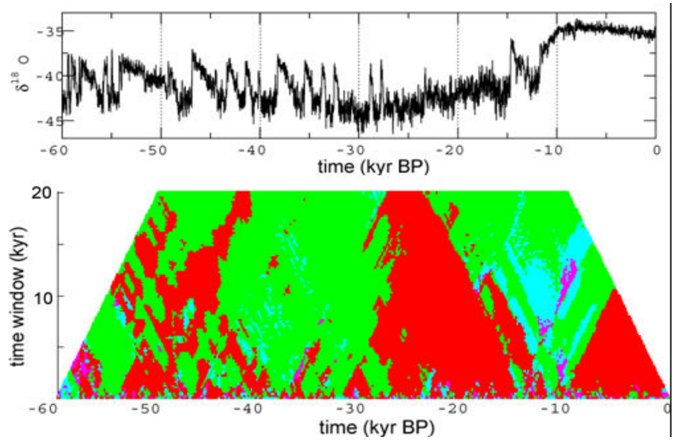
GRIP



GRIP

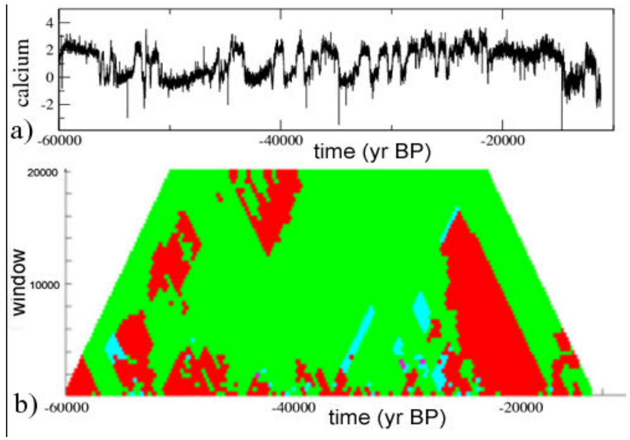


NGRIP



Calcium data: bifurcation at 27-28 kyr BP

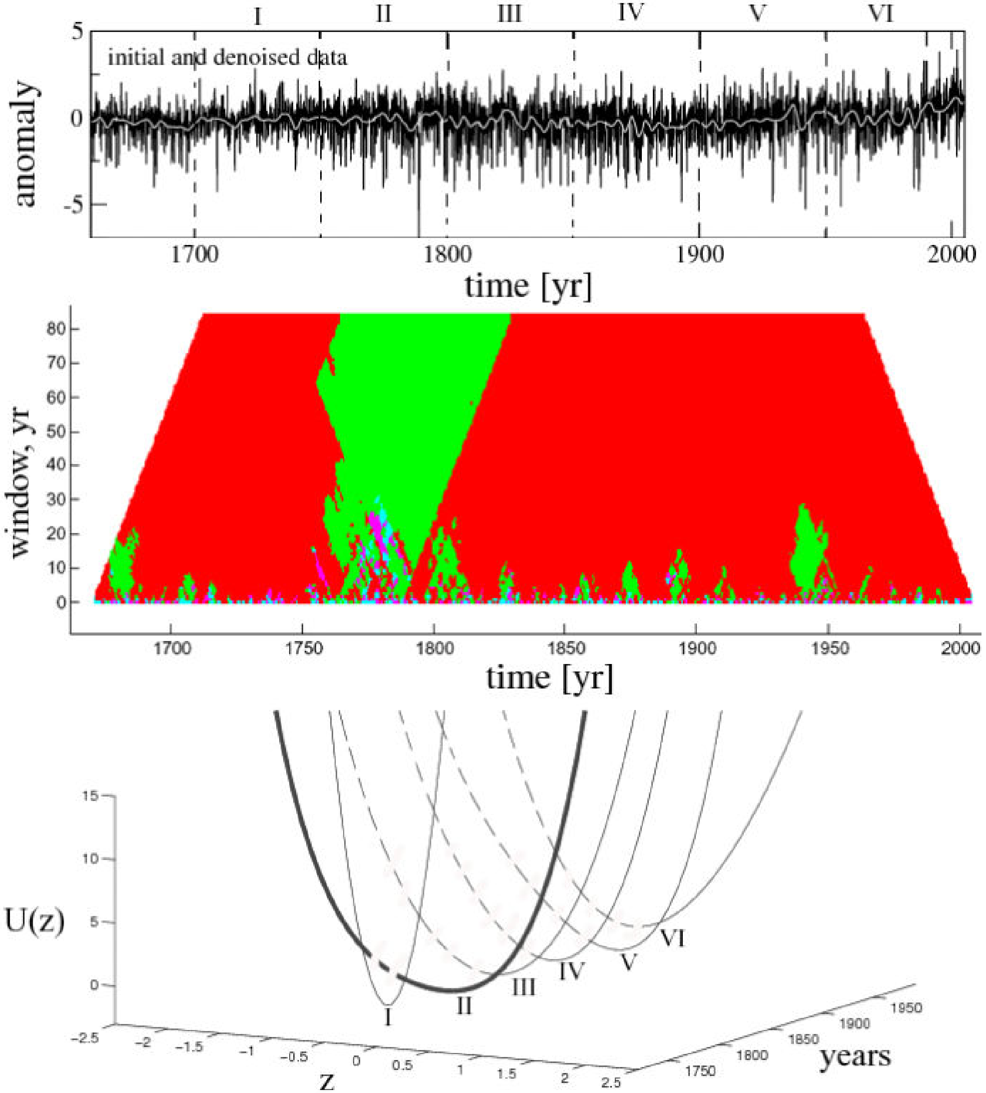
GRIP



GICC05 time scale, resolution 20yr

Annual resolution

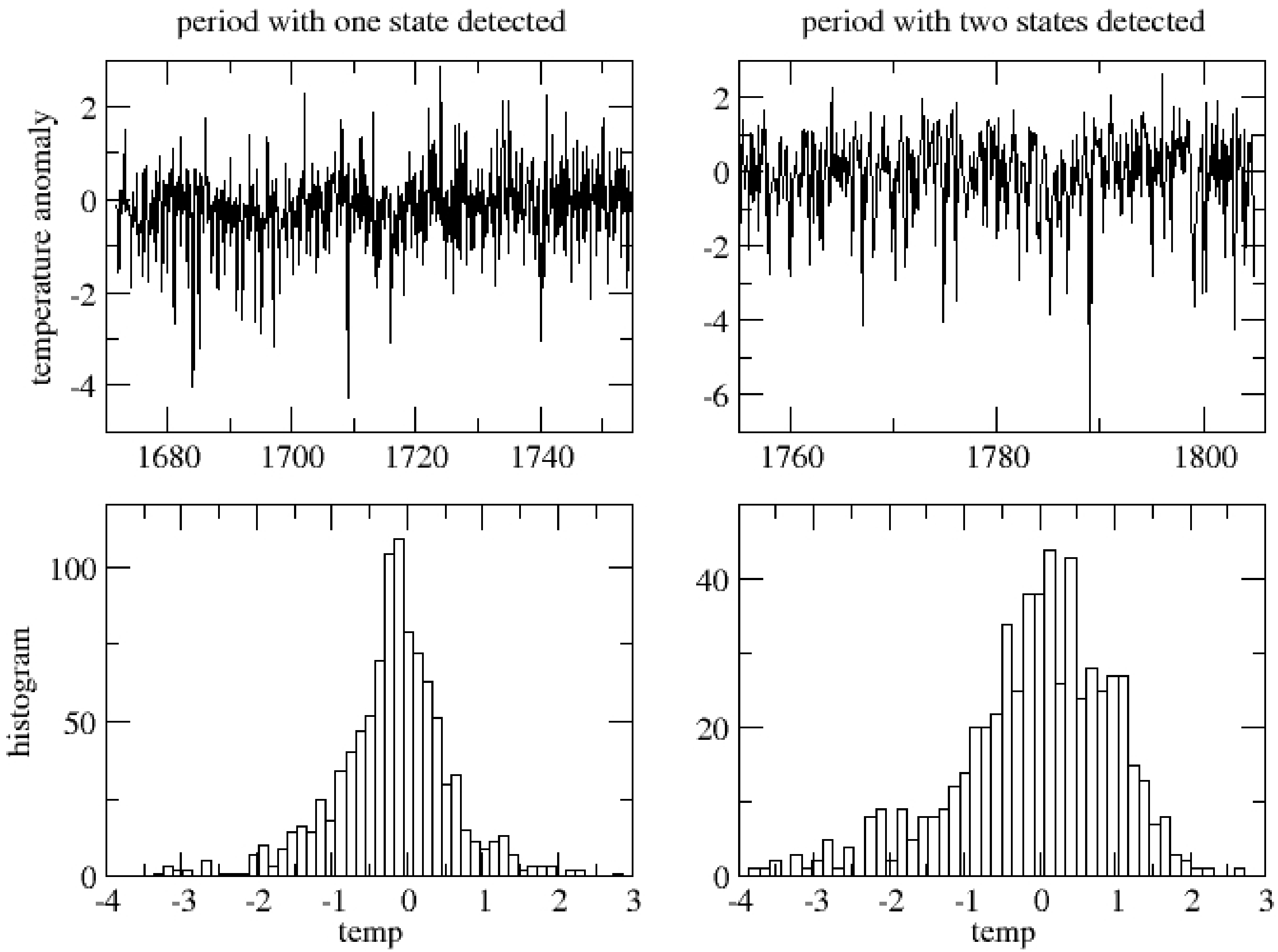
European monthly temperature anomaly (1659-2004)



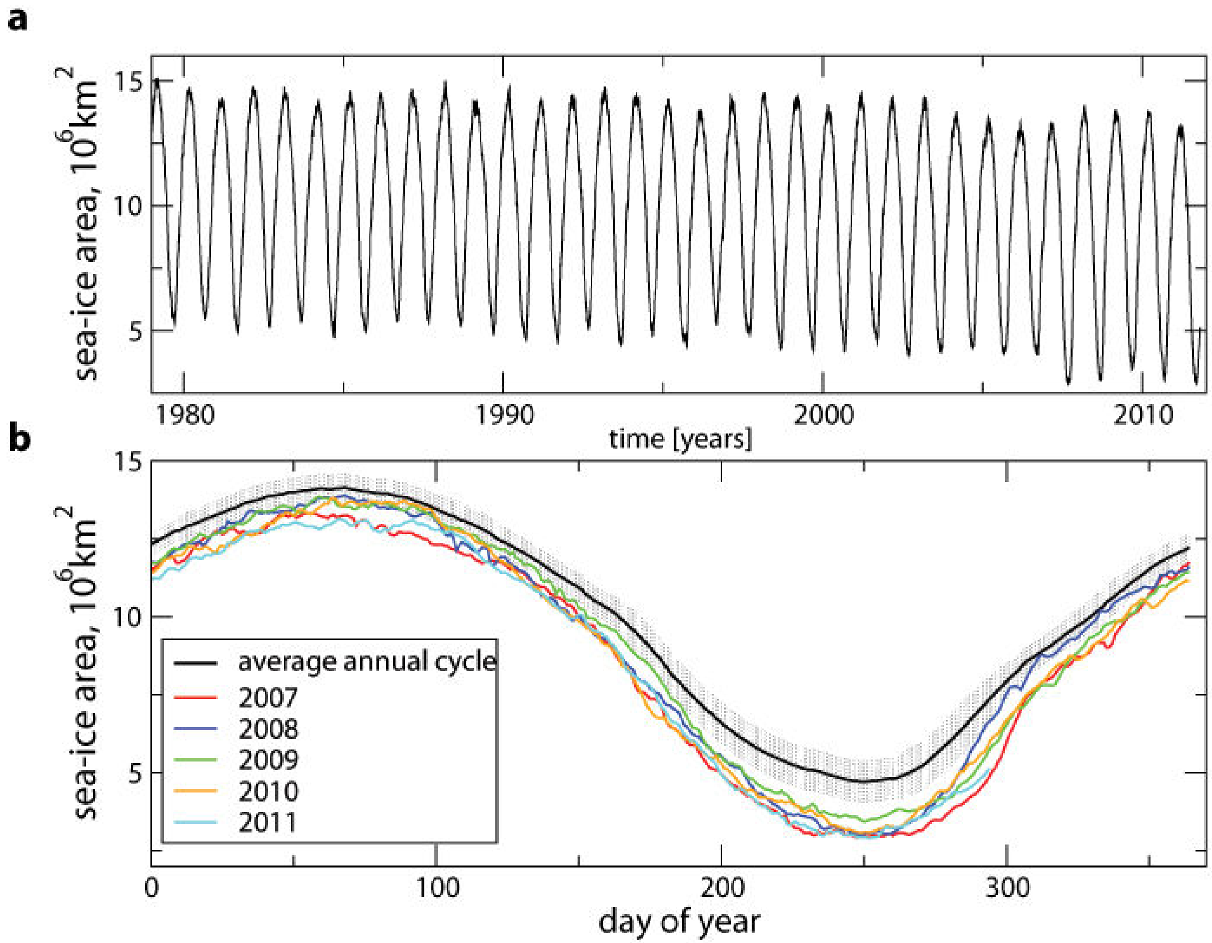
A change of the number of states around year 1770. There were observed climatic anomalies in the second half of the 18th century named after Baron de Malda, who recorded observational notes, "Malda anomaly`` (Dalton minumum was later), when *the Mediterranean climate was "strange", with thunderstorms, floods, droughts, and severe winters.* The potential analysis shows appearing instability of the climate, when another, colder state was about to appear, but later stabilised and formed 1-well potential again.

Luterbacher et at, Science (2004)
Barriendos & Llasat, Climate change (2003)

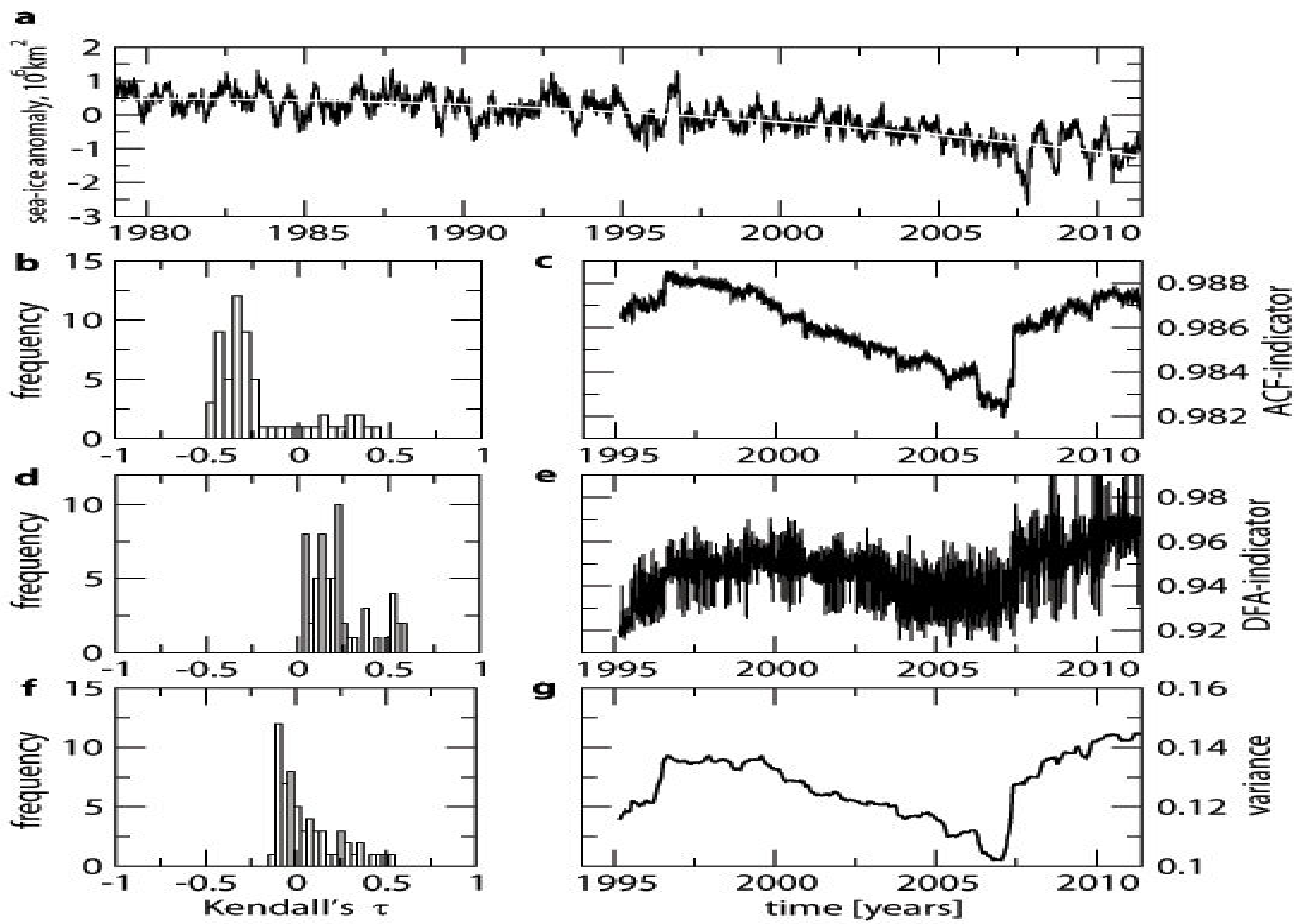
European monthly temperature anomaly: histograms



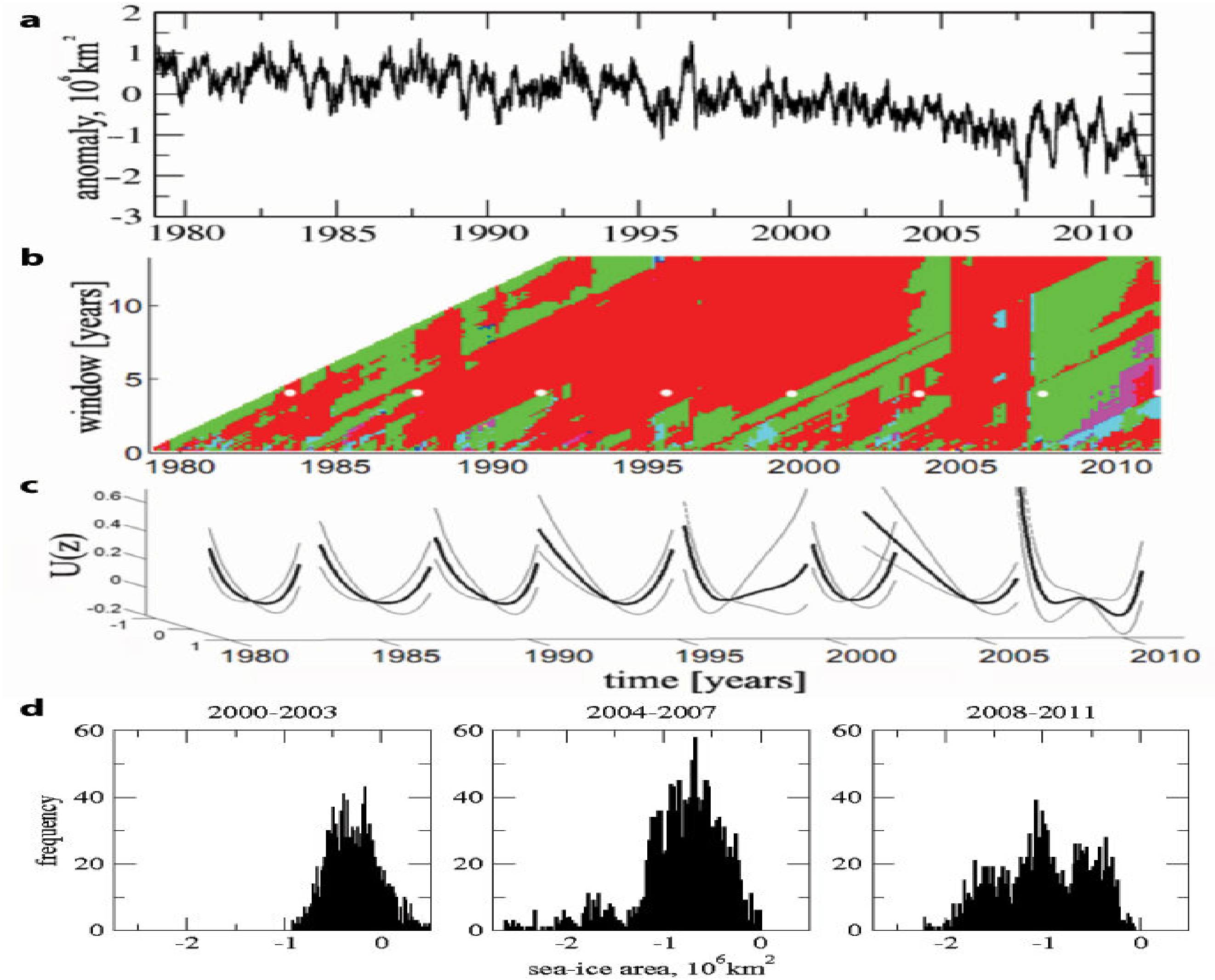
Application of tipping point toolbox: Arctic sea-ice extent



Arctic sea-ice extent: indicators



Arctic sea-ice extent: potential



Work in progress
(jointly with AWI, Germany)

- Visit in February 2012
- More analyses of paleodata
- Estimation of uncertainties
- Potential forecast
- Paper soon to follow

Summary

- **Tipping point toolbox** combines techniques of degenerate fingerprinting and potential analysis
- **Degenerate fingerprinting** with its set of propagators allows us to anticipate tipping points and distinguish climate transitions and bifurcations
- **Potential analysis** provides information about the structure of the system potential, its bifurcations and transitions. The method is useful for in-depth analysis of bifurcations.
- Further applications of the method in statistical physics (Vaz Martins et al,PhysRev E2010. Various time series can be studied.

1) Livina & Lenton, GRL 2007
2) Lenton et al, PhilTrans RoyalSoc 2009
3) Livina et al, Climate of the Past 2010
4) Vaz Martins et al, Phys Rev E
5) Livina et al, Climate Dynamics, 2011
6) Lenton et al, PhilTrans RoyalSoc, in press
7) Livina et al, Physica A, 2011
8) Lenton et al, Clim. Past Discuss, 2012
9) Livina & Lenton, submitted

Thank you!