Linearization of Finite-Time Processes

A novel approach to the analysis of transient dynamics

Linear Theory

The central concept: hyperbolicity $\cong$ splitting into subspaces of monotonically growing/decaying trajectories

Definition 1 (cf. [Berger, Doan & Siegmund, J Diff Eq, 246(3), 2009]). The linear process Φ admits an *exponential monotonicity dichotomy (EMD) on* $\mathbb{I}$ *(w.r.t.* $|\cdot|$*)* if

$$\exists\, Q \text{ projection in } \mathbb{R}^n: \quad \overline{\lambda}(\operatorname{im} Q, \Phi) < 0 < \underline{\lambda}(\ker Q, \Phi).$$

Results

Hyperbolicity/EMD on $\mathbb{I}$ is robust in the linear processes on $\mathbb{I}$, in the linear r.h.s. of ODEs and in the initial values of general ODEs (with the same projection), see Fig. 1

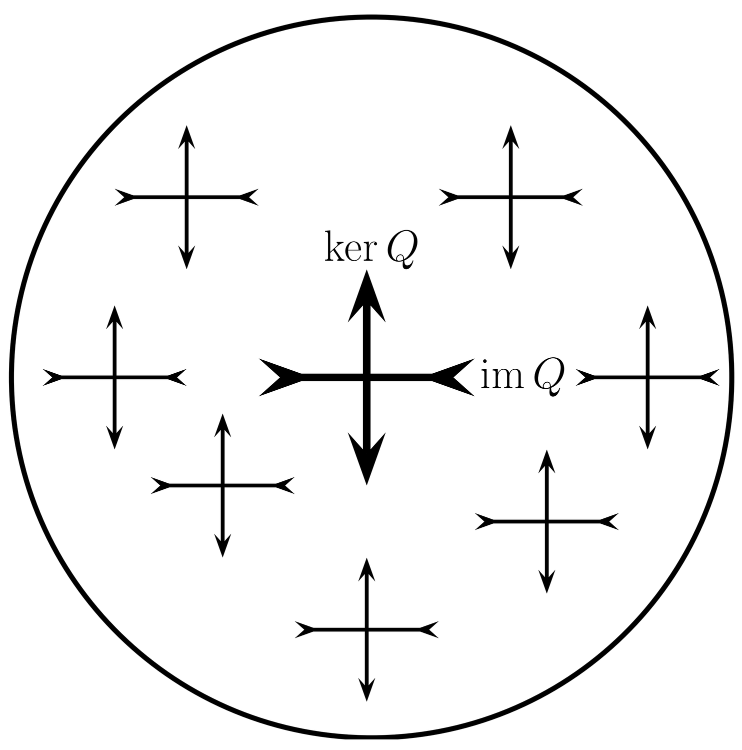

Fig. 1: Robustness Theorem (for initial values)

Hyperbolicity radius, stability radius for processes and linear r.h.s. of ODEs

Spectral Theorem for linear processes, cf. [Doan, Palmer & Siegmund, J Diff Eq, 250(11), 2011]

Linearization Theory

Problem: What can be concluded from properties of the linearized system on local properties of the nonlinear system?

Results

Local Stable / Unstable Manifold Theorem: stable and unstable subspaces are locally contained in the domain of attraction and repulsion, resp., see Fig. 2

Local Stable / Unstable Cone Theorem: stable and unstable cones are locally contained in the domain of attraction and repulsion, resp., see Fig. 2

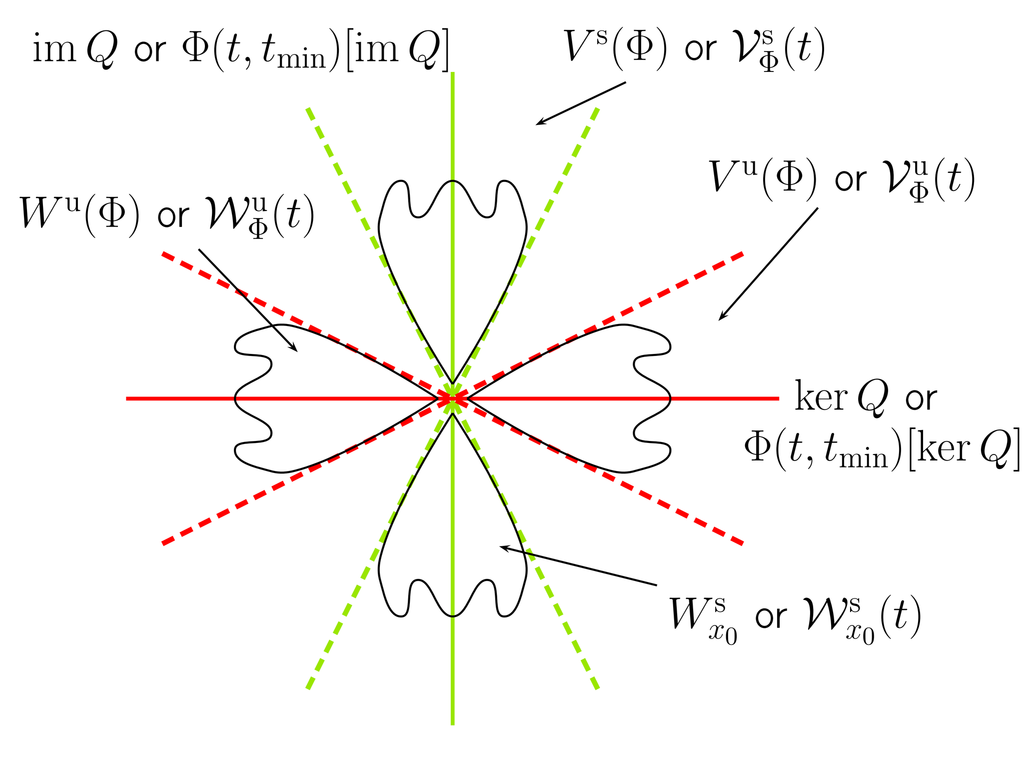

Fig. 2: Local Stable Manifold / Cone Theorem

Theorem of Linearized Finite-Time Attraction / Repulsion: trajectories with attracting and repelling linearization attract and repell their neighborhood, resp.

Hartman-Grobman-like Theorem: the Local Stable / Unstable Cone Theorem holds for each time-fiber of the respective extensions

Definitions

- $\mathbb{I} \subset \mathbb{R}$ – compact (time-set), $t_{\min} := \min \mathbb{I}$
- $\Phi\colon \mathbb{I} \times \mathbb{I} \times \mathbb{R}^n \to \mathbb{R}^n$ – linear process on I, with $t \mapsto \Phi(t, t_{\min}) \in L(\mathbb{R}^n)$ Lipschitz continuous
- *Logarithmic difference quotient:*

$$\Delta(|\Phi(\cdot, t_{\min})x|)(t, s) := \frac{\ln|\Phi(t, t_{\min})x| - \ln|\Phi(s, t_{\min})x|}{t - s}$$

- *Growth rates:* $X \subseteq \mathbb{R}^n$ – subspace:

$$\underline{\lambda}(X, \Phi) := \inf\{\Delta(|\Phi(\cdot, t_{\min})x|)(t, s);\ t, s \in \mathbb{I},\ t \neq s,\ x \in X\},$$
$$\overline{\lambda}(X, \Phi) := \sup\{\Delta(|\Phi(\cdot, t_{\min})x|)(t, s);\ t, s \in \mathbb{I},\ t \neq s,\ x \in X\},$$

  are called *lower* and *upper growth rates of* X *under* Φ, resp.
- Δ is continuous in x, t, s; $\underline{\lambda}, \overline{\lambda}$ are continuous in X and Φ.
- Linear processes are metrized via supremum distance of trajectories and of growth rates (quasi C^1-distance)

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