Early warning signals of intermediate complexity

Steven Lade and Thilo Gross

Max Planck Institute for the Physics of Complex Systems

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- Incorporate data and knowledge with a generalised model
- Critical transition as fold bifurcation*; fast-slow time separation
- Calculate eigenvalues -> warning signal

Generalised modelling

(Detailed simulations)
Global circulation models
Integrated assessment models
Agent based modelling
…

← complementary →

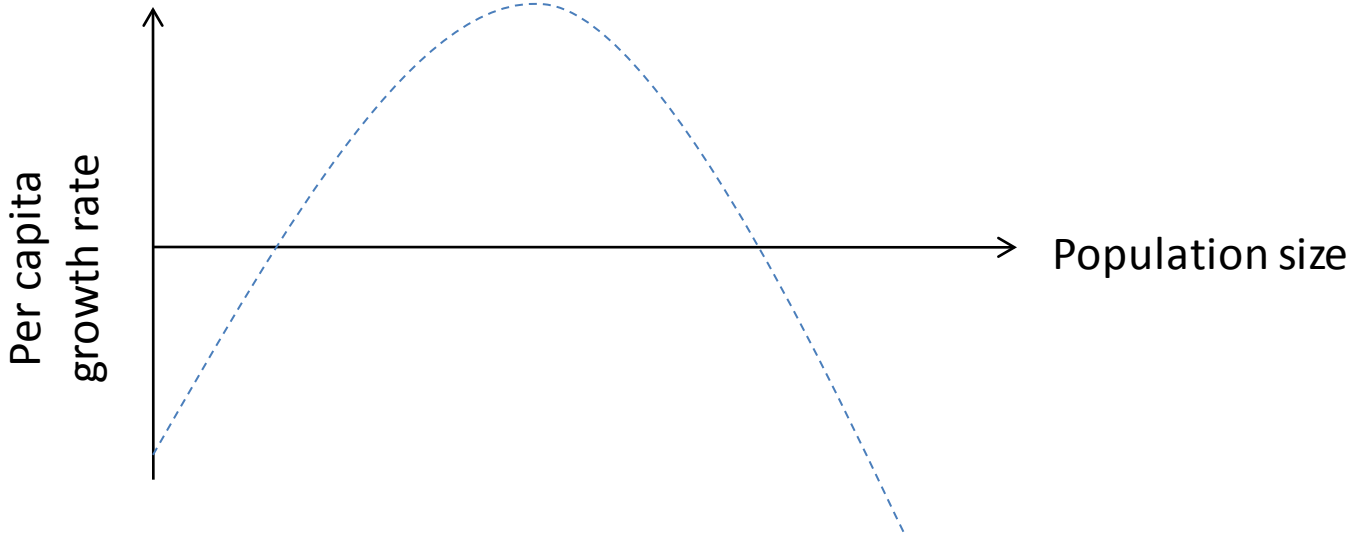
(Time series analysis)
Variance
Autocorrelation
Skewness
…

Outline

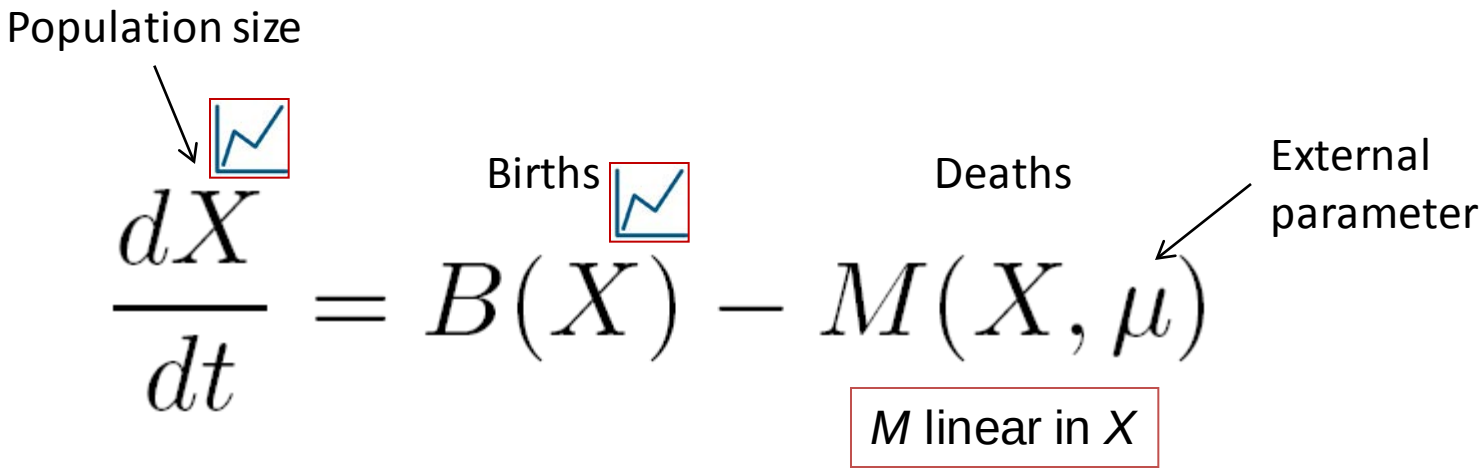
1. Simple growth model with Allee effect (introduction)
2. Fishery model (realistic)
3. Tri-trophic food chain (versatility)

Allee effect

- Per capita growth rate increasing with population size
- Interesting population dynamics… including critical transitions



Generalised model for Allee effect



Distinguish processes, but do not specify functional forms

These two series of measurements and one assumption are sufficient to calculate an early warning signal

Early warning signal for Allee effect

$$\frac{dX}{dt} = B(X) - M(X, \mu)$$

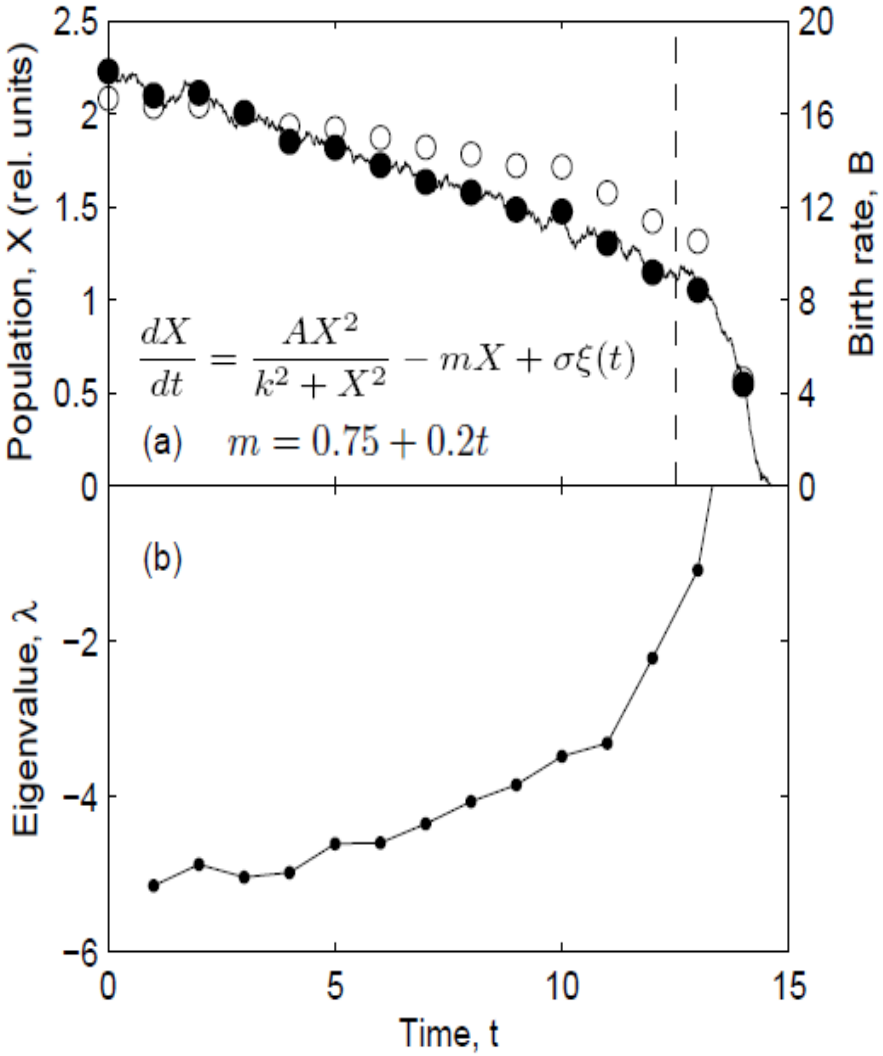
$$\lambda = B'(X) - M'(X, \mu)$$

1. $B'_i = \dfrac{B_i - B_{i-1}}{X_i - X_{i-1}}$
2. $M_i = \dfrac{X_i - X_{i-1}}{t_i - t_{i-1}} - B_i$
3. $M'_i = \dfrac{M_i}{X_i}$
4. $\lambda_i = B'_i - M'_i$

Early warning signal is $\mathrm{Re}(\lambda)$ approching zero from below.

Early warning signal for Allee effect

- Clear early warning signal
- Signal from only 15 time points



Fishery model

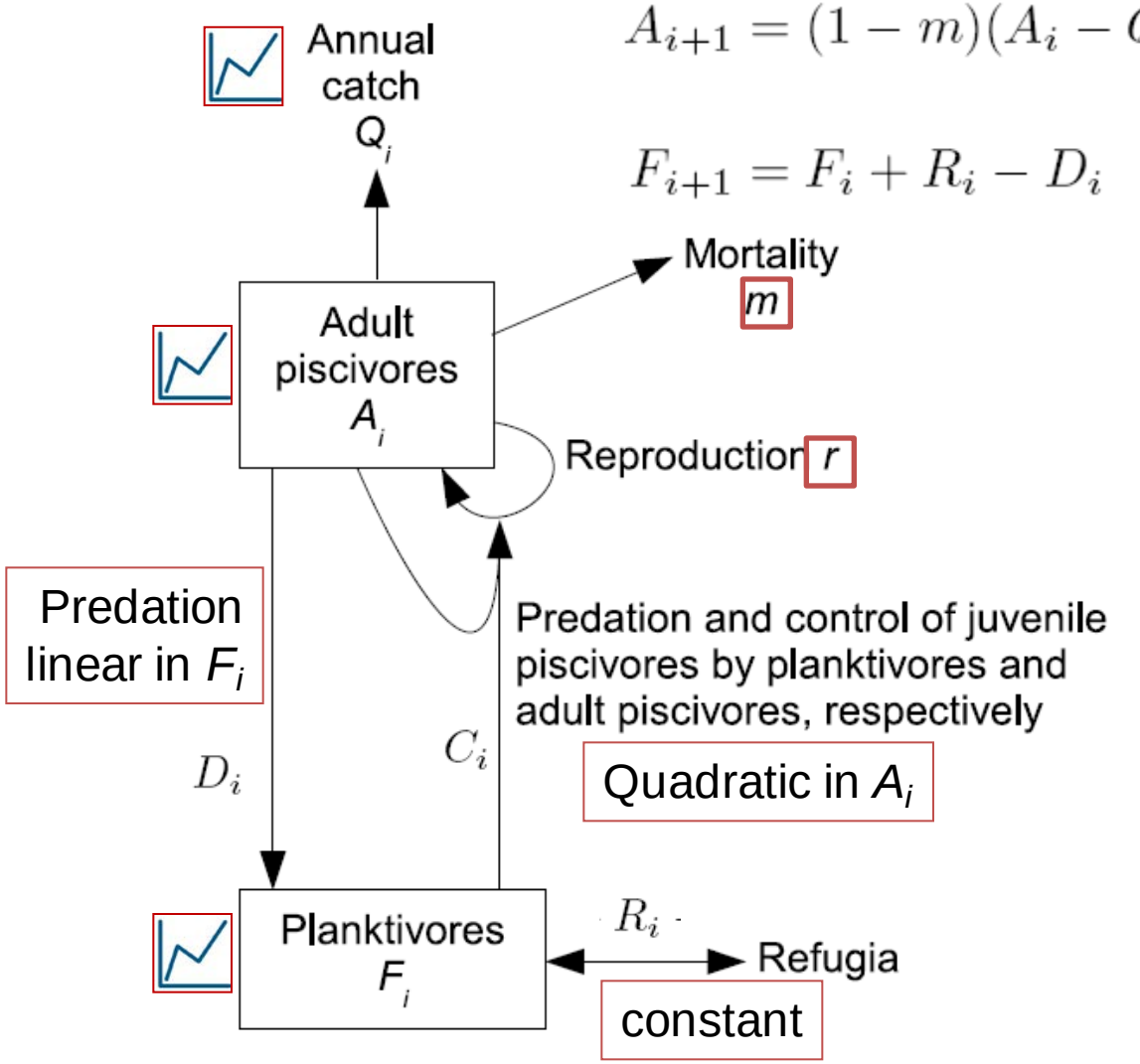
- Piscivores being harvested. Planktivores also important for dynamics.
- Fishery managers are concerned that increased harvesting may cause a collapse from high-piscivore regime to low-piscivore regime

Fishery model

(discrete time)

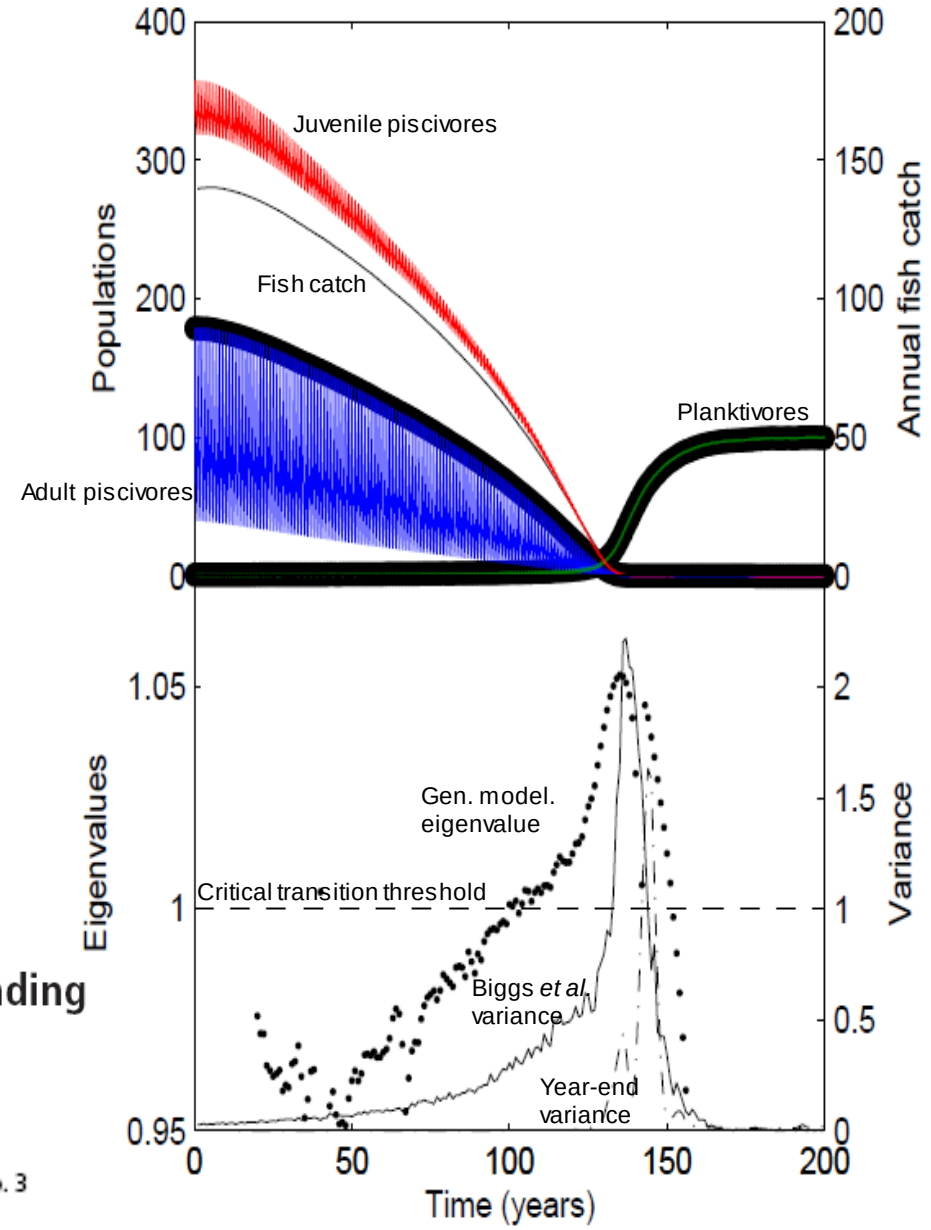
$$A_{i+1} = (1-m)(A_i - Q_i) + rA_i - C_i$$

$$F_{i+1} = F_i + R_i - D_i$$



Fishery collapse

- Biggs *et al.* is a hybrid discrete-continuous model
- Use year-end data
- Polynomial smoothing
- Successful even with explicitly modelling neither juvenile population nor continuous-time intra-year dynamics



Turning back from the brink: Detecting an impending regime shift in time to avert it

Reinette Biggs[a,1], Stephen R. Carpenter[a,2], and William A. Brock[b]

[a]Center for Limnology and [b]Department of Economics, University of Wisconsin, Madison, WI 53706

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Tri-trophic food chain

Linear in X_2

$$\frac{dX_1}{dt} = A(X_1) - G(X_1, X_2)$$

Linear in X_2 Linear in X_3

$$\frac{dX_2}{dt} = G(X_1, X_2) - H(X_2, X_3)$$

Linear in X_3 Linear in X_3

$$\frac{dX_3}{dt} = H(X_2, X_3) - M(X_3, \mu),$$

Extendable to incomplete conversion effciency

$$\frac{dX_1}{dt} = A_n X_1(K_n - X_1) - \frac{BX_1^2 X_2}{K_3 + X_1^2} + \sigma_1 \xi_1(t)$$

$$\frac{dX_2}{dt} = \frac{BX_1^2 X_2}{K_3 + X_1^2} - \frac{A_p X_2^2 X_3}{K_p + X_2^2} + \sigma_2 \xi_2(t)$$

$$\frac{dX_3}{dt} = \frac{A_p X_2^2 X_3}{K_p + X_2^2} - mX_3 + \sigma_3 \xi_3(t),$$

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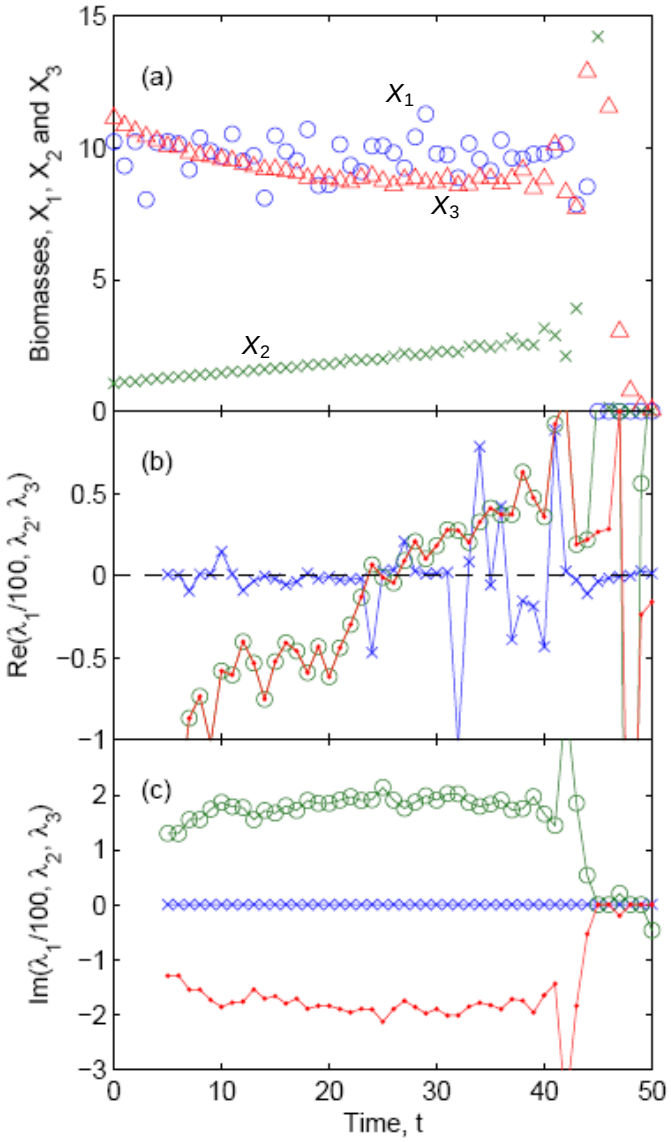
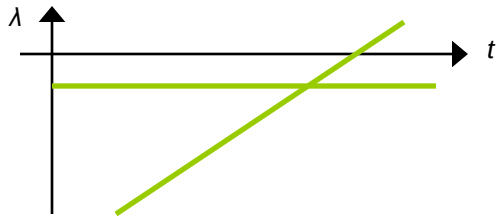
Tri-trophic food chain simulation



- Critical transition
- Clear warning signal
- Bifurcation delay
- Warning of transition
- Gradient-based method can distinguish some types of critical transition

Future work

- Bayesian filtering framework
  - Quality of prior knowledge
  - Compare suitability of generalised models
  - Confidence in an early warning signal
  - Reduce sensitivity to noise
- Normal form calculation (Thompson & Sieber)
  - Calculate probabilities over time of future transitions
- Spectrum of eigenvalues



- Observational data from a past transition?

Conclusions

- Early warning signals of intermediate complexity
- Generalised modelling framework successfully predicted critical transitions
- Based on combining multiple types of data with system-specific knowledge
- Complementary to existing approaches like variance and autocorrelation
- Lade & Gross (2012), *PLoS Comp. Biol.* 8(2) e1002360