Intermittent transformative non-stationary dynamics in complex systems.

Henrik Jeldtoft Jensen
Complexity & Networks Group
and
Department of Mathematics

Collaborators:

Tangled Nature

Collaborators:

Paul Anderson, Kim Christensen, Simone A di Collobiano, Matt Hall, Dominic Jones, Simon Laird, Daniel Lawson, Paolo Sibani

Tangled Economy

David Robalino, Xiaoye Chen

Nicky Zachariou, Kim Christensen, Eduardo Vigas, Misako and Hideki Takayasu

Outline

1. Complex systems dynamics
2. Generic model - ecology as paradigm
3. Intermittency
4. Record dynamics
5. Economics - current investigations
6. Summary & conclusions

Follows the combined O'Keeffe-Einstein principle

O'Keeffe:

Nothing is less real than realism. Details are confusing. It is only by selection, by elimination, by emphasis, that we get at the real meaning of things.

Einstein:

Make everything as simple as possible, but not simpler.

The essential characteristics of a complex system

System

- Many interacting components
- Emergence
- Evolution

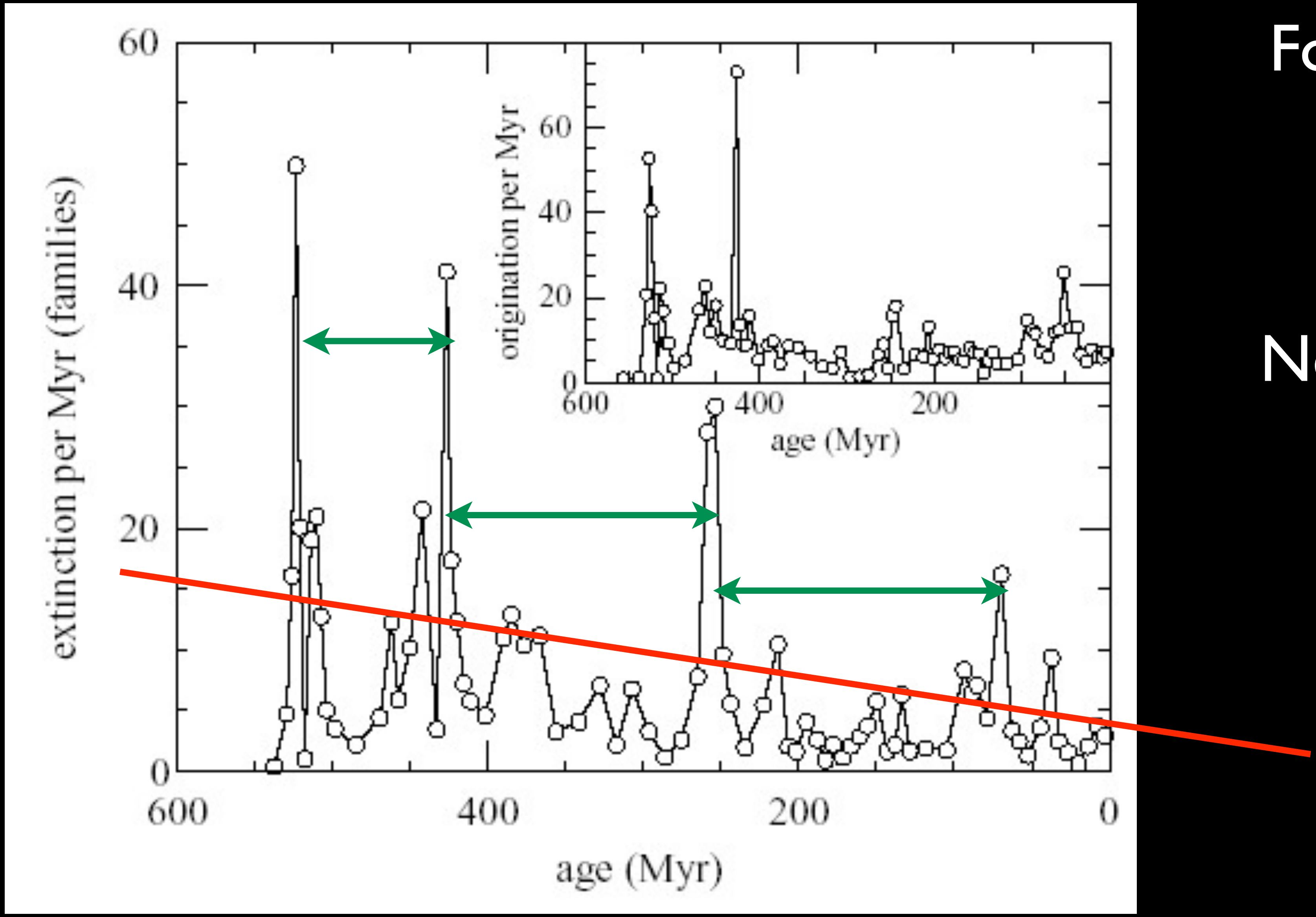
Math model

- High dimensional
- Multiple levels
- Non-stationary

Models

- Individuals interacting according to type, *S*, and current environment
- Fixed rate of micro dynamics
- Emergent macro dynamics: $n(S,t)$
    - Intermittency
    - Adaptation
    - Evolving network structures
- Topology: E.g. degree distribution, connectance
- Stability

Non-stationary macro dynamics

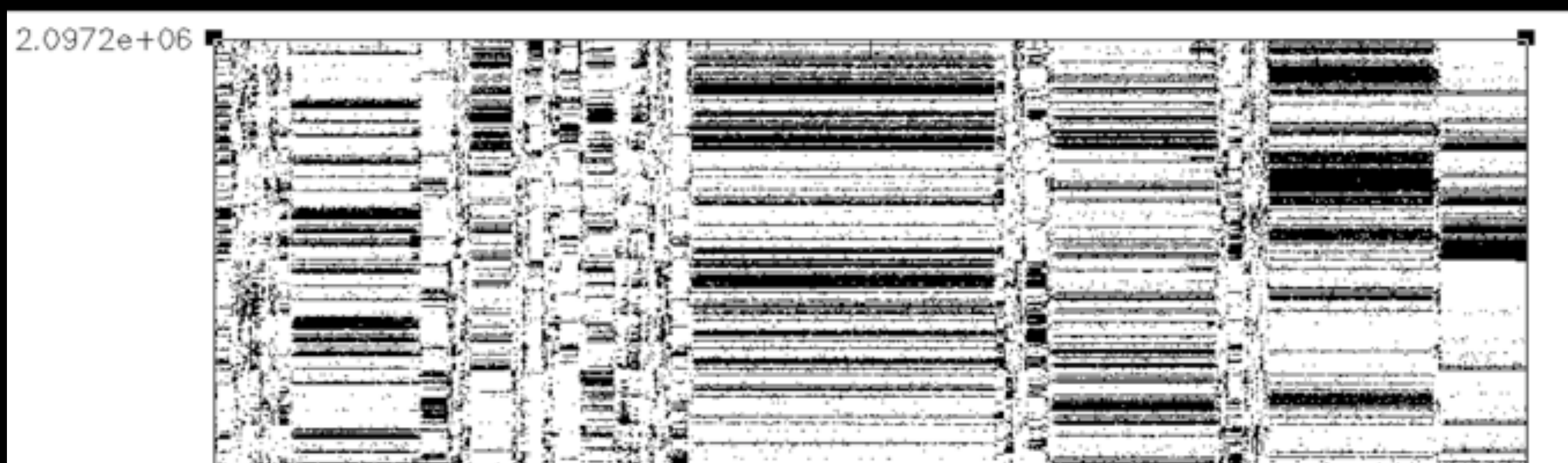


Fossil record: Decreasing extinction rate.

From:

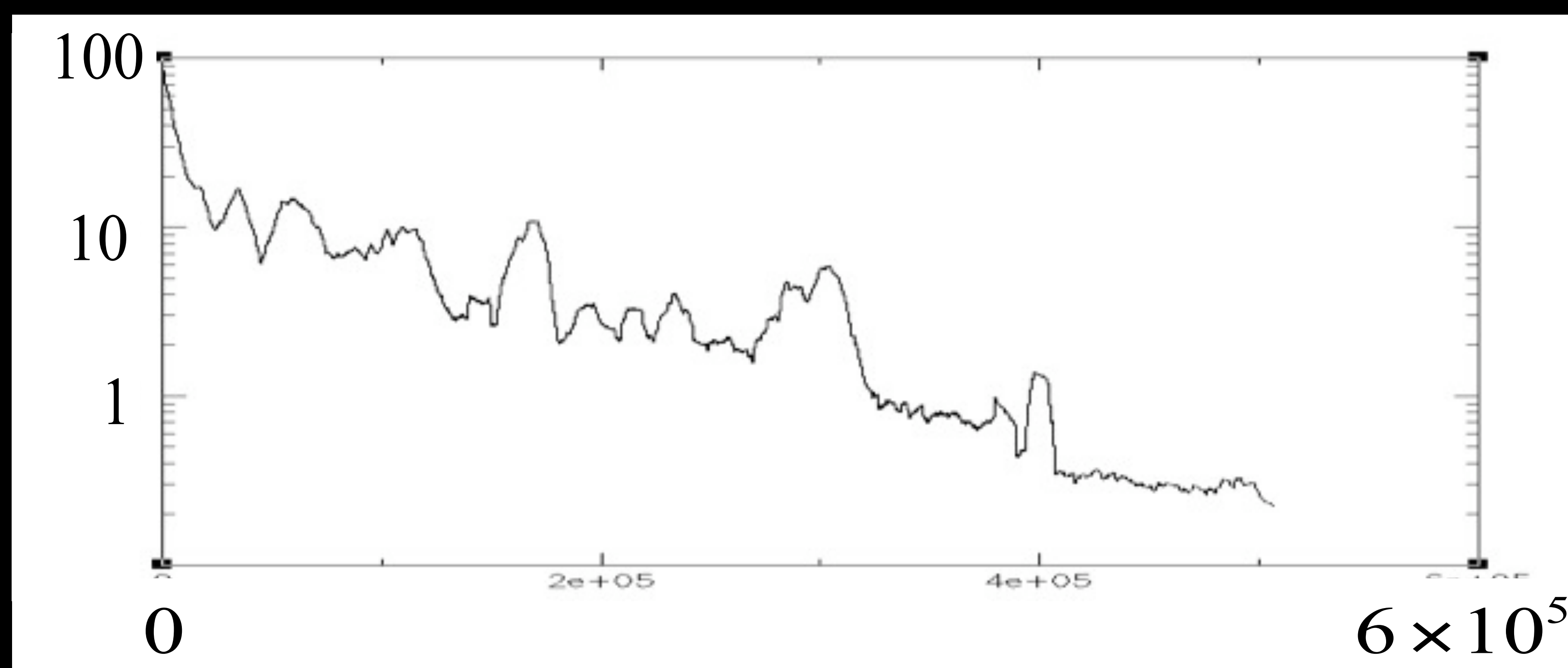
Newman and Sibani, Proc. Roy. Soc. B. **266**, 1593 (1999)

Intermittency & decreasing extinction rate

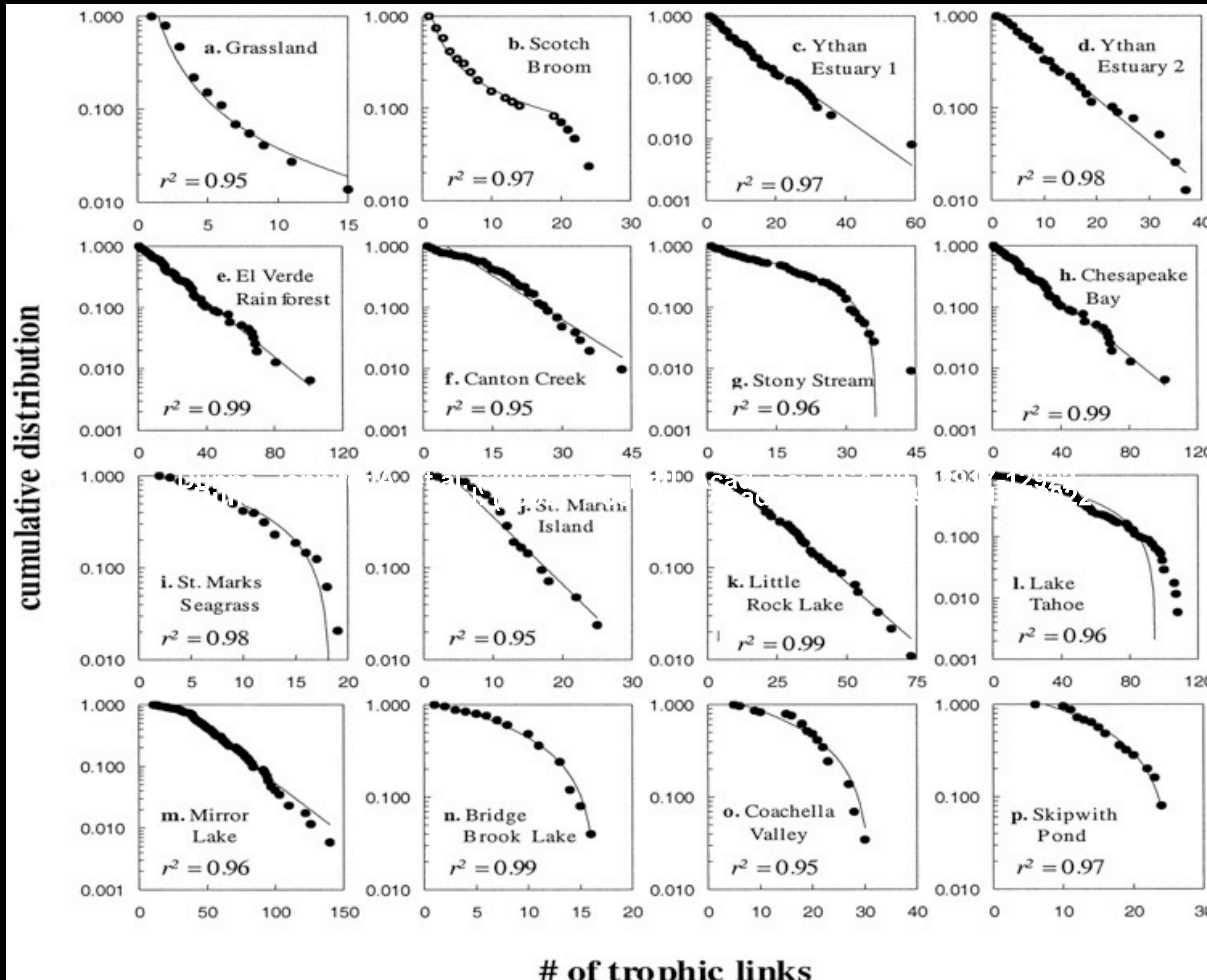


Matt Hall

# of transitions in window



Degree distribution



Dunne, Jennifer A. et al. (2002) Proc. Natl. Acad. Sci. USA 99, 12917-12922

The evolved degree distribution

Correlated

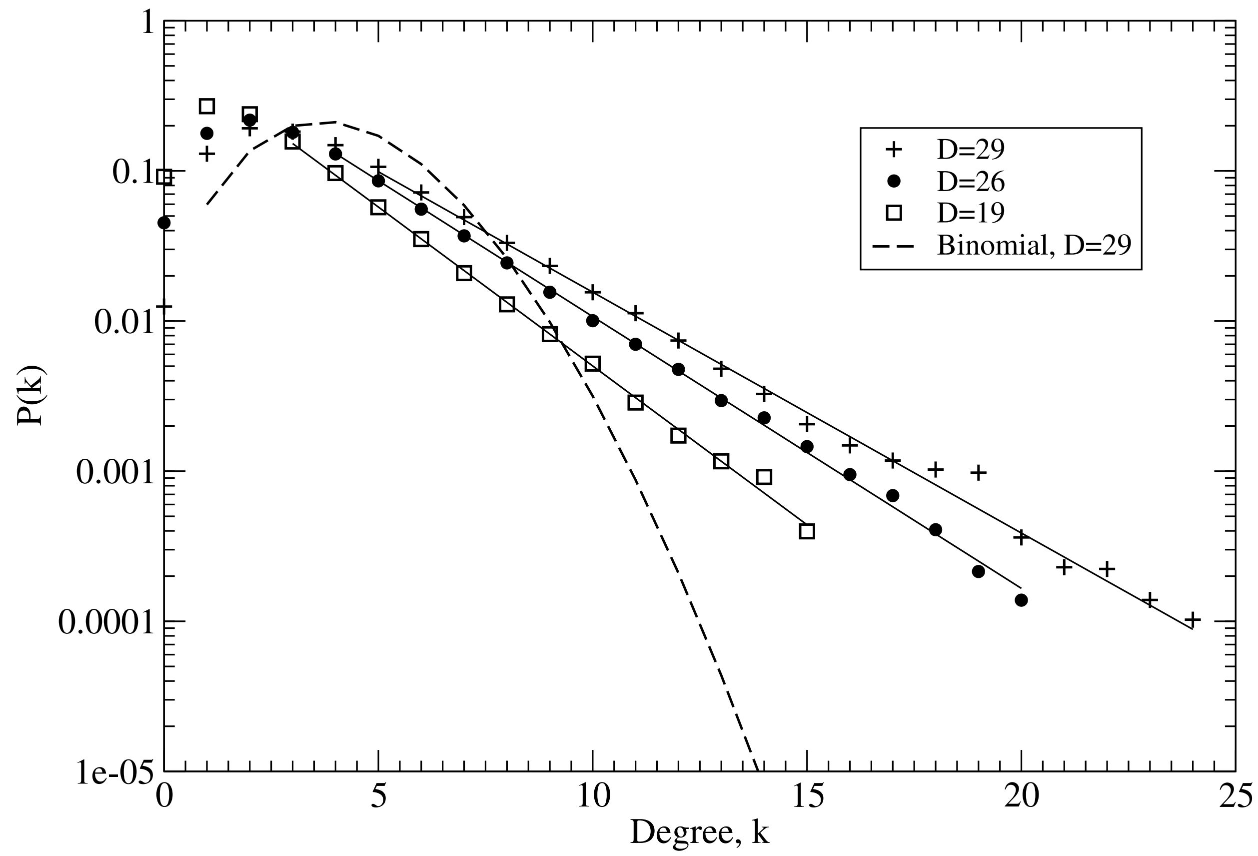
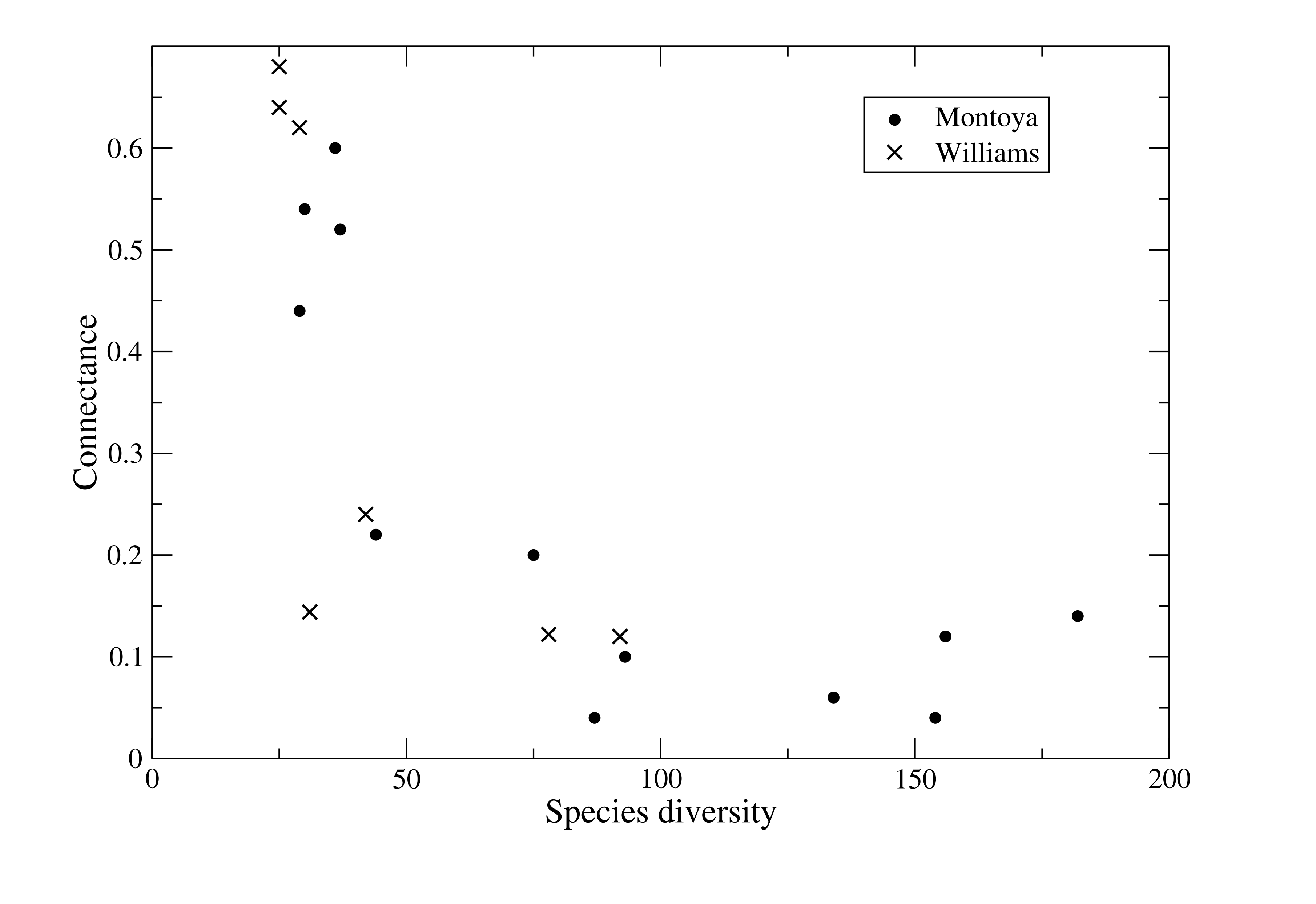


Figure 1: Degree distributions for the Tangled Nature model simulations. Shown are ensemble averaged data taken from all networks with diversity, $D = \{19, 26, 29\}$ over 50 simulation runs of 10^6 generations each. The exponential forms are highlighted by comparison with a binomial distribution of $D = 29$ and equivalent connectance, $C \simeq 0.145$ to the simulation data of the same diversity.

Exponential becomes 1/k in limit of vanishing mutation rate

From Laird & Jensen, Ecol. Model. In Press
See also Laird & Jensen, EPL, **76**, 710 (2006)

Connectance



Montoya JM, Sole RV *Topological properties of food webs: from real data to community assembly models*, OIKOS **102**, 614-622 (2003)

Williams RJ, Berlow EL, Dunne JA, Barabasi AL, Martinez ND *Two degrees of separation in complex food webs*, PNAS **99**, 12913-12916 (2002)

The evolved connectance

Correlated

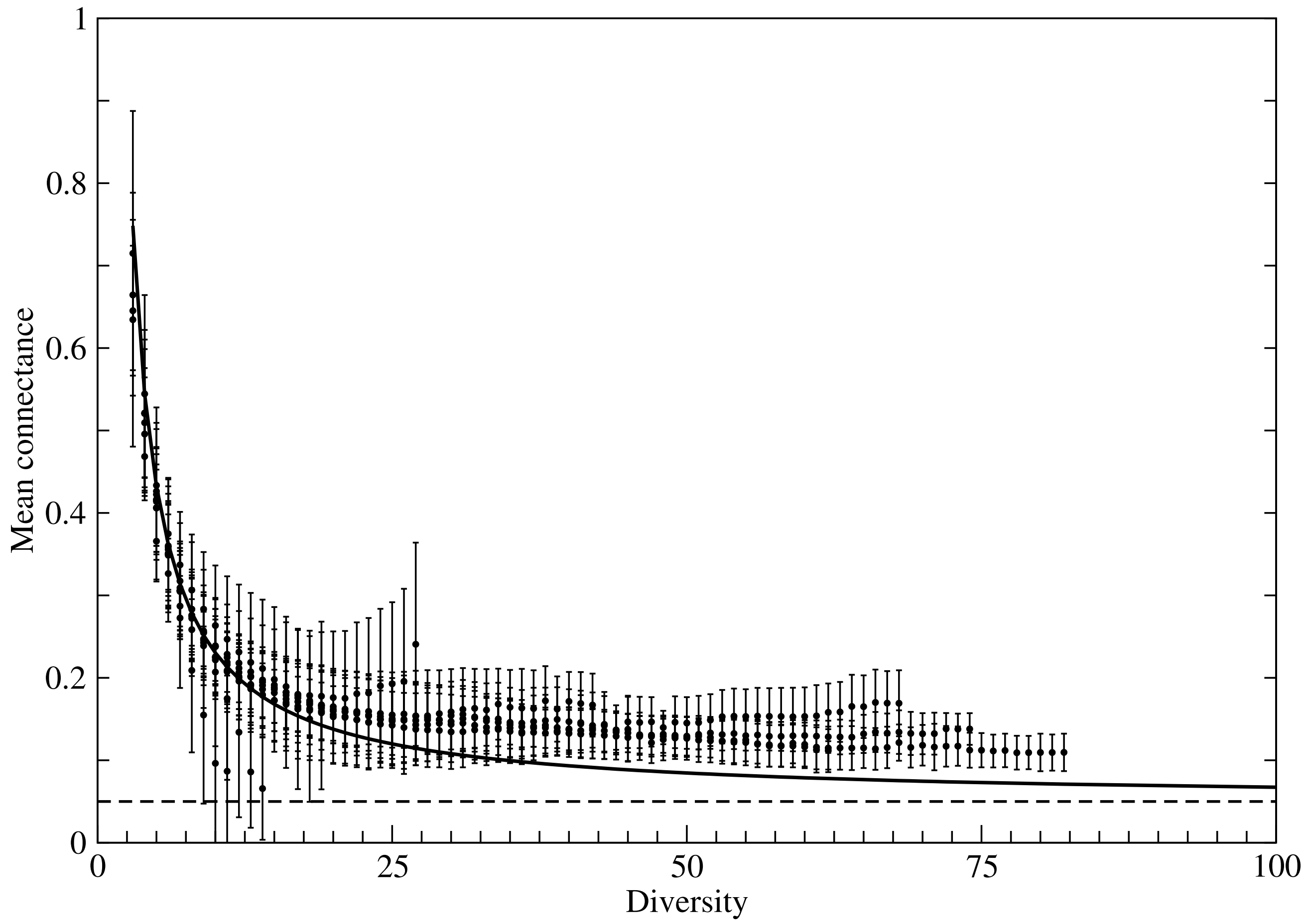


Figure 4: Plot of ensemble-averaged mean connectances, $< C >$ against species diversity. Error bars represent the standard error. The lower dotted line marks the null system connectance, $C_J = 0.05$, which the evolved systems clearly surpass. The overlaid functional form is that given by Eq.(8) using the correct background connectance, $C_J = 0.05$ and with a value of, $s = 5.5$ for the selection parameter.

From Laird & Jensen, Ecol Compl. **3**, 253 (2006)

Model

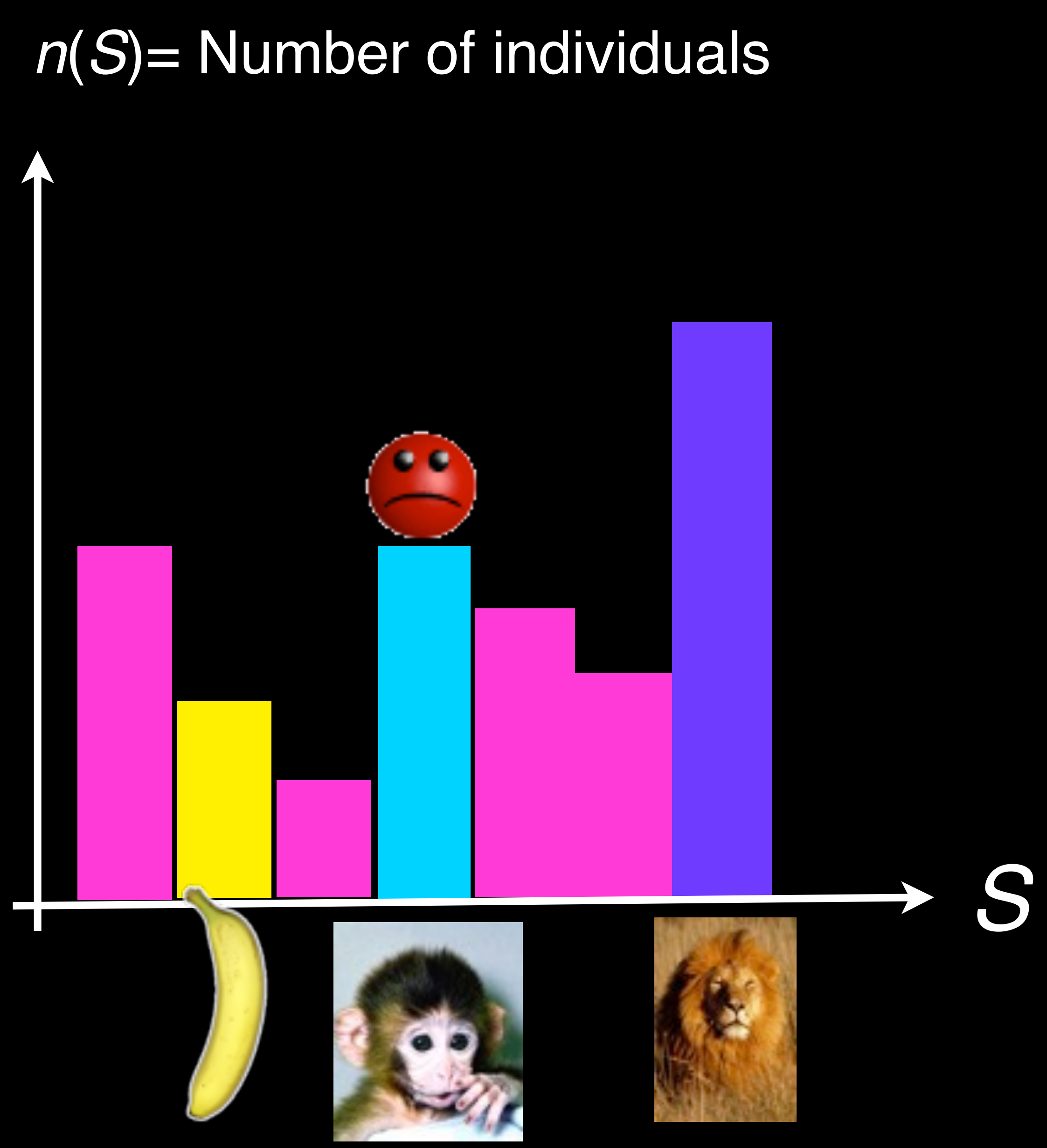
Collaborators:

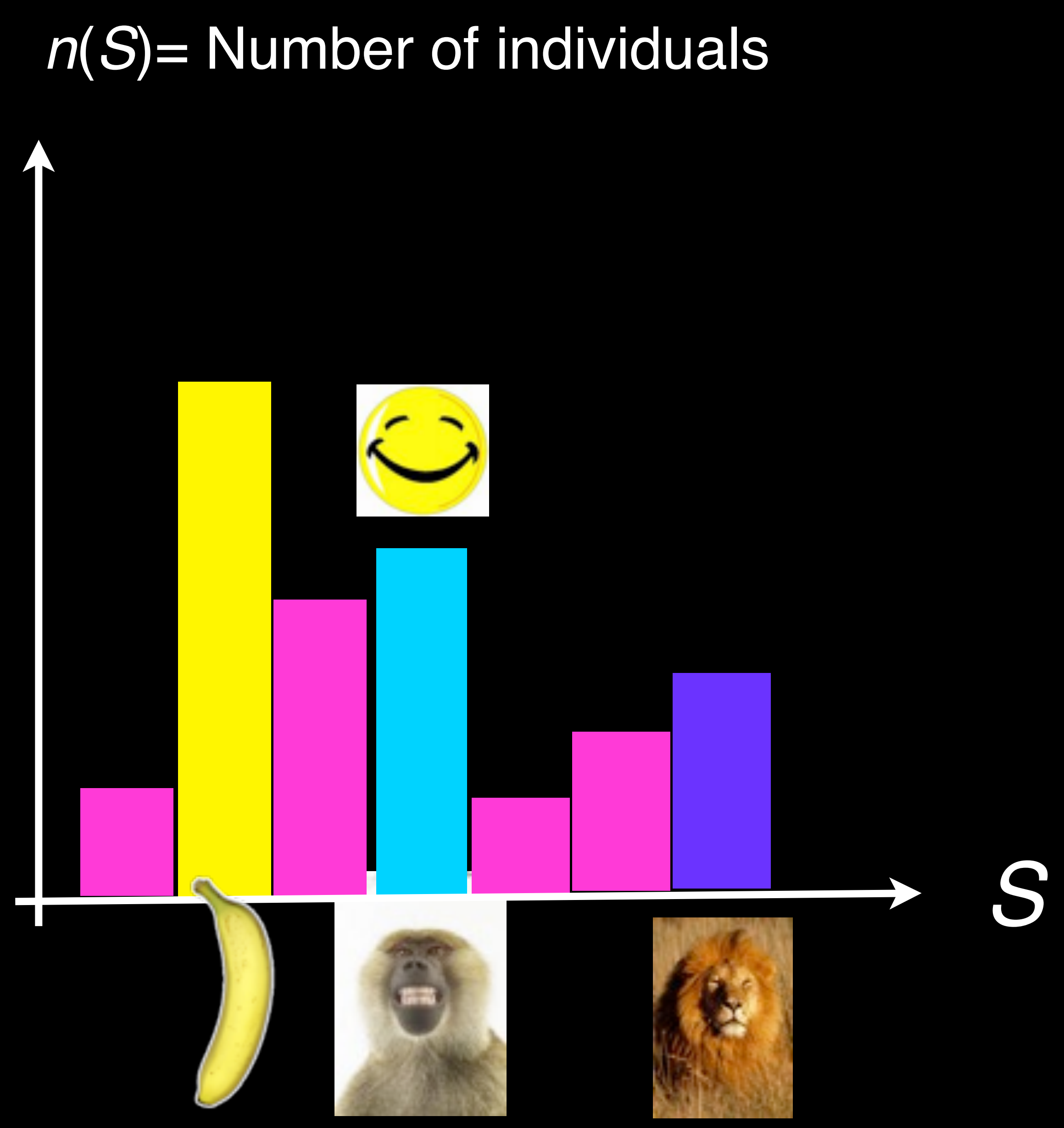
Simon Laird, Daniel Lawson, Paul Anderson, Kim Christensen, Matt Hall, Simone A di Collobiano, Paolo Sibani, Dominic Jones, Andrea Cairoli

Tangled Nature - paradigm

Individuals reproducing in type space

Your success depends on who you are amongst

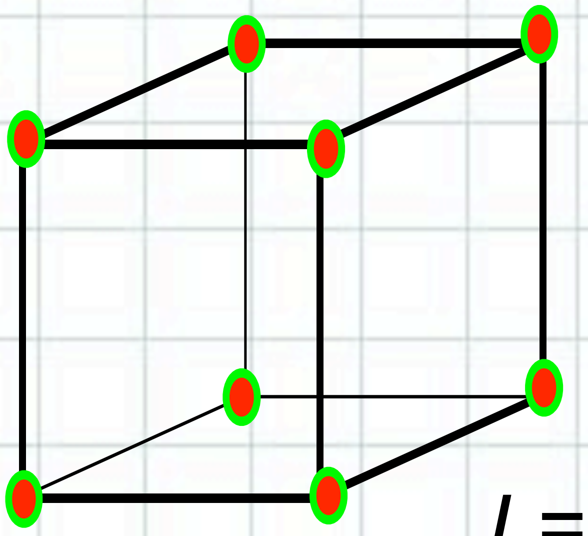




Definition

Individuals $\mathbf{S}^{\alpha} = (S_1^{\alpha}, S_2^{\alpha}, \ldots, S_L^{\alpha})$, where $S_i^{\alpha} = \pm 1$

and $\alpha = 1,2,\ldots, N(t)$



Dynamics – a time step

Annihilation

Choose indiv. at random, remove with probability

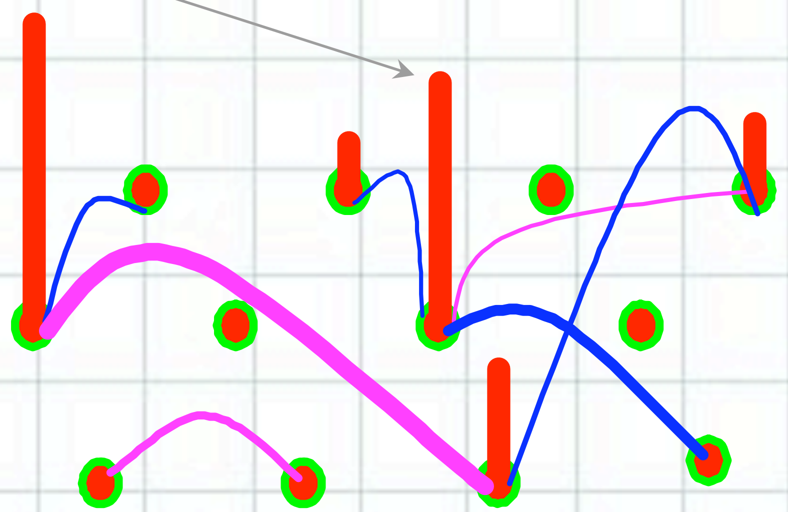
$$p_{kill} = const$$

Reproduction:

- Choose indiv. at random
- Determine

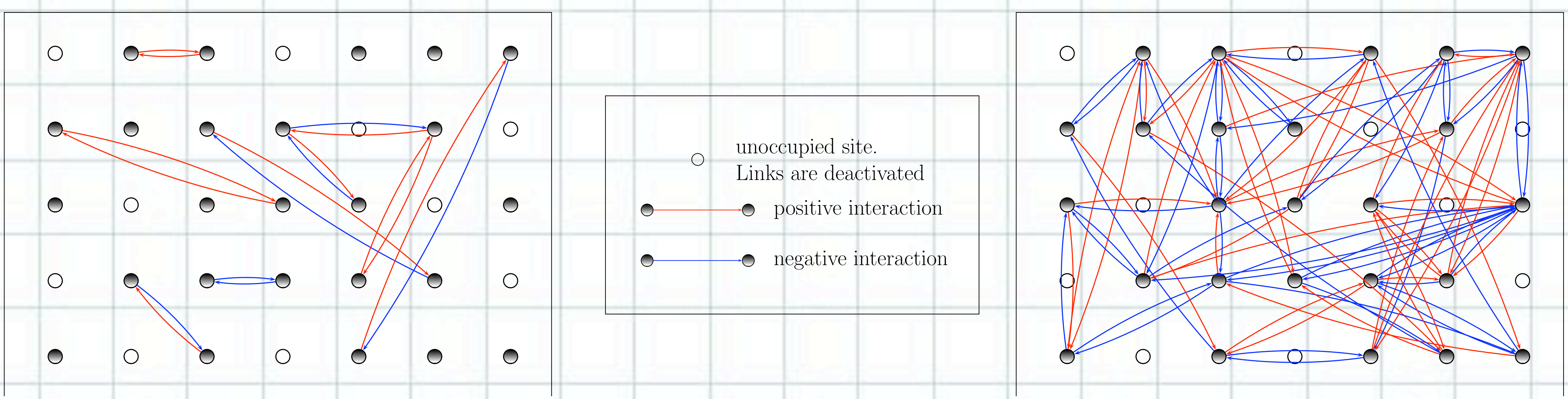
$$H(\mathbf{S}^\alpha, t) = \frac{k}{N(t)} \sum_{\mathbf{S}} J(\mathbf{S}^\alpha, \mathbf{S}) n(\mathbf{S}, t) - \mu N(t)$$

$n(\mathbf{S}, t) =$ occupancy at the location $\mathbf{S}$



The coupling matrix J(S,S’)

- Either consider J(S,S’) to be uncorrelated
- or to vary smoothly through type space
- and sparse or dense



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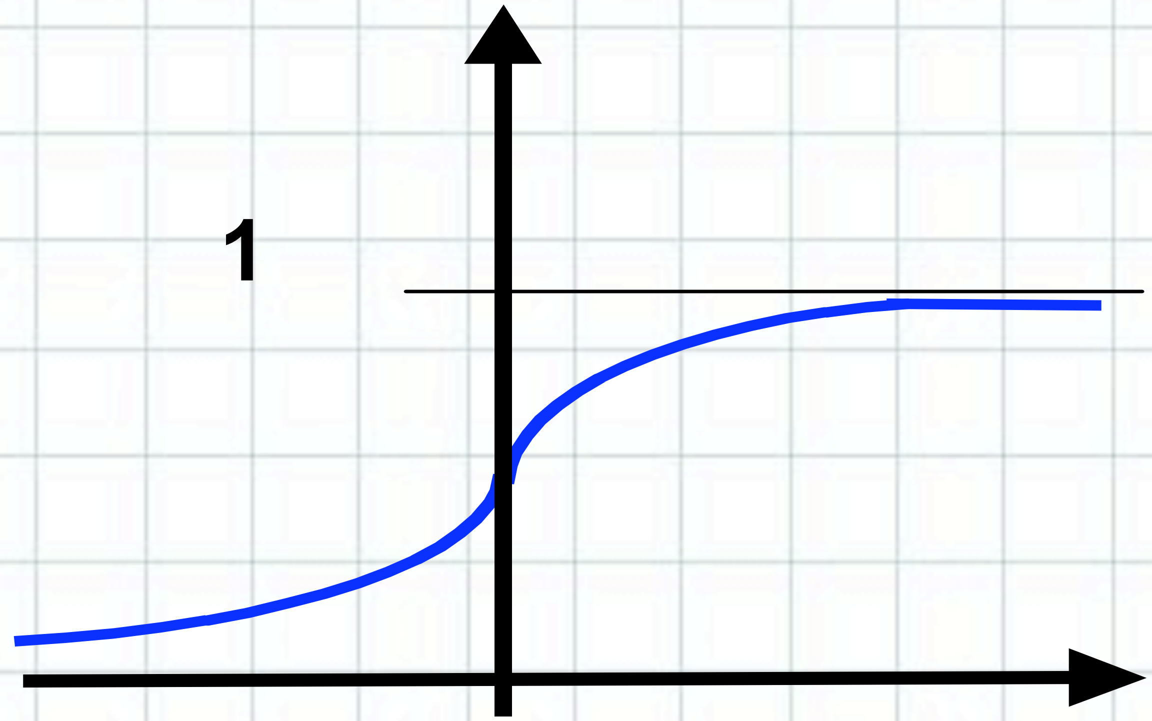
Imperial College London

$H(\mathbf{S}^{\alpha}, t)$ → reproduction probability

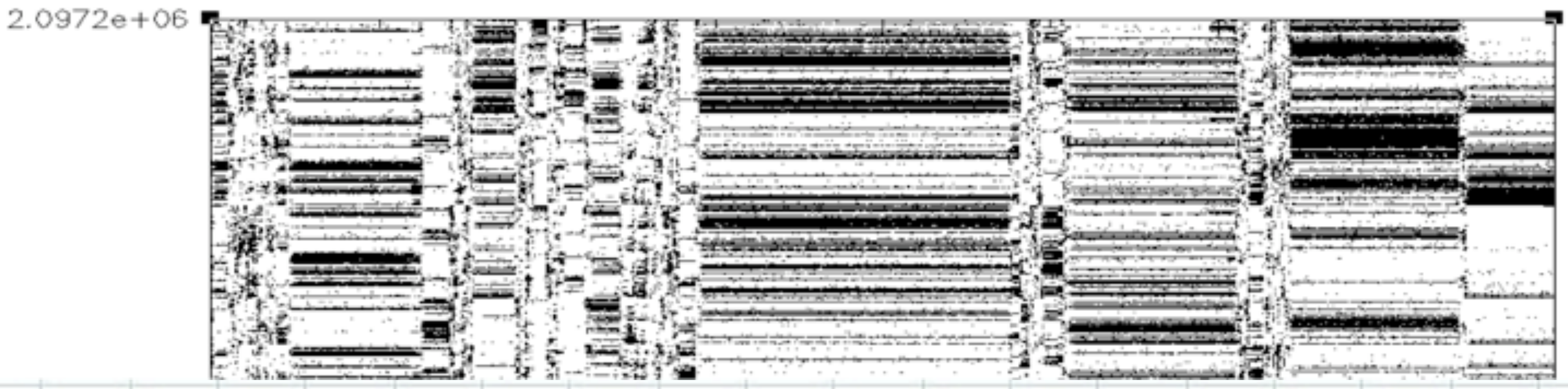
Smooth function $\mathbb{R} \rightarrow [0, 1]$

$$p_{off}(\mathbf{S}^{\alpha}, t) = \frac{\exp[H(\mathbf{S}^{\alpha}, t)]}{1 + \exp[H(\mathbf{S}^{\alpha}, t)]} \in [0, 1]$$



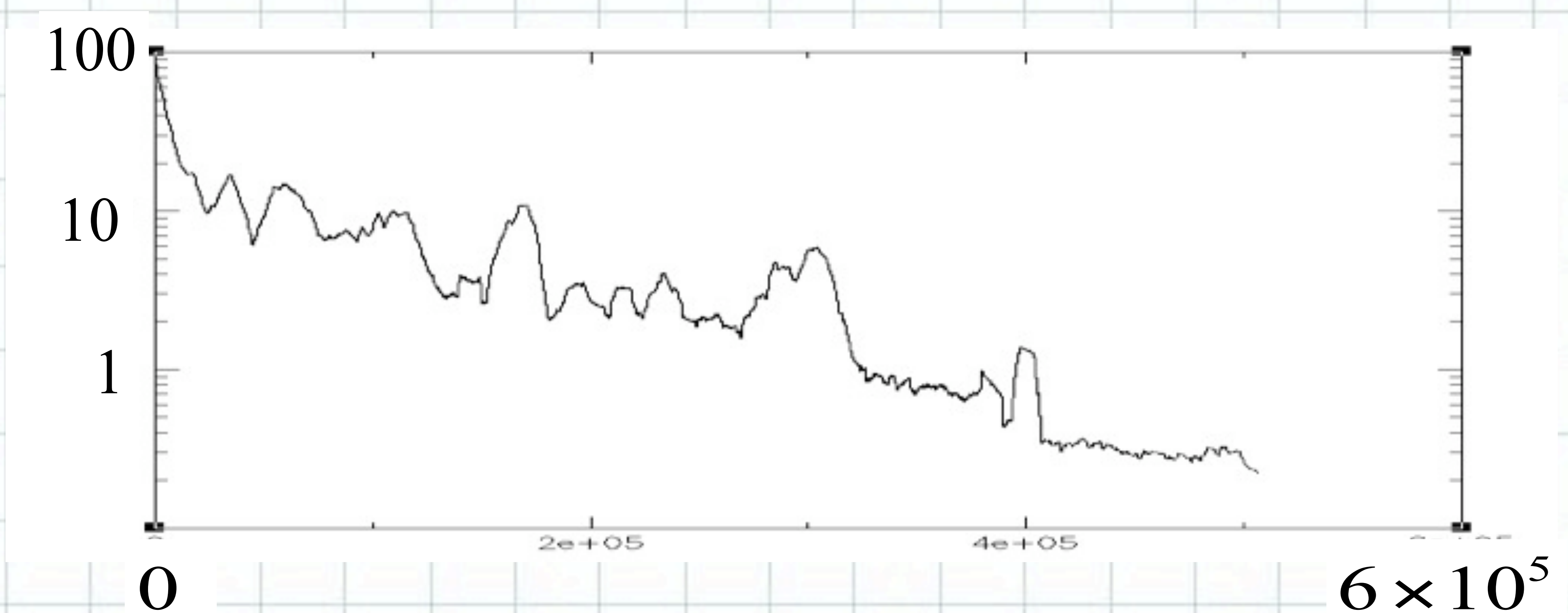
$H(\mathbf{S}^{\alpha}, t)$

Intermittency:



# of transitions in window

Matt



1 generation =

$N(t)/p_{kill}$

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Imperial College London

Time evolution of

Distribution of active coupling strengths

Paul Anderson

Non correlated

Low connectivity

High connectivity

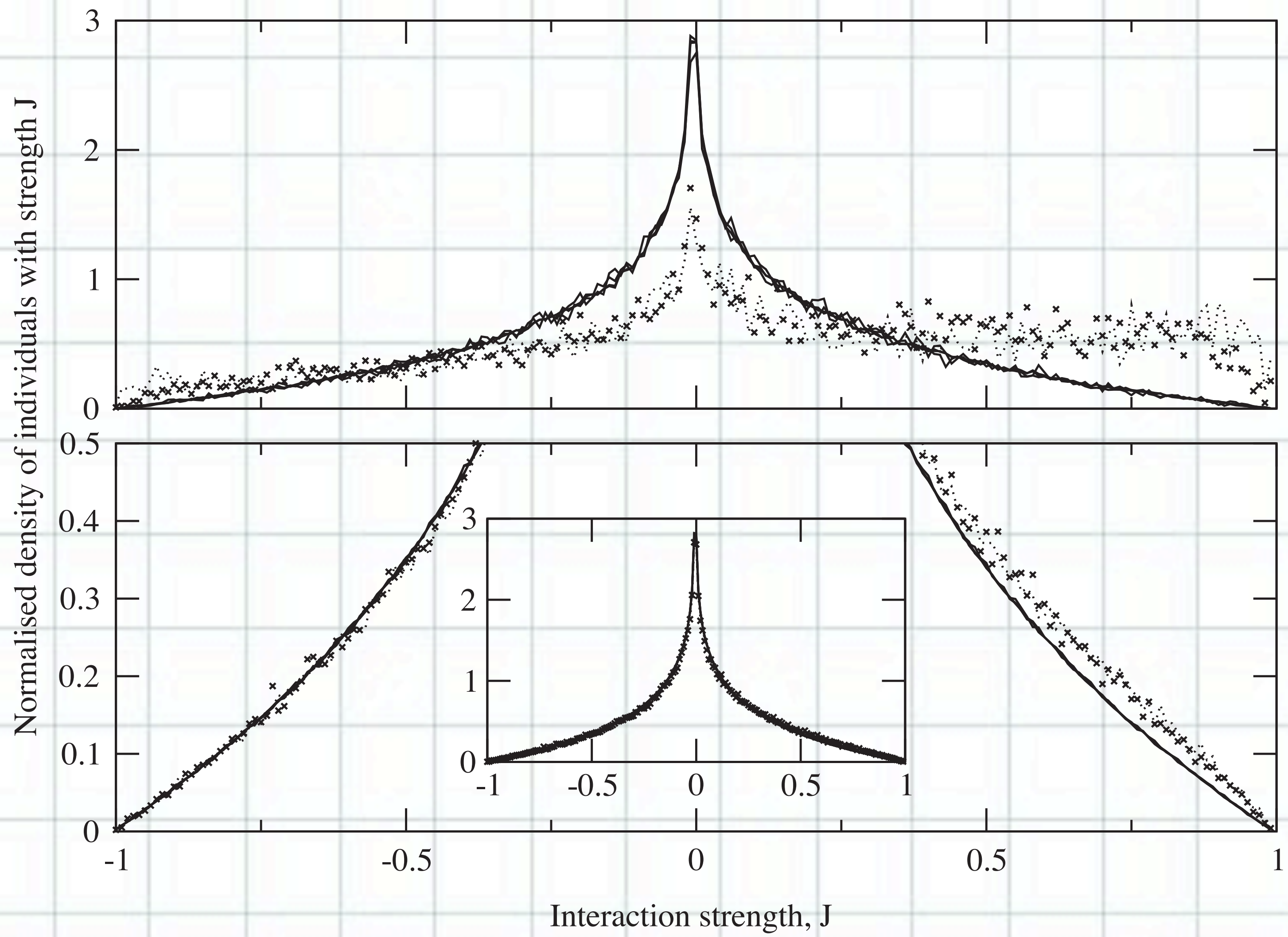
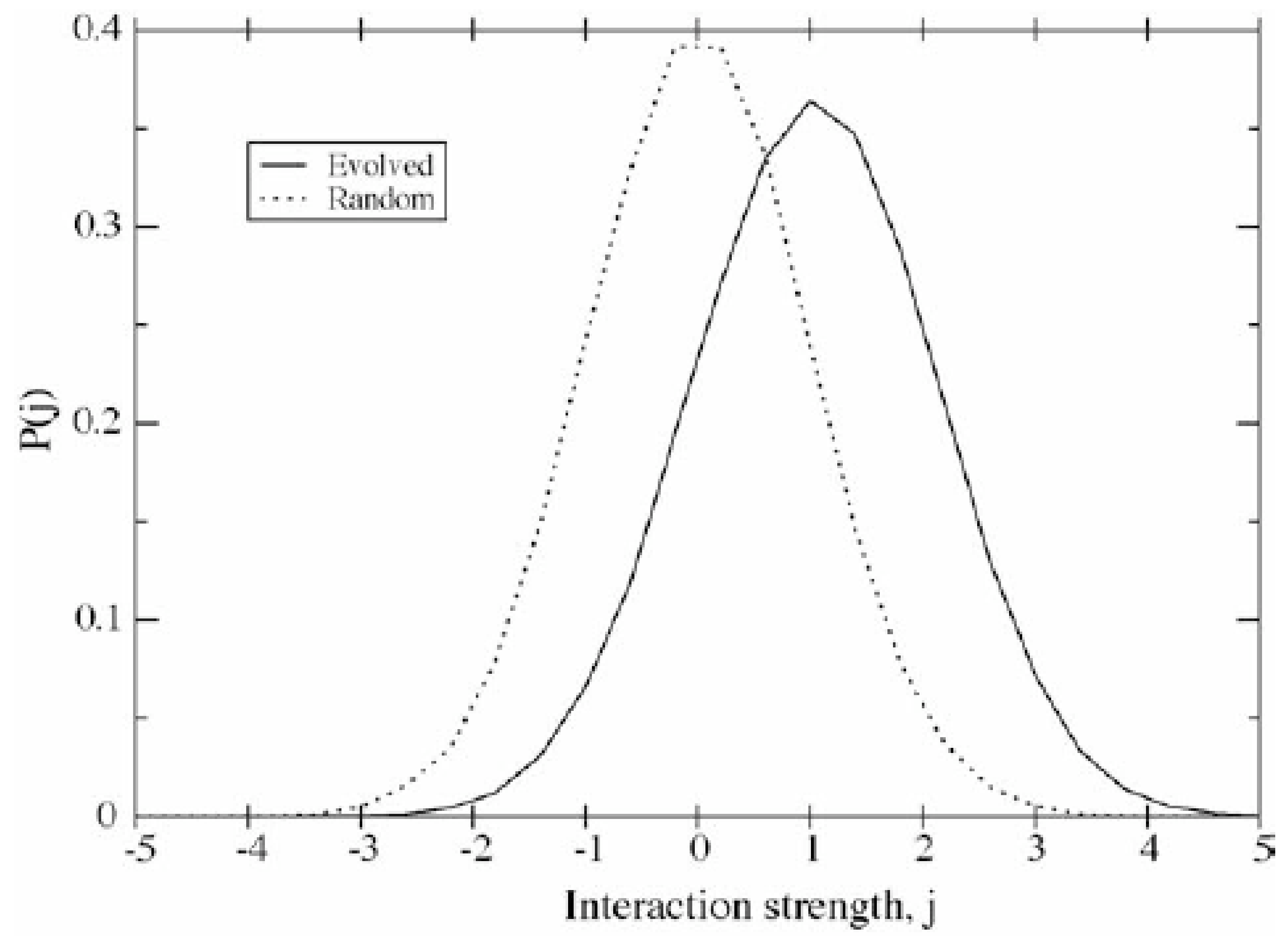


Fig. 3. Interaction distributions. Top: Distribution of interaction strengths between individuals for $\theta = 0.005$. Bottom: $\theta = 0.25$. Inset: Entire distribution. Solid lines, random; crosses, simulation at $t = 500$; dotted lines, simulation at $t = 500,000$. All plots are normalized so that their area is one. For high θ, a significant increase in positive interactions is seen. For low θ, a change is seen but for trivial reasons.

Time evolution of

Distribution of active coupling strengths

Correlated



High connectivity

Simon Laird

Time evolution of

Species abundance distribution

Non Correlated

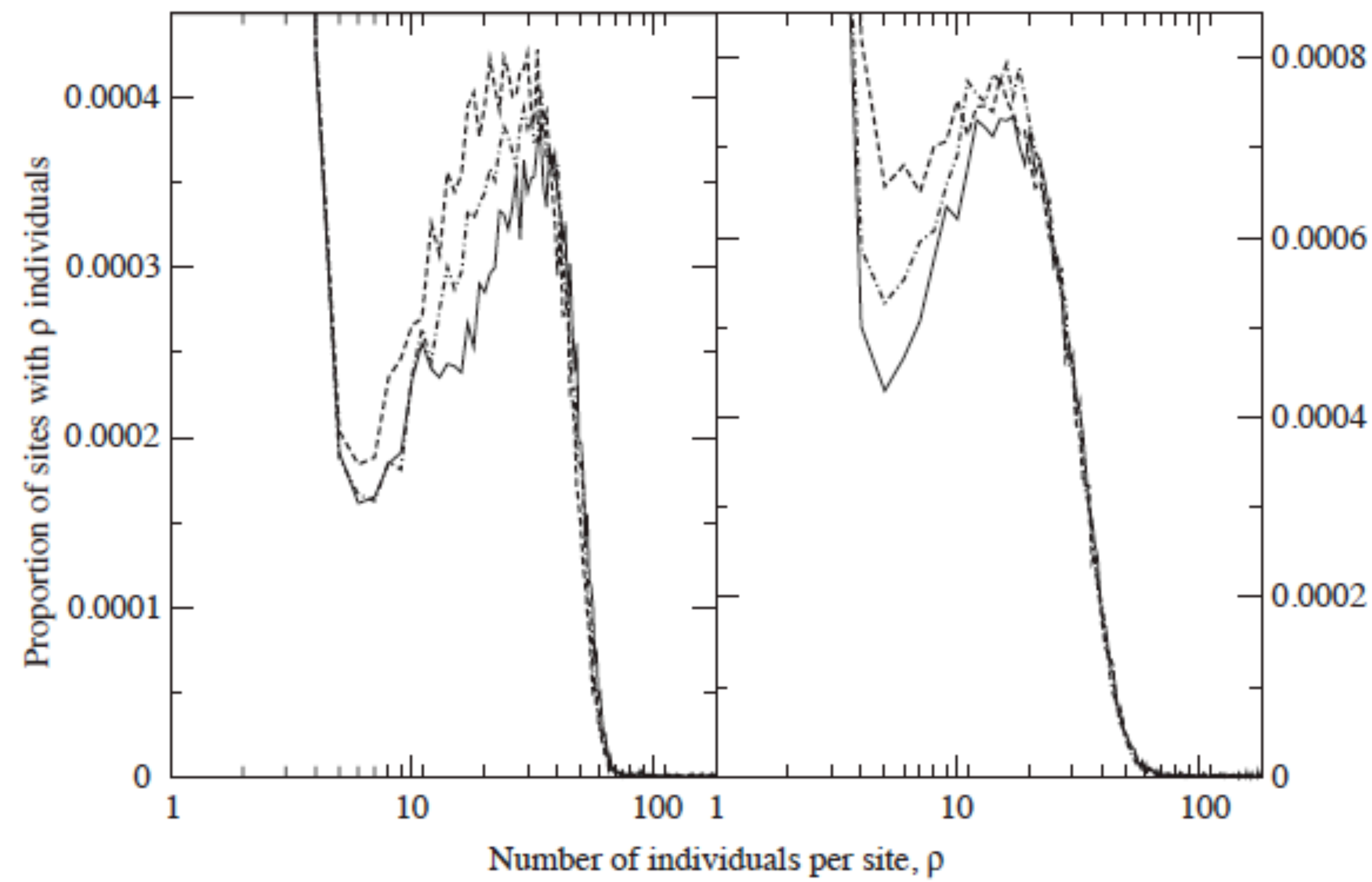


Fig. 5. Species abundance distributions. Species abundance distributions for the simulations only. Dashed line, $t = 500$; dashed–dotted line, $t = 5000$; solid line, $t = 500,000$. Low θ on the left, high θ on the right. The ecologically realistic log-normal form is only seen for high θ.

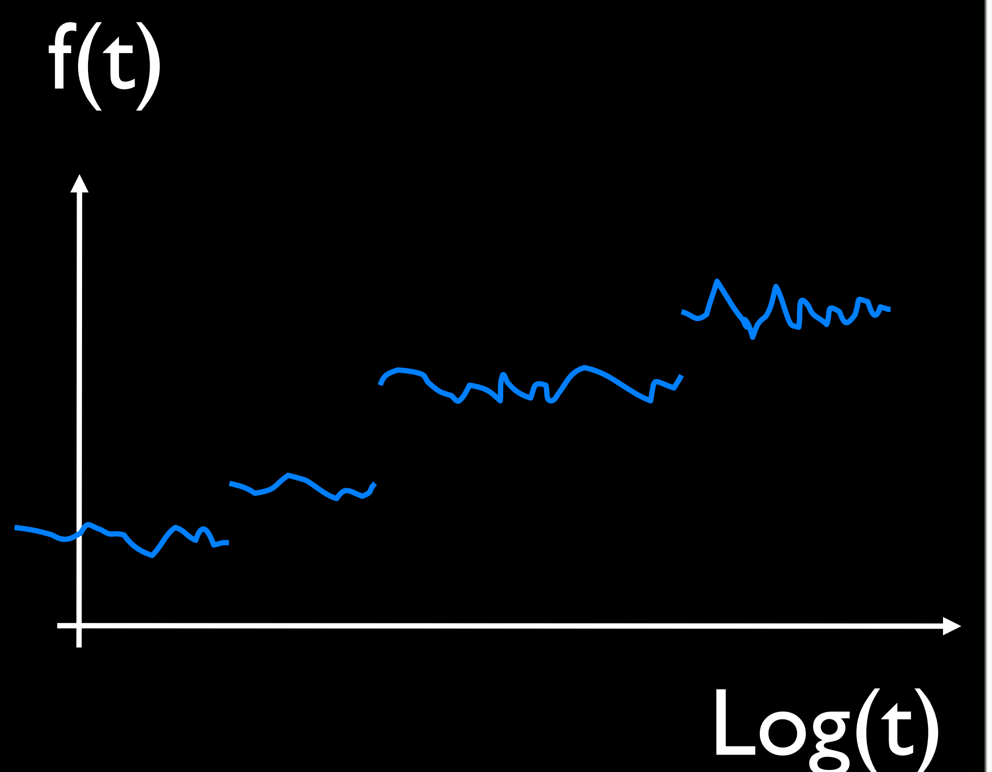
Low connectivity High connectivity

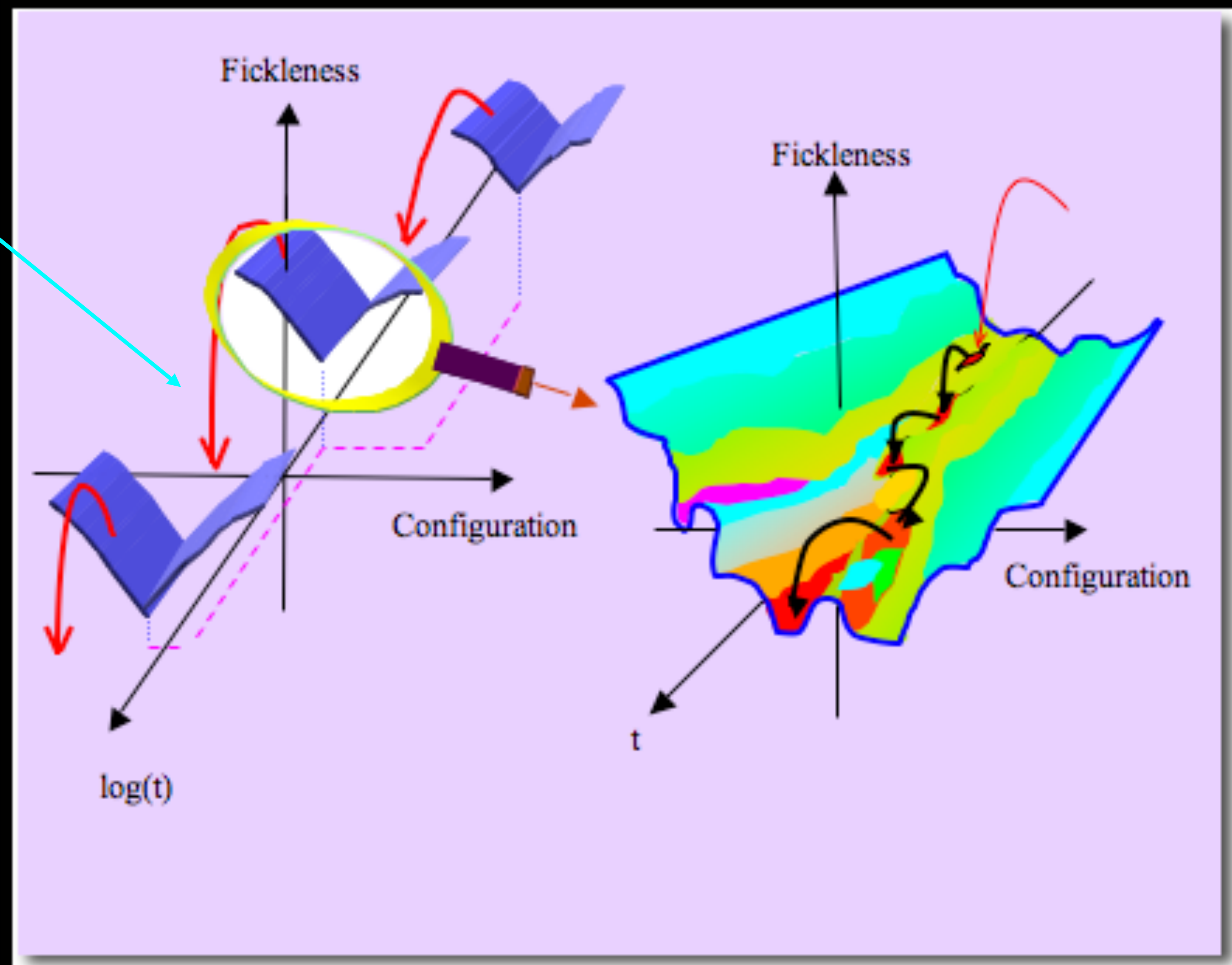
Complex dynamics:

Intermittent, non-stationary

Jumping through collective adaptation space: quake driven

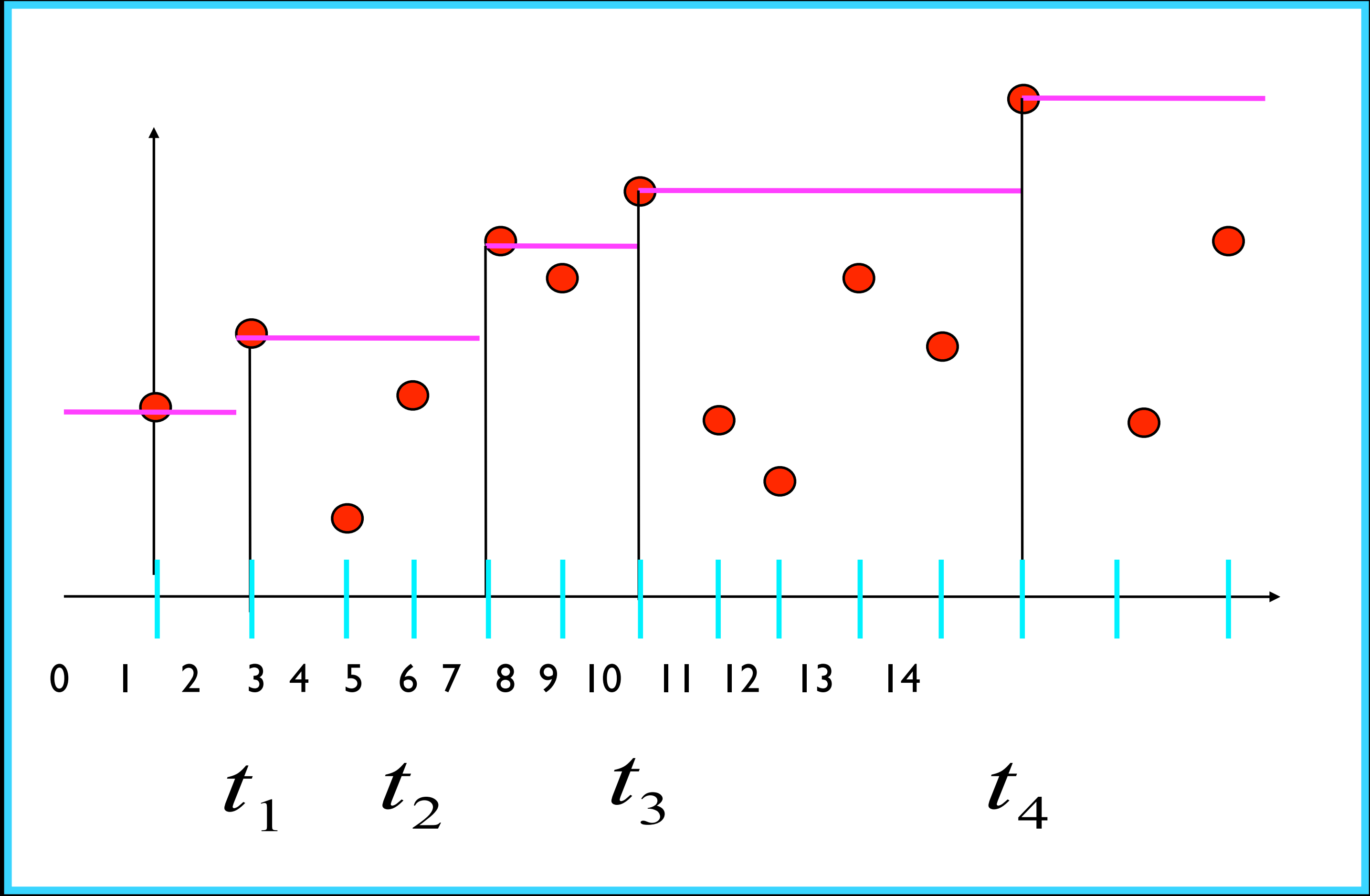
Transitions





Henrik Jeldtoft Jensen

Record dynamics



Distribution of the number of records during t time steps independent of the nature of the fluctuating signal:

$$P_1(t) = \frac{1}{t}$$ The first out of t is the biggest

$$P_{(1,m)}(t) = \frac{1}{(m-1)t}$$ Two records during t: one at t=1 with prob (m-1) & one at t=m with prob 1/t.

$\Downarrow$

$$P_2(t) = \sum_{m=2}^{t} \frac{1}{(m-1)t} \approx \frac{\ln t}{t}$$ Two records during t

$\Downarrow$

$$P_n(t) \approx \frac{(\ln t)^{n-1}}{(n-1)!}\frac{1}{t} = e^{-}\lambda\frac{\lambda^{n-1}}{(n-1)!}$$ with $\lambda = \ln t$

log Poisson

Record dynamics

$$\tau = \ln(t_k) - \ln(k_{k-1}) = \ln(\frac{t_k}{t_{k-1}})$$

exponentially distributed

- Poisson process in logarithmic time
- Mean and variance

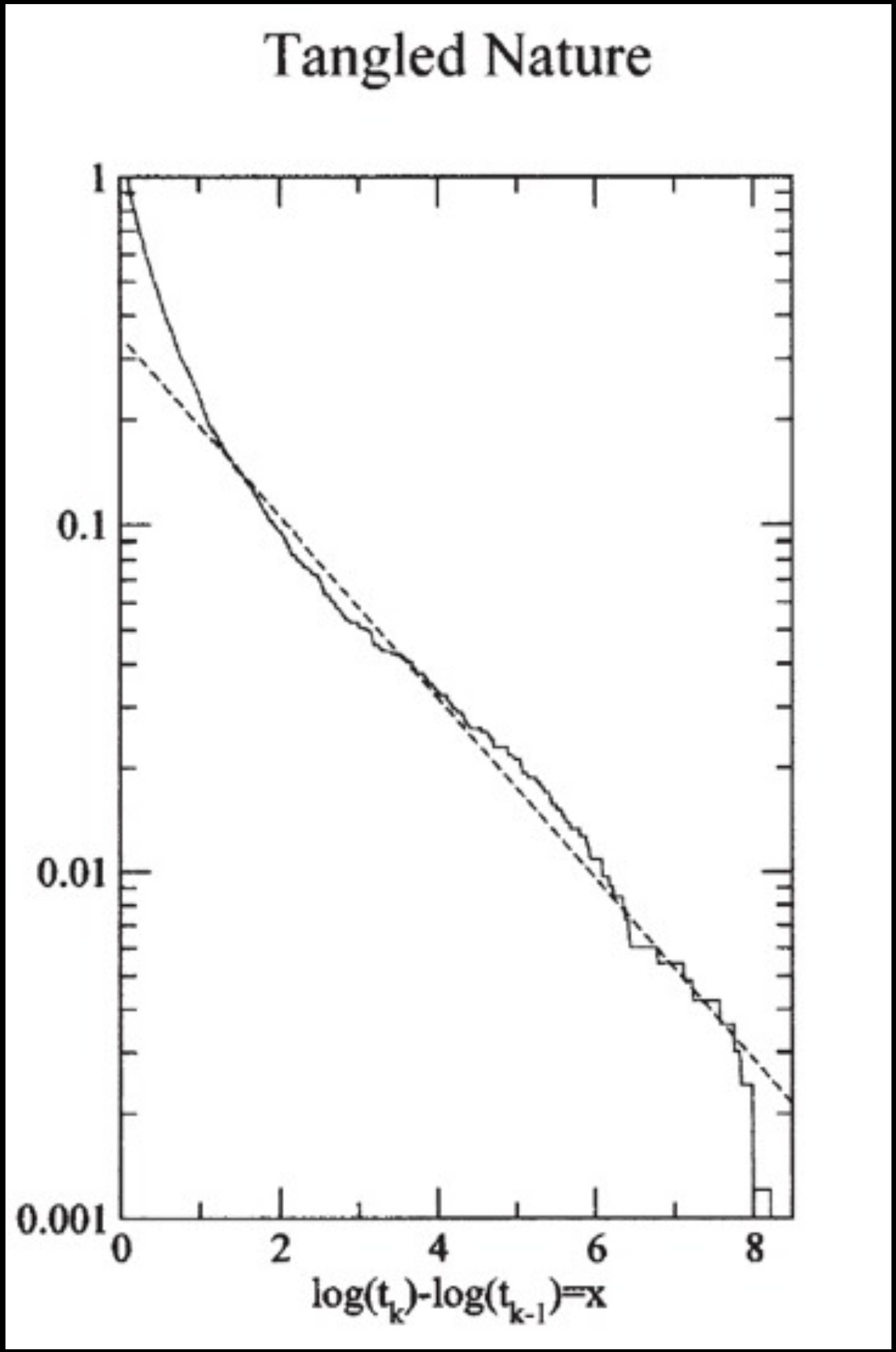
$$\langle Q \rangle \propto \ln t \;\; \text{and} \;\; \langle (Q - \langle Q \rangle)^2 \rangle \propto \ln t$$

- Rate of records constant as function of ln(t)
- Rate decreases $\propto 1/t$
- Non-stationary "1/f fluctuations"

Record dynamics:

Tangled Nature

Cumulative distribution of transitions



Paul Anderson

Record dynamics

- Spin glasses (Internal energy)
- Relaxation of magnetic field in superconductors
  (Flux lines entering the sample)
- Hungry ants (Exit times)
- Earthquakes (Omori law)
-

Stability

Tangled Nature -

Population size

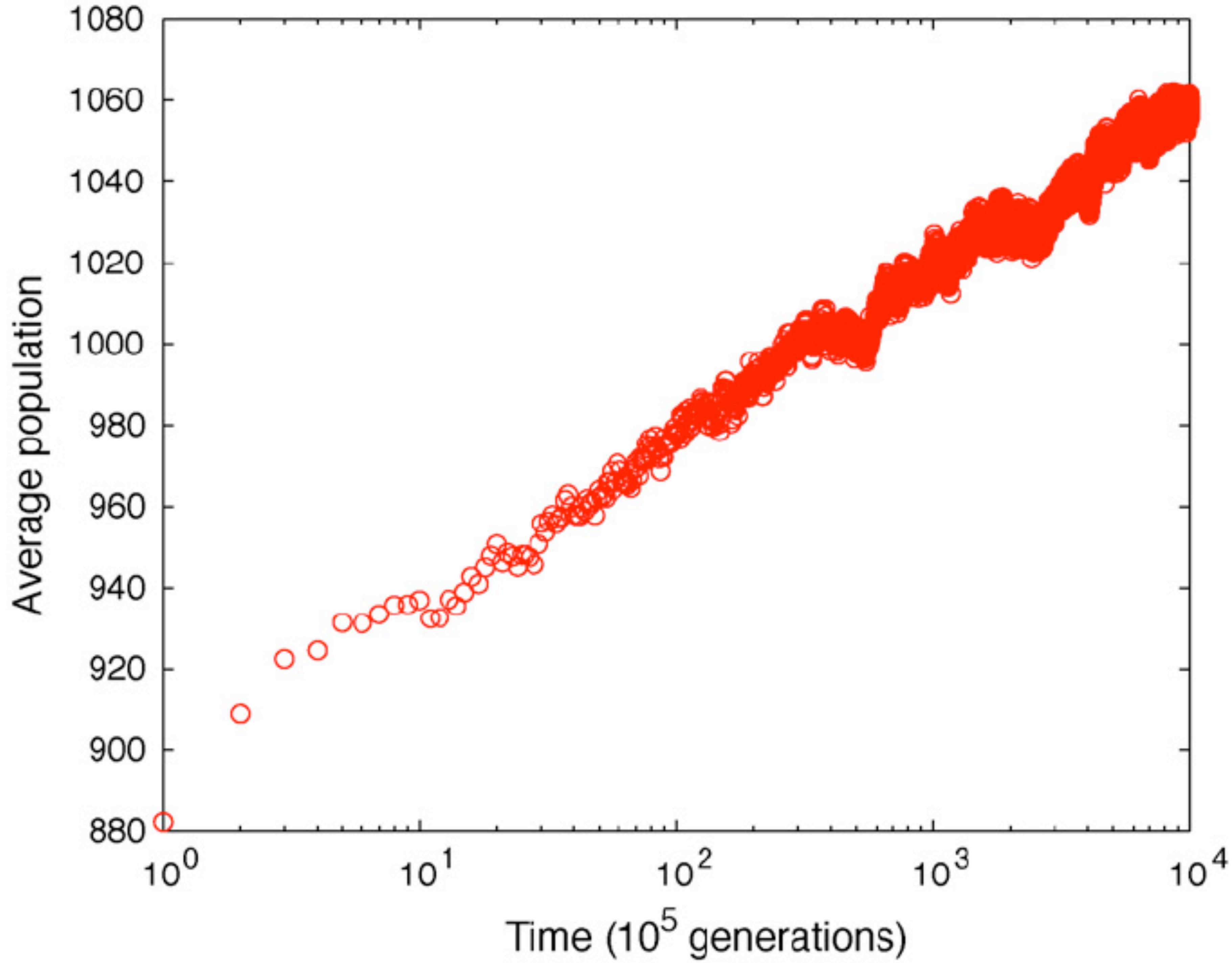


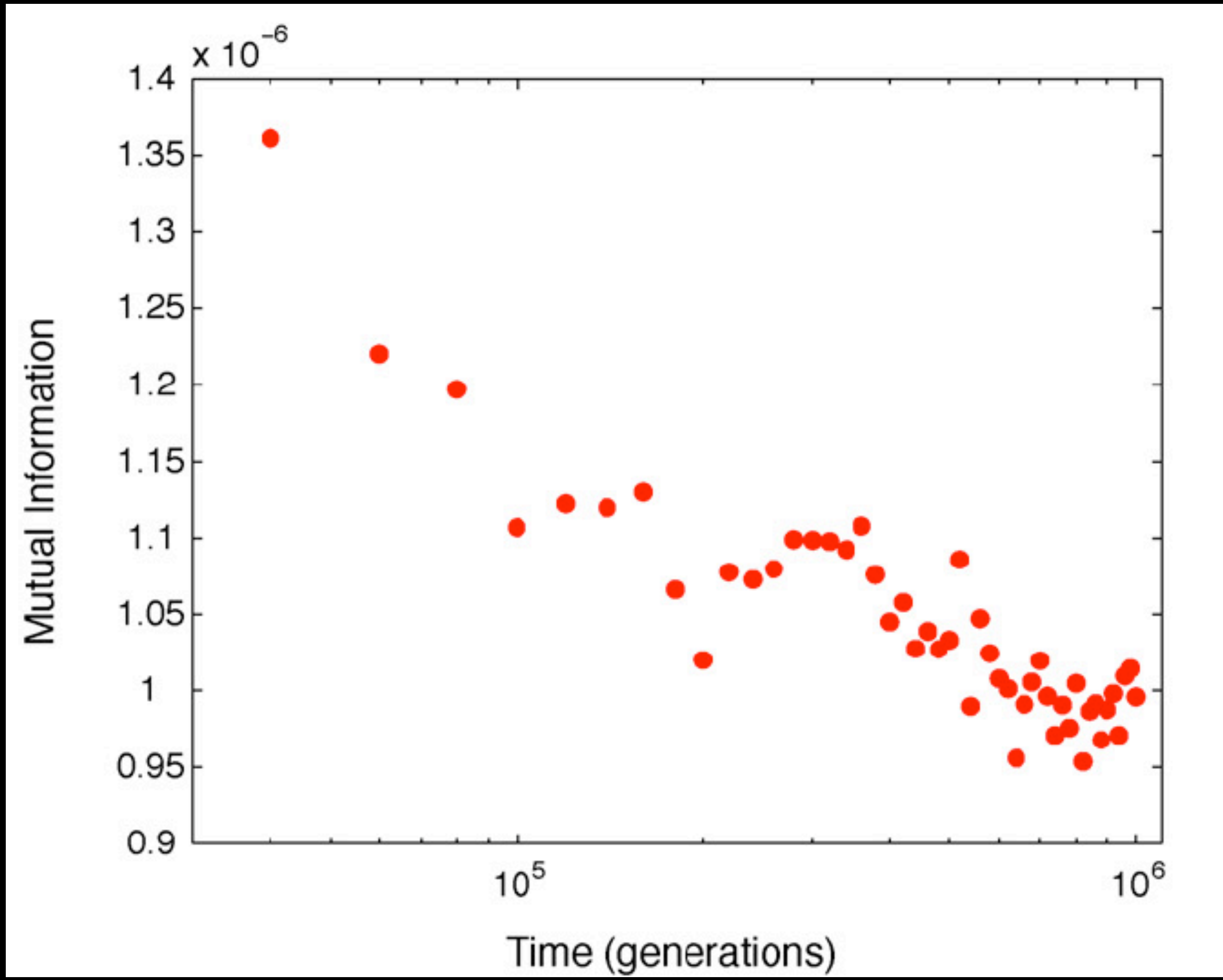
Fig. 3. The mean population (averaged over an ensemble of 1000 runs) increases logarithmically in time.

Dominic Jones

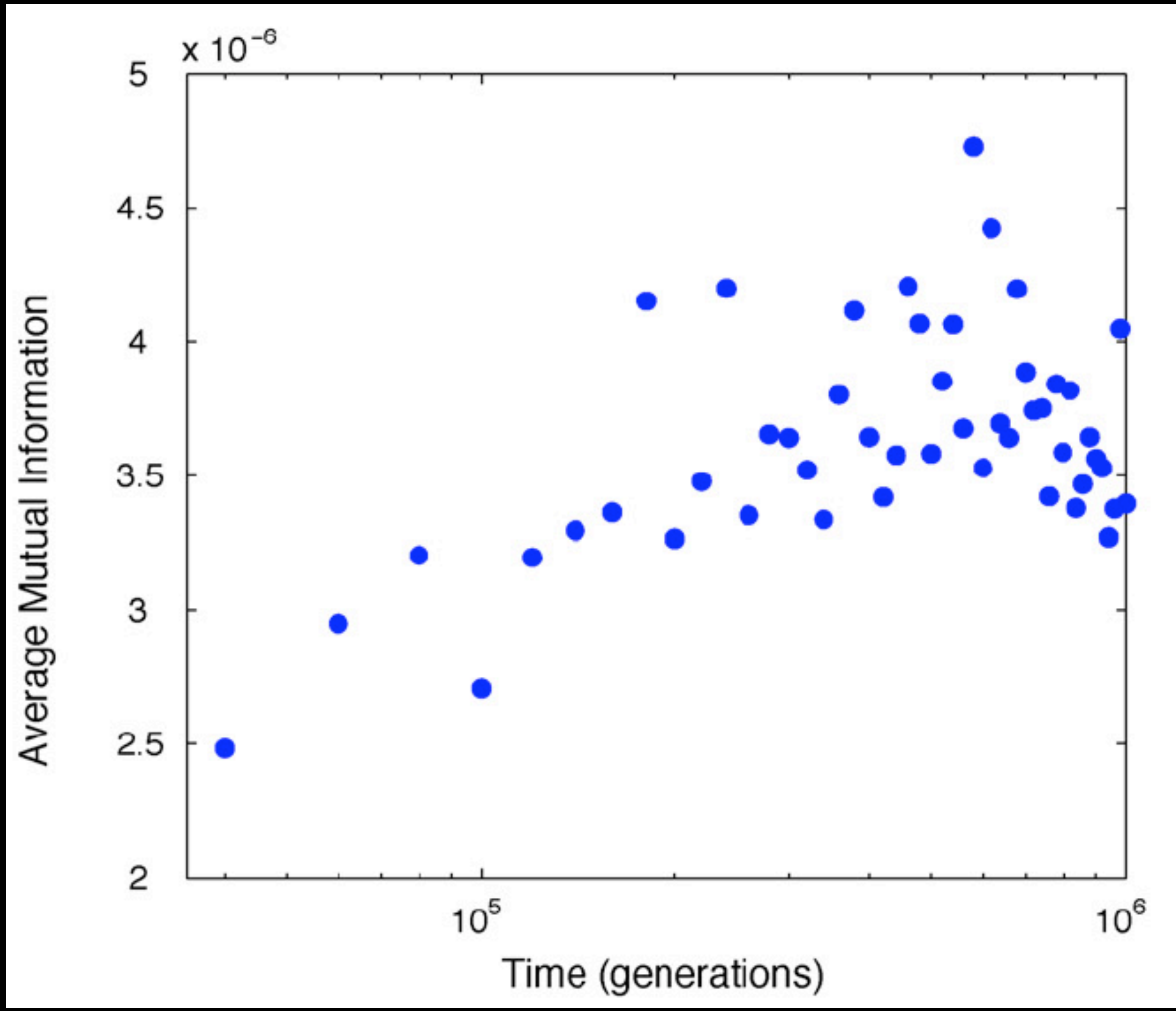
Dynamics - correlations

$$I = \sum_{J_1, J_2} P(J_1, J_2) \log[\frac{P(J_1, J_2)}{P(J_1)P(J_2)}]$$

Total



Core



Dominic Jones

Origin of adaptation? Effect of mutation

Let $H = \tilde{J} - \mu N$, then the effect of a mutation is

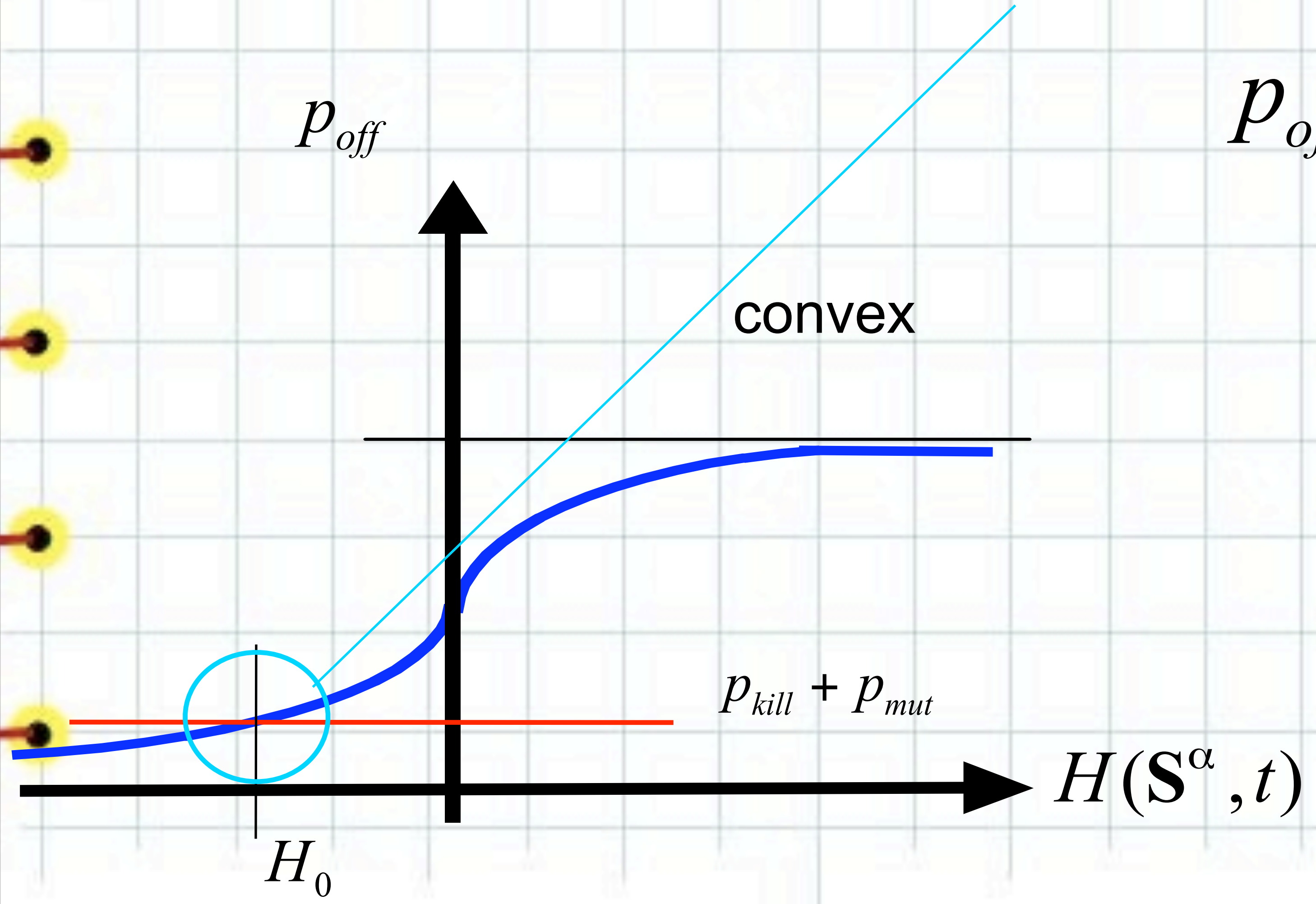
$$H \mapsto H + \delta \tilde{J}.$$

→ Symmetric fluctuations $prob(\delta \tilde{J}) = prob(-\delta \tilde{J})$

leads to asymmetri

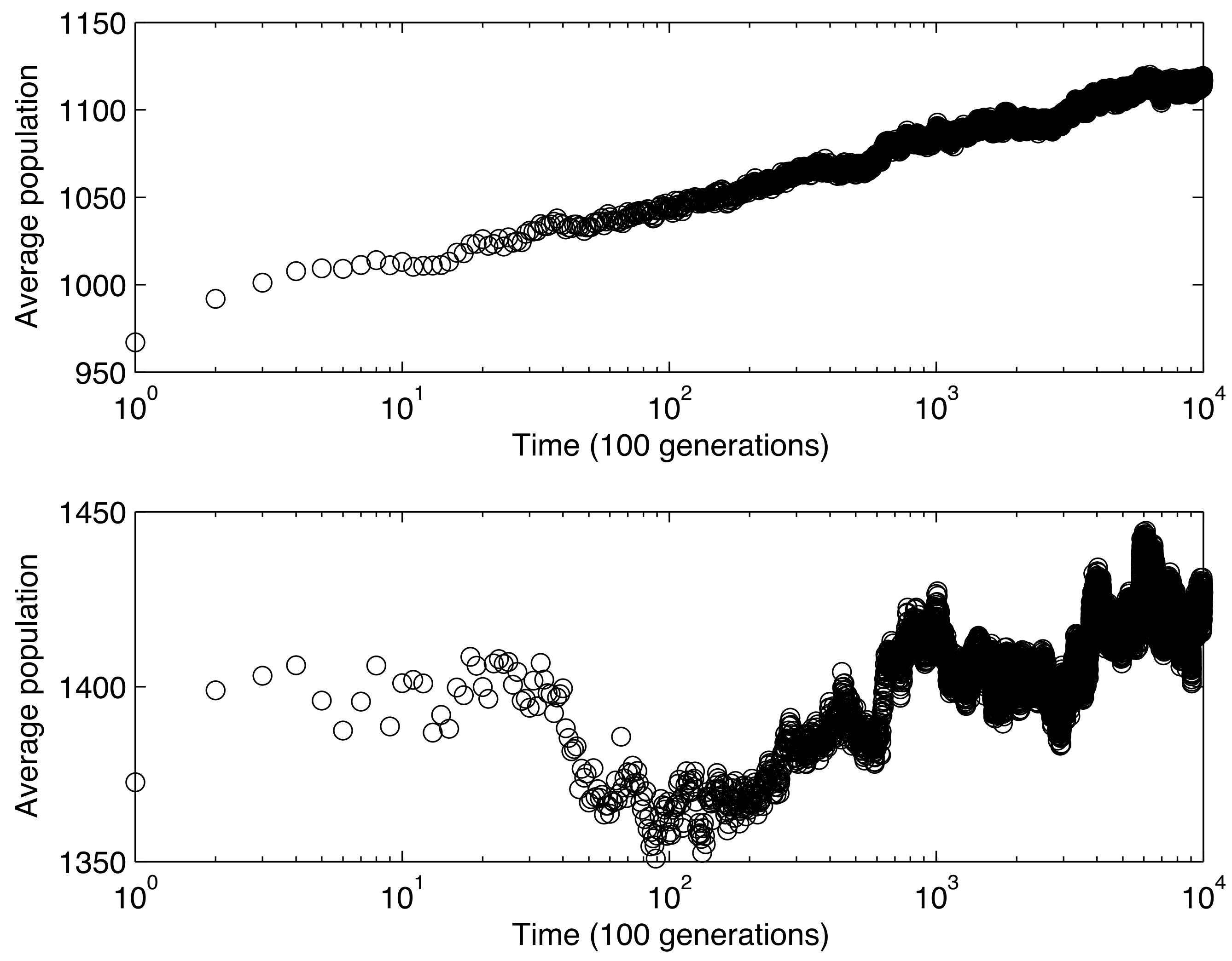
$$p_{off}(H_0 + \delta \tilde{J}) - p_{kill} >$$

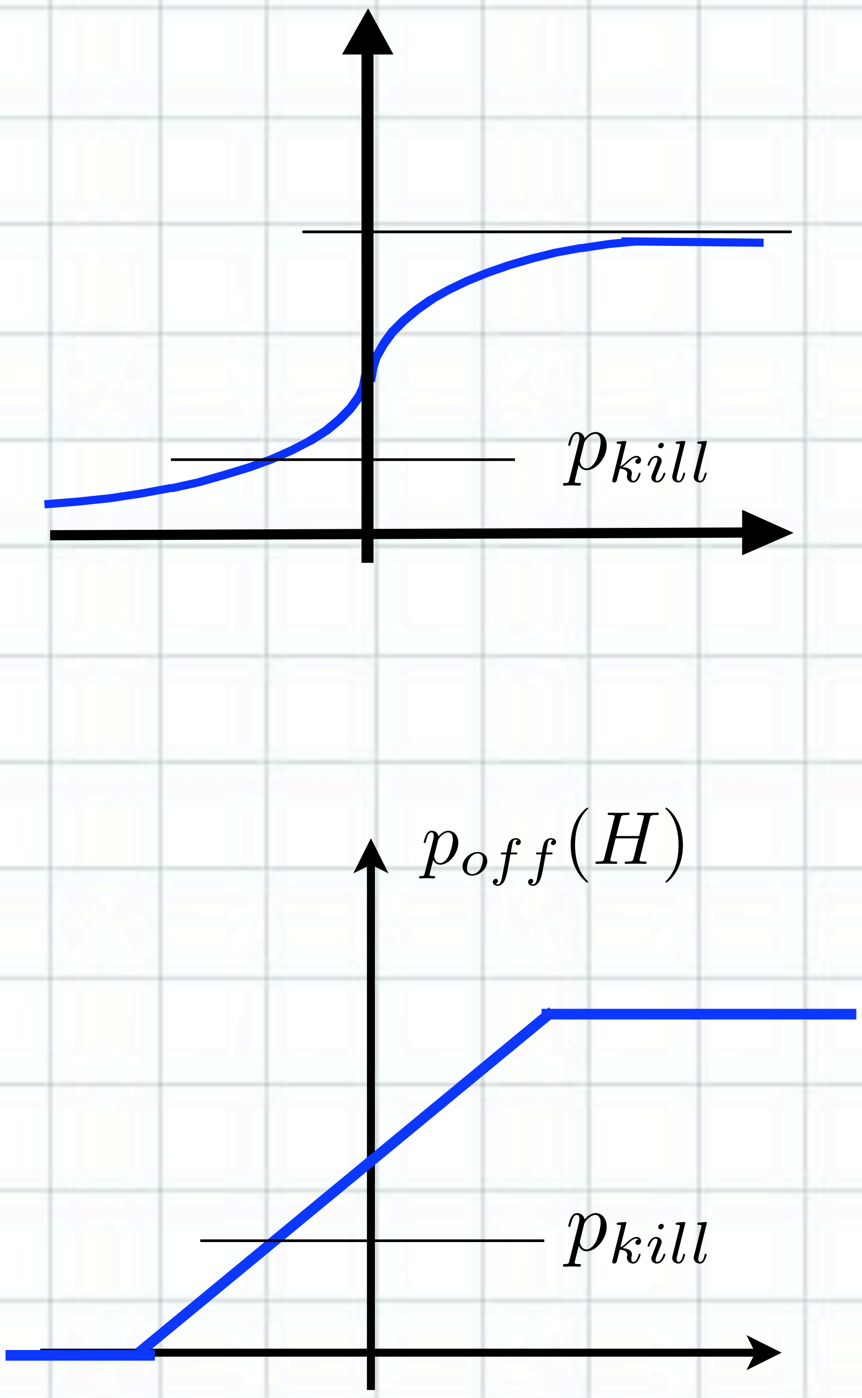
$$p_{kill} - p_{off}(H_0 - \delta \tilde{J})$$



Origin of adaptation?

Dominic Jones

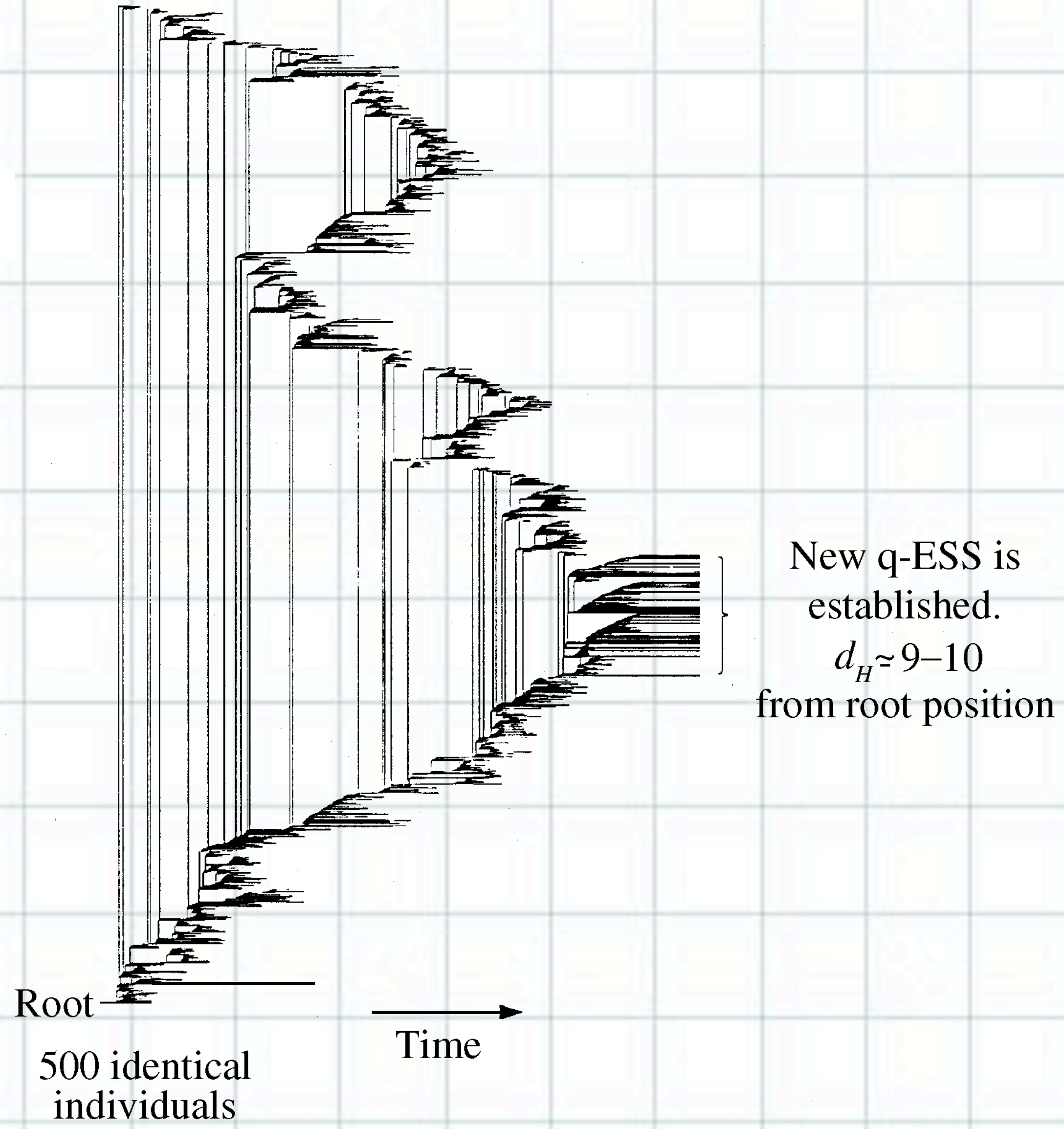




Macro dynamics - the transitions

Non correlated

Graph courtesy to
Matt Hall



Stability of the meta-stable configurations

Consider simple adiabatic approximation.

Stability of genotype S assuming: $n(S',t)$ independent of t for $S' \neq S$

Consider $$\frac{\partial n(S,t)}{\partial t} = [p_{off}(n(S,t),t) - p_{kill} - p_{mut}]\frac{n(S,t)}{N(t)}$$

Stationary solution $n_0(S)$ corresponds to $p_{off}(n_0(S)) - p_{kill} - p_{mut} = 0$

Fluctuation $$\delta = n(S,t) - n_0(S)$$

Fulfil $$\dot{\delta} = A\frac{n_0}{N_0}\delta$$

$$\text{with } A = -(1 - p_{mut})(p_{off})^2 e^{-H_0}(\frac{J}{N_0^{\,2}} + \mu) < 0$$ **i.e. stability**

Transitions between meta-stable configurations caused by co-evolutionary collective fluctuations

$n(S',t)$ needs to be considered

dependent of t for $S' \neq S$

Henrik Jeldtoft Jensen

Imperial College London

The error threshold

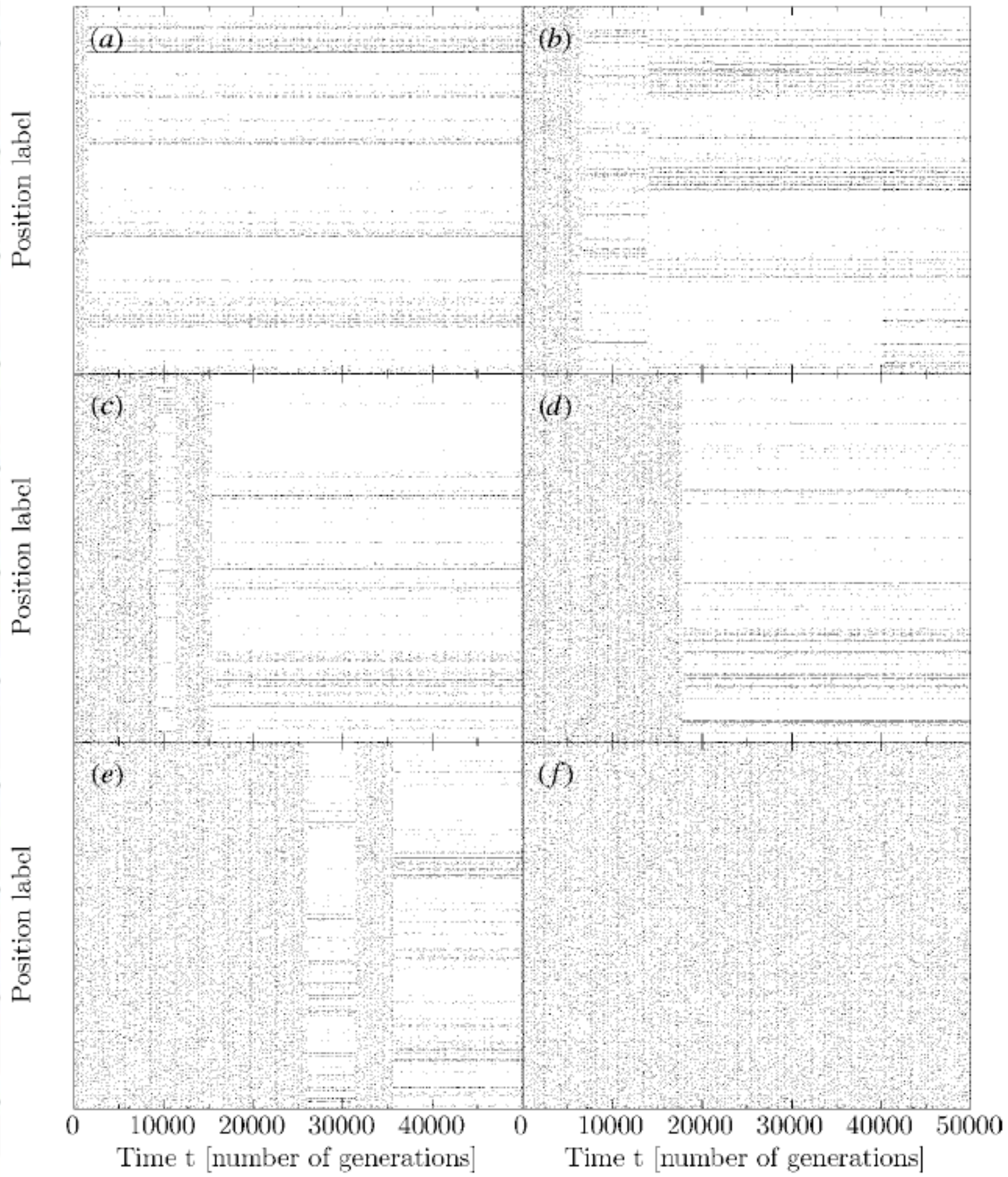
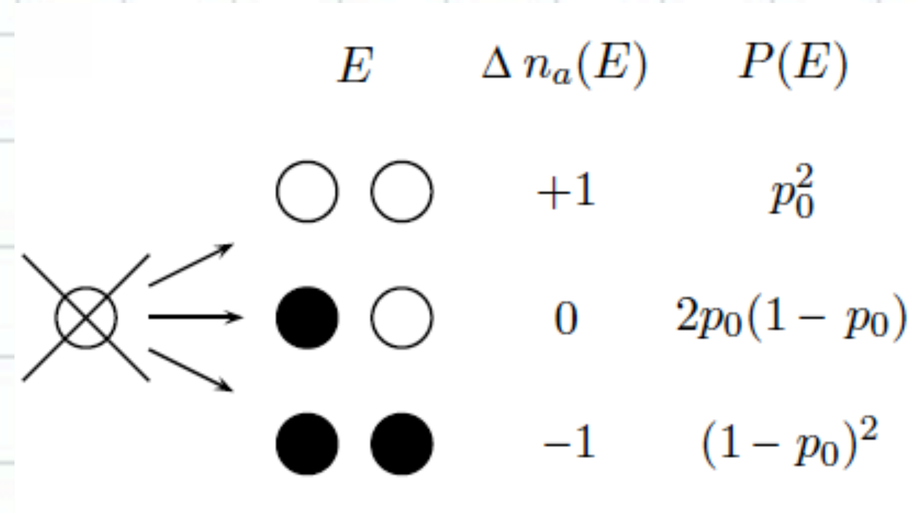


Figure 3. Occupation plots for different values of the mutation rate. The y-axis refers to an arbitrary enumeration of all positions in genotype space. Occupied positions are indicated by a black dot. Results shown are for $p_{\rm kill} = 0.2$, $\mu = 1/1000 \cdot \ln\left(\frac{1-p_{\rm kill}}{p_{\rm kill}}\right)$ and $C = 0.05$. (*a*) Mutation rate: $p_{\rm mut} = 0.009$. The initial transient is extended. (*b*) Mutation rate: $p_{\rm mut} = 0.009\,25$. The initial transient has the same extension of any q-ESS state. (*c*) Mutation rate: $p_{\rm mut} = 0.0095$. The transition between two q-ESS state are extended. (*d*) Mutation rate: $p_{\rm mut} = 0.01$. The initial transient is very extended. (*e*) Mutation rate $p_{\rm mut} = 0.0104$. The initial transient and any transitions are extensively hectic. (*f*) Mutation rate $p_{\rm mut} = 0.0108$. There is no q-ESS state.

Too large mutation rate prevents qEES to establish.

Mean field analysis:

$$p_0 = (1 - p_{mut})^L$$



Number of individuals on site *a*

$$\Delta n_a = +1 p_o^2 + (-1)(1 - P_0)^2 = 2p_0 - 1$$

$$\Downarrow$$

$$n_a(t+1) = n_a(t) + \frac{n_a(t)}{\sum_a' n_{a'}(t)} [p_{off}^a(t)(2p_0 - 1) - p_{kill}]$$

Collobiano, Christensen and Jensen, J Phys A **36**, 883, (2003) Assume steady state, time average and use

$$\langle \frac{n_a(t)}{\sum_a' n_{a'}(t)} p^a_{off}(t) \rangle = \langle \frac{n_a(t)}{\sum_a' n_{a'}(t)} \rangle \langle p^a_{off}(t) \rangle$$

then we obtain

$$p_q = \frac{p_{kill}}{2(1 - p_{mut})^L - 1}$$

for the on average off-spring probability for those site which are able to counterbalance the kill by off-spring production.
Leading to a corresponding weight function *H* for the wild-types in the q-ESS

$$H_q = \ln(\frac{p_q}{1 - p_q}) = \ln(\frac{p_{kill}}{2(1 - p_{mut})^L - 1 - p_{kill}})$$

In the hectic states we assume the a simple balance between reproduction and killing

$$p_{off} = p_{kill}$$

or $$H_{hectic} = \ln(\frac{p_{kill}}{1 - p_{kill}})$$

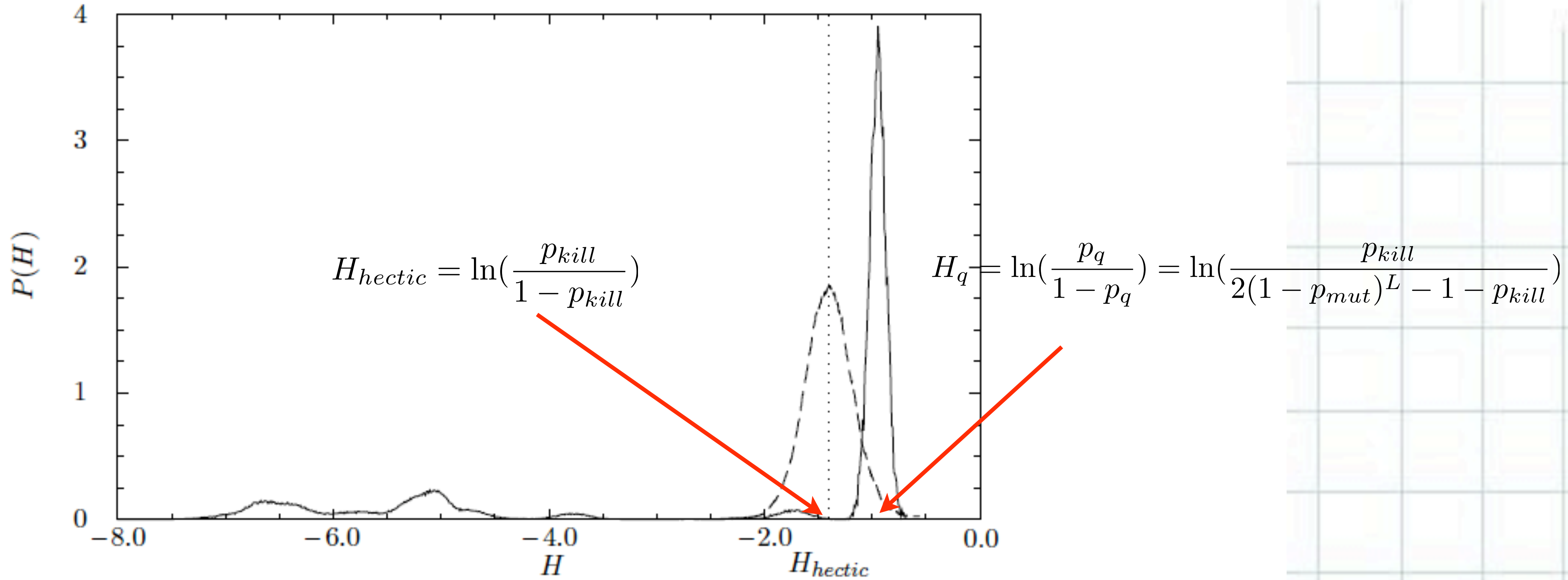


Figure 2. The probability density function of the weight function $H = \ln\left(\frac{p_{\rm off}}{1-p_{\rm off}}\right)$ during a q-ESS state of a simulation (solid line) and during a transition between two q-ESS states (dashed line). During a q-ESS state (solid line) positions range in two sets: unfit positions, for which the weight function is lower than −3.0 and fit positions, for which the fitness is greater then the average value $\langle H \rangle = \ln\left(\frac{1-p_{\rm kill}}{p_{\rm kill}}\right) \approx -1.38 = H_{\rm hectic}$, indicated by a vertical dotted line. During a transition (dashed line) the fitness of all positions is normally distributed around $H_{\rm hectic}$ where all positions reproduce (on average) at the same rate, equal to the killing rate. Note the support of the weight function in the hectic phase exceeds H_q, ensuring that the positions in genotype space are able to fulfil the q-ESS balance equation (13). The parameters (for precise definitions, see [14, 15]) are $p_{\rm kill} = 0.2$, $\mu = 1/1000 \cdot \ln\left(\frac{1-p_{\rm kill}}{p_{\rm kill}}\right) \approx 0.0014$, $C = 10.0$ and $p_{\rm mut} = 0.008$.

A hectic transition can only develop into a q-ESS if hectic peak overlaps with q-EES peak

$$H_{hectic} + \alpha k \geq H_q$$

We assume width of hectic peak proportional to with of distribution of *J* given by *k*

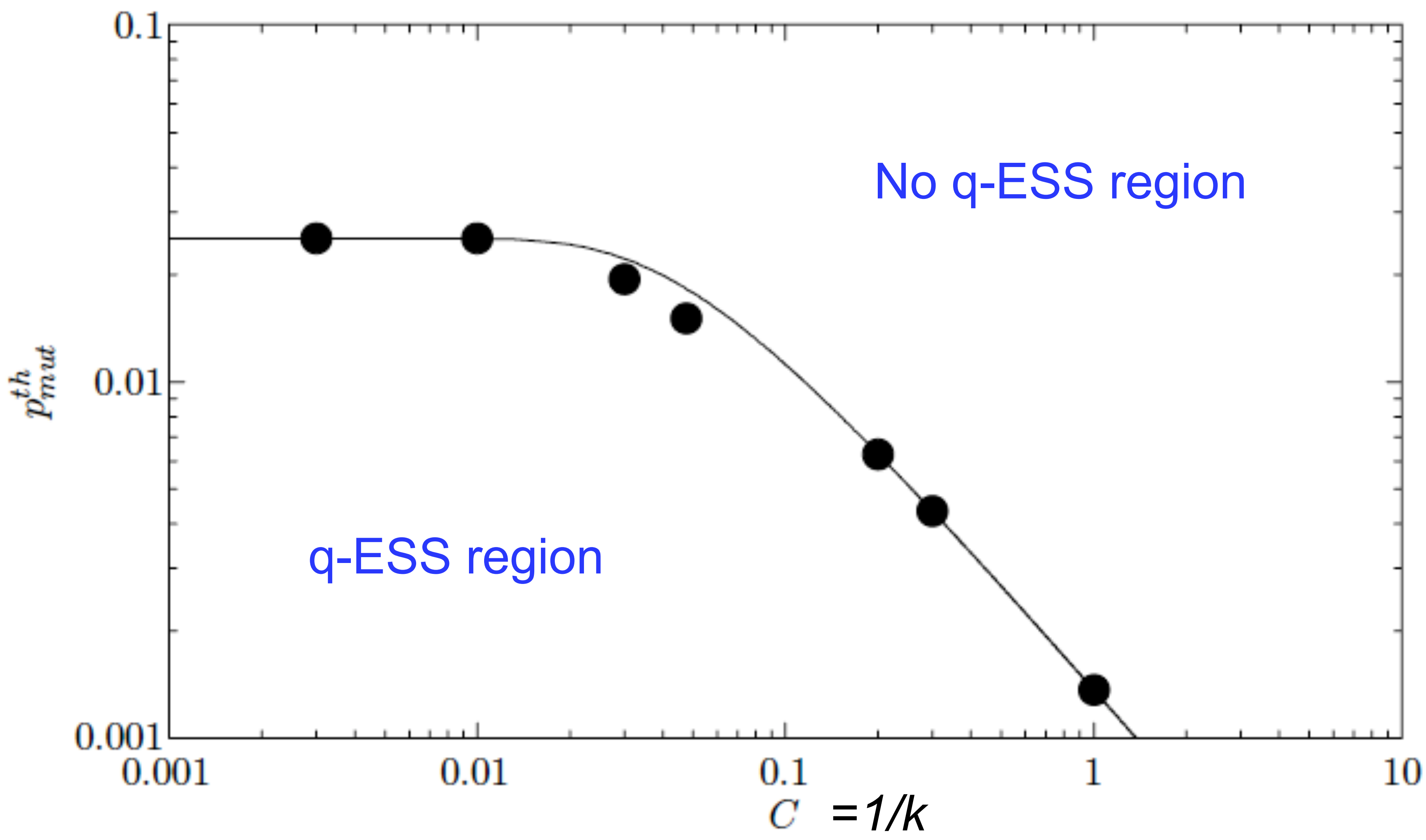


Figure 4. The computational determination of the error threshold. The loss of q-ESS states occurs for mutation rates above the solid circles. The data, compared with the theoretically predicted error threshold $p_{\mathrm{mut}}^{\mathrm{th}}$ (solid line), indicate a value of $\alpha = 0.07$, see equation (18). The parameters of the simulations are $L = 20$, $\mu = 0.005$ and $p_{\mathrm{kill}} = 0.2$.

α is used as fitting parameter

In progress

Dynamical systems approach

Mean field

→ Fixed point analysis

→ Stability matrix

Economics

David Robalino

Stochastic dynamics

Use correlated J(S$_1$,S$_2$)

Consider **S** to label economical entities, say companies of capital C(**S**,t)

Dynamics:

Replacement

$$n(\mathbf{S},t) \rightarrow C(\mathbf{S},t)$$

Define $$P_{gain}(S,t) = \frac{\exp[H(S,t)]}{1+\exp[H(S,t)]}$$

let

$$\mathrm{J}^{+}(\mathbf{S}) = \sum_{\mathbf{S}'} J(\mathbf{S},\mathbf{S}')\theta[\mathbf{J}(\mathbf{S},\mathbf{S}')]$$

$$\mathrm{J}^{-}(\mathbf{S}) = \sum_{\mathbf{S}'} J(\mathbf{S},\mathbf{S}')\theta[-\mathbf{J}(\mathbf{S},\mathbf{S}')]$$

With probability $P_{gain}(\mathbf{S},t)$: $$C(\mathbf{S},t+1) = C(\mathbf{S},t)(1+c_g\frac{J^{+}(\mathbf{S})}{J^{Tot}(\mathbf{S})})$$

With probability $1-P_{gain}(\mathbf{S},t)$: $$C(\mathbf{S},t+1) = C(\mathbf{S},t)(1+c_l\frac{J^{-}(\mathbf{S})}{J^{Tot}(\mathbf{S})})$$

Comparison between data and model:
Volume as GDP

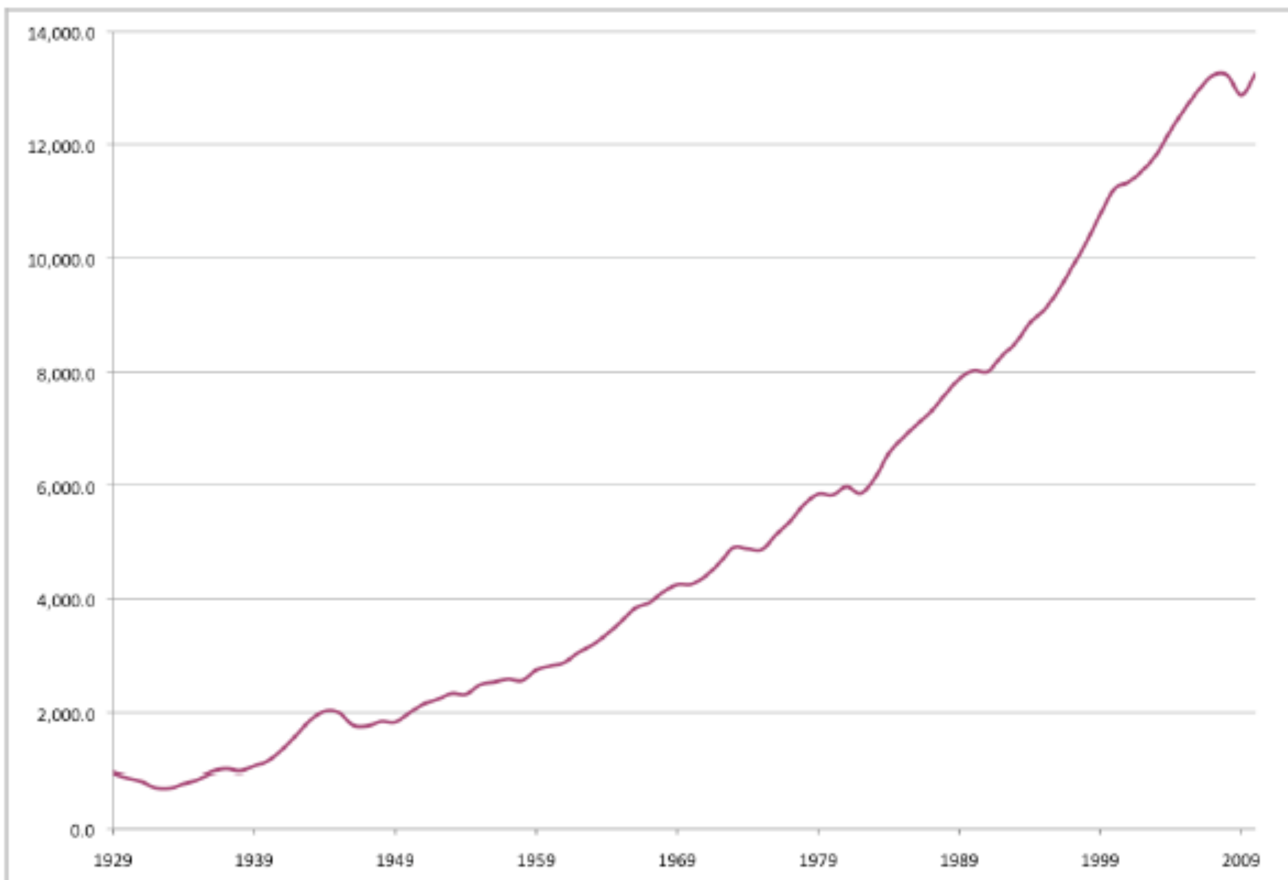


Figure 11: US GDP 1929-2010 corrected for inflation. (Source *Bureau of Economic Analysis*)

In model

$$\mathrm{GDP}(t) = \sum_{\mathbf{S}} C(\mathbf{S}, t)$$

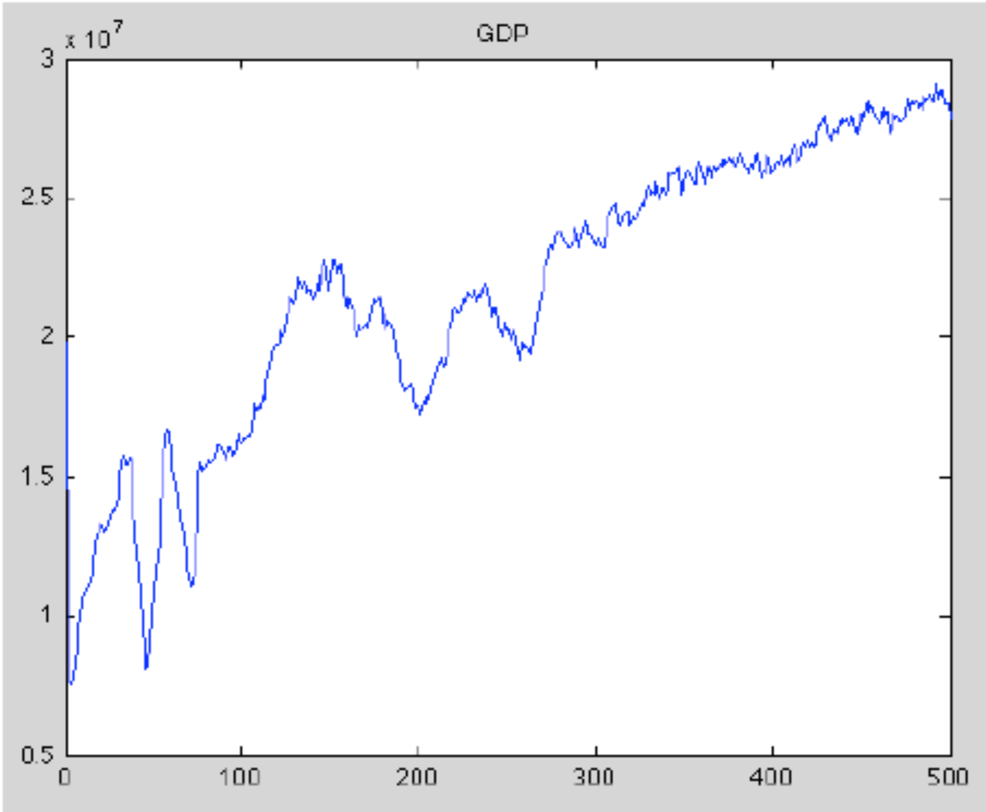


Figure 12: Model GDP (Iterations ×10).

David Robalino,
MSci Thesis

Comparison between data and model:
Growth rate

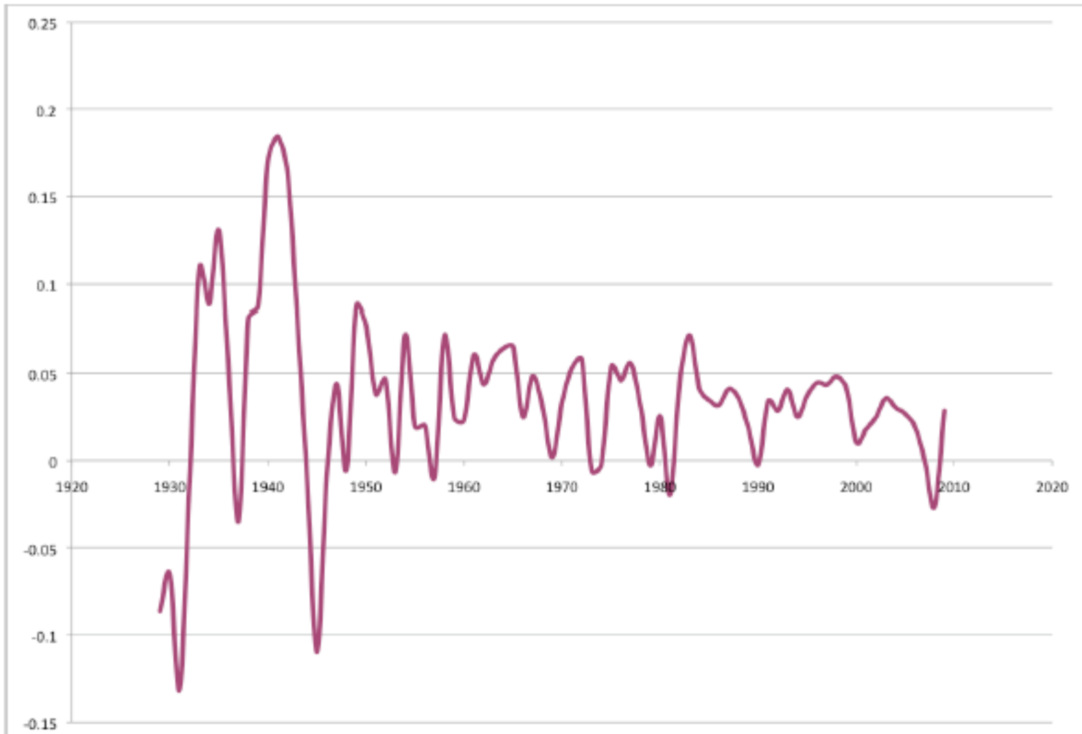


Figure 13: US GDP growth 1929-2010 corrected for infration (Source *Bureau of Economic Analysis*).

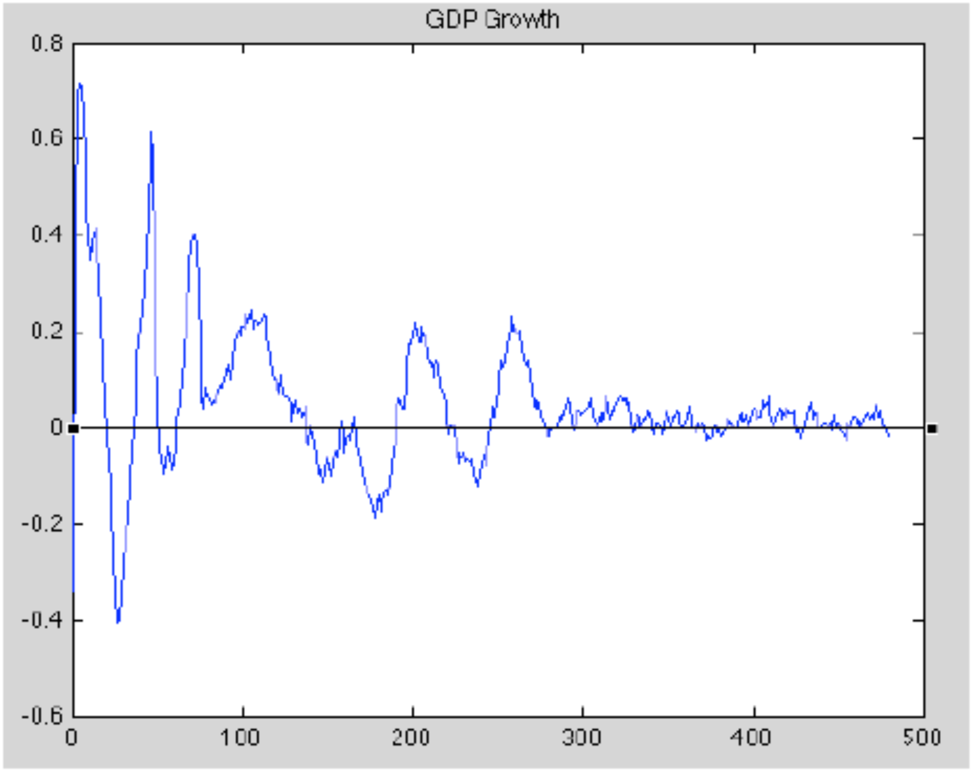


Figure 14: Model GDP growth (Iterations ×10).

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Comparison between data and model:
Size of companies

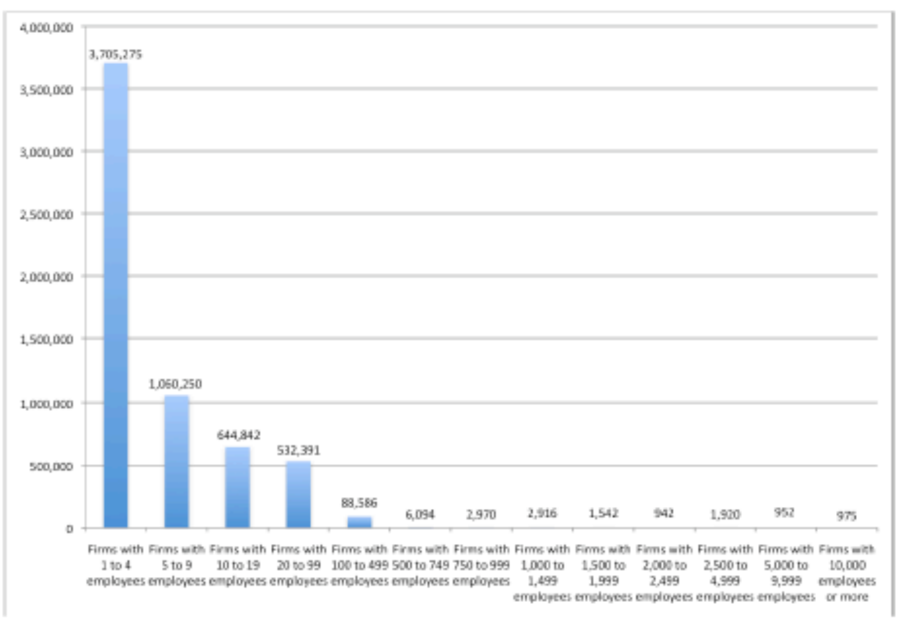

Figure 15: US firms by number of employees 2007 (Source *U.S. Census Bureau*).

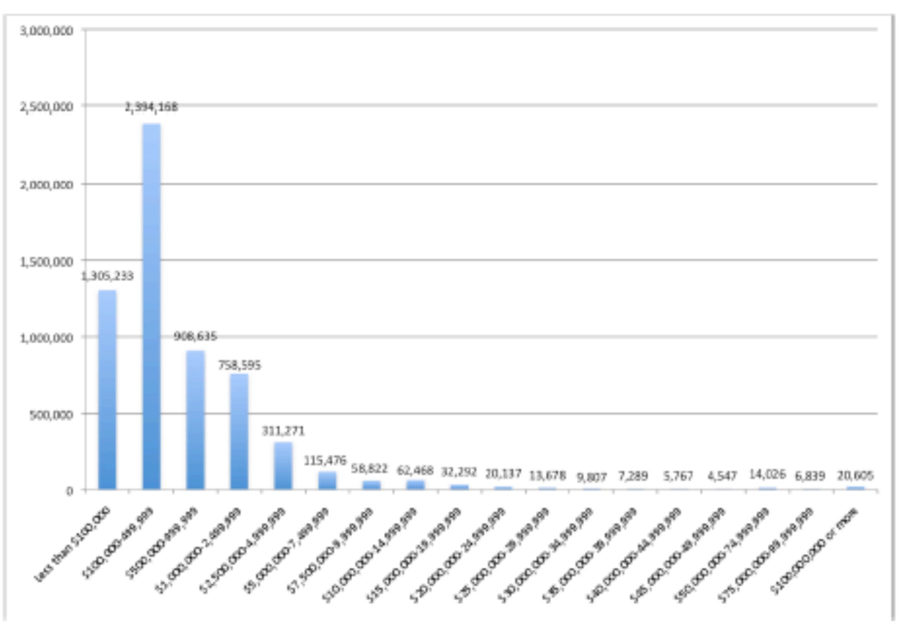

Figure 16: US firms by receipts size 2007 (Source *U.S. Census Bureau*).

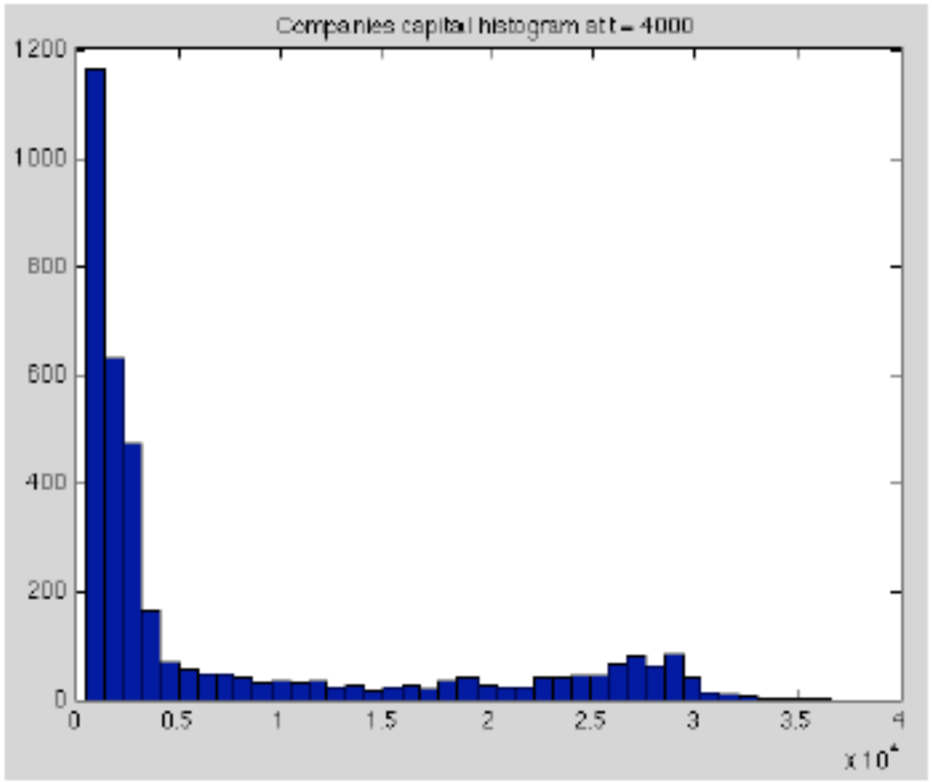

Figure 17: Model histogram of companies capital

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Comparison between data and model:
Company age

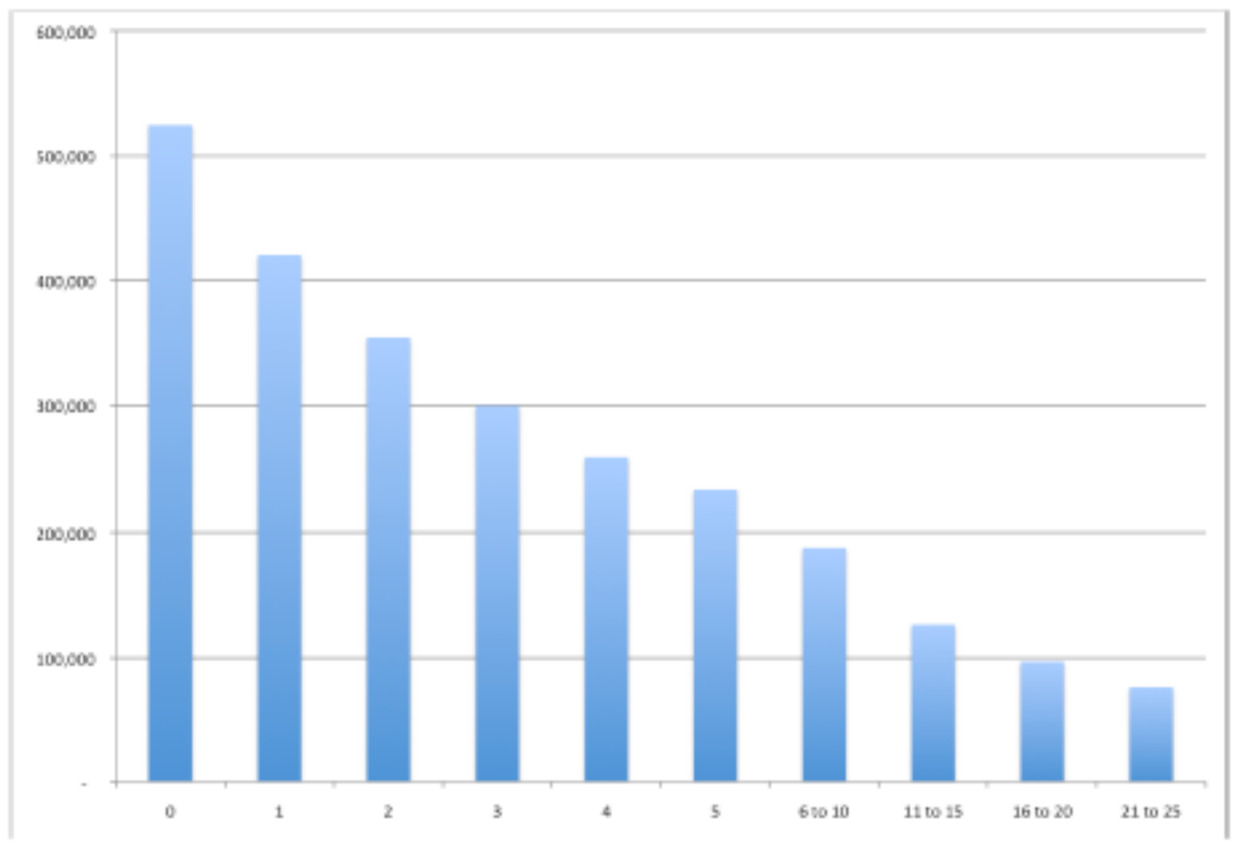

Figure 18: US firms by age in 2007 (Last 4 cols are averages; Source *U.S. Census Bureau*).

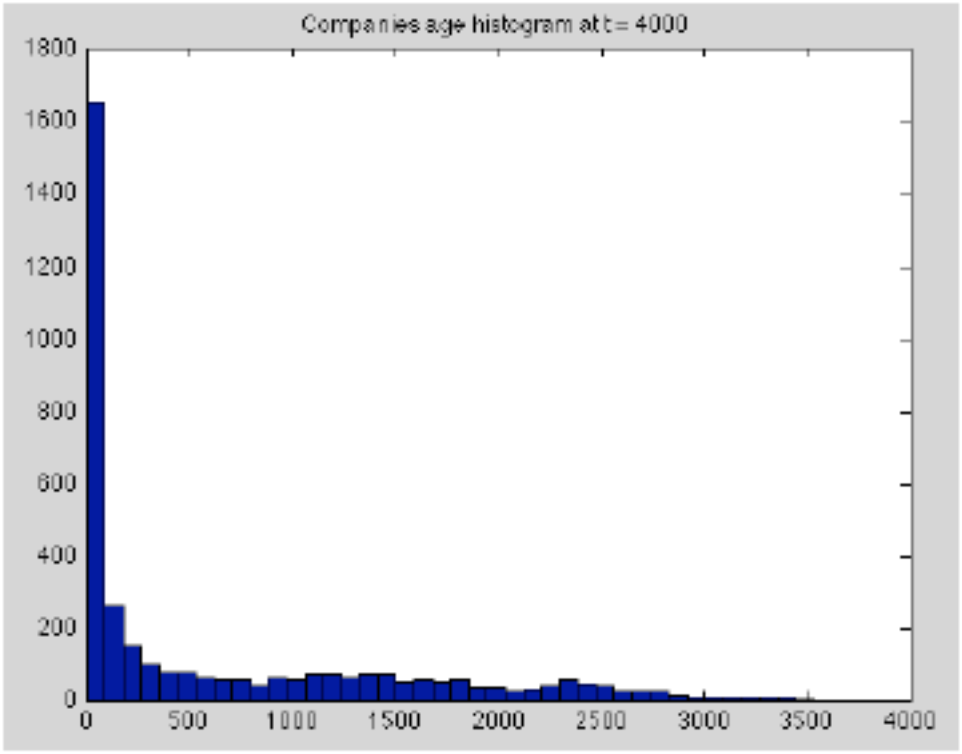

Figure 19: Model Companies by age at t=4000.

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Comparison between data and model:
Number of companies

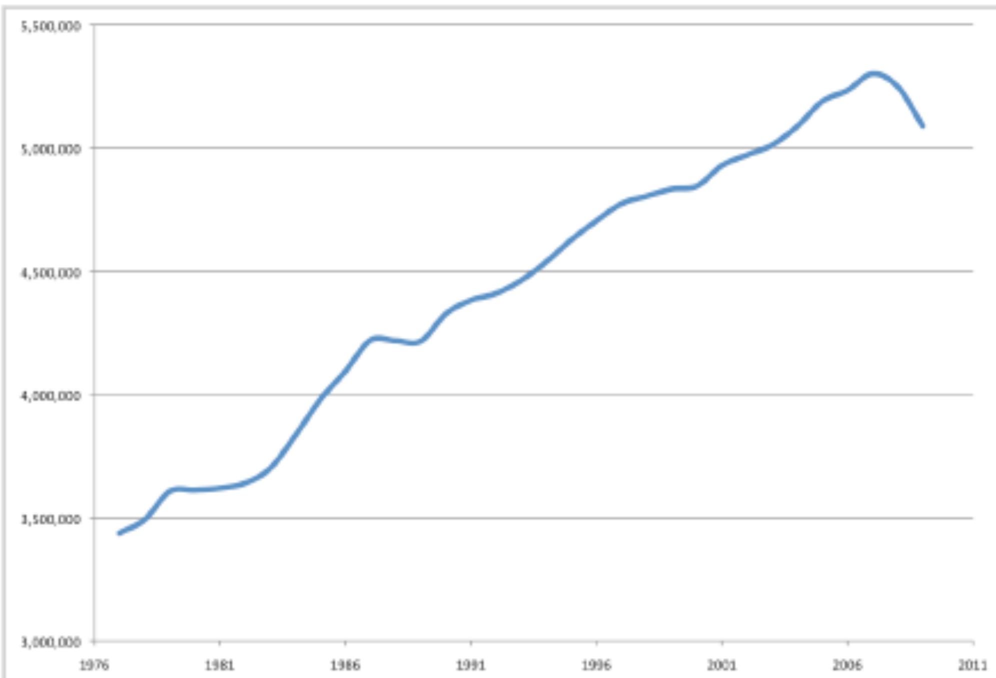

Figure 22: US Number of Firms (Source *U.S. Census Bureau*).

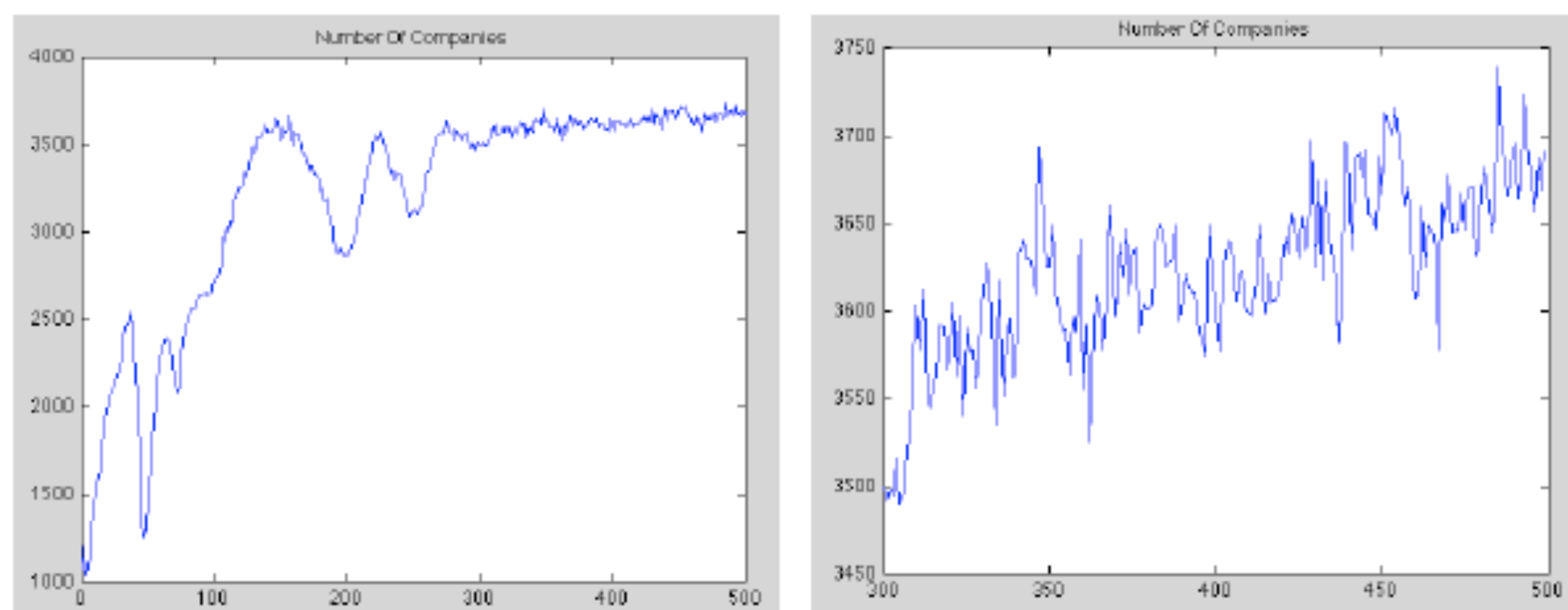

Figure 23: Model number of companies (Iterations ×10).

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Economics

Xiaoye Chen

Deterministic dynamics
with
stochastic node addition

Robustness versus diversity

Simple dynamical network model of N nodes and k edges:

- Each node i = one company possessing:
  - C_i = Cash - liquidity
  - M_i = Material - goods or services
  - P_i = position in "production chain"
  - $A_i = C_i + M_i(P_i+1)$ = assets
  - E_i = Maximum fraction of cash it uses in trade each turn. Represents the amount of risk a company is willing to take.
  - I_i = in degree
  - O_i = out degree

Summer project by Xiaoye Chen

- Each link a_{ij} = 1 represents flow direction of cash between two companies

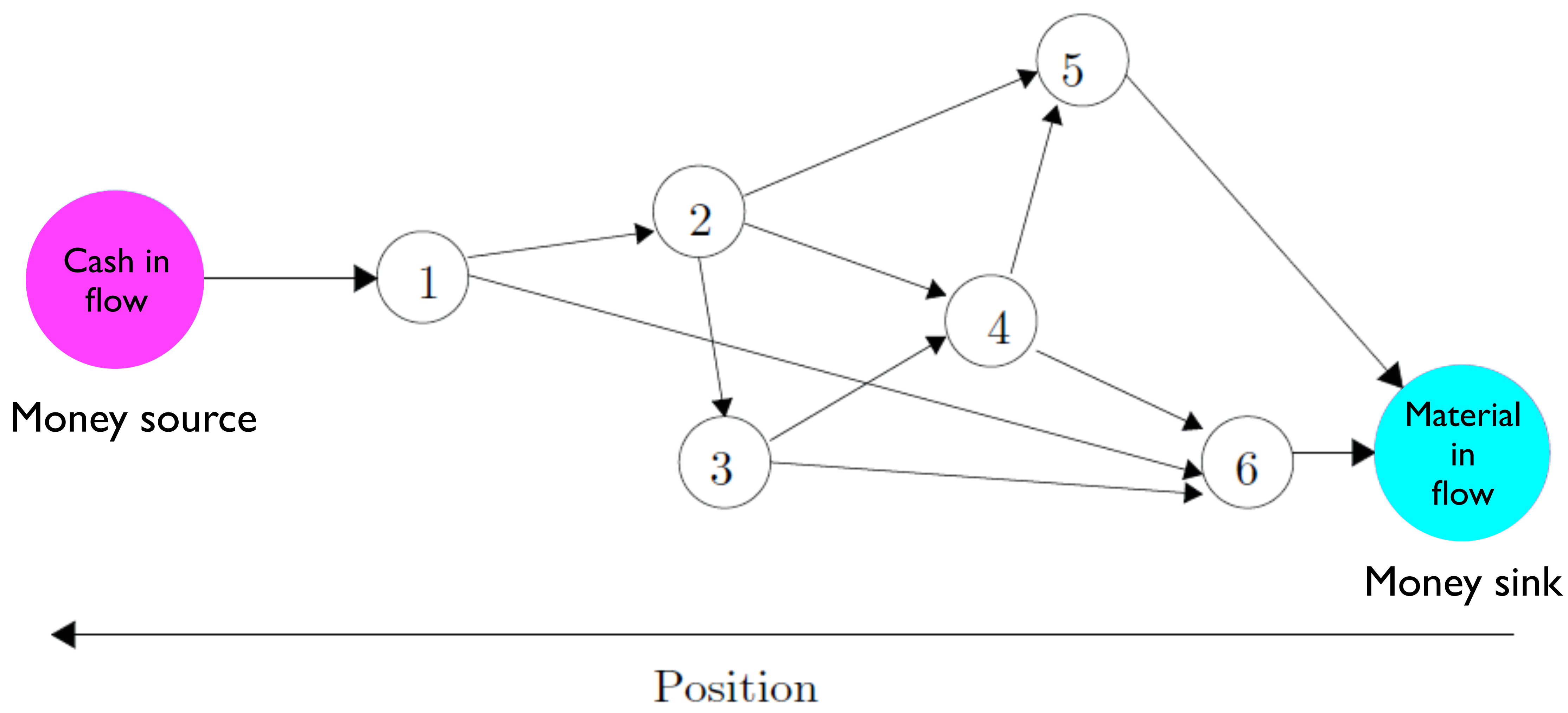


Figure 1: Schematic of network, showing the direction of links.

Dynamics: nodes are created and disappear

1. Initially N_o nodes. And $K=N_0m$ edges

2. New companies:

stochastic

-> Select company preferentially according to assets.
New subsidiary created by transferring 1/2 the assets of mother company.

-> New company is linked to mother company + preferential attachment according to assets.

3. Bankruptcy

-> A company with negative cash is bankrupt. It is merged with another company selected preferentially on cash.
The cash of the company is decreased accordingly.
The bankrupt is removed from the system.

Time step

1. Fixed cash payment: running cost
2. Trade with adjacent companies i & j:

$T_{ij} = \min\{C_i E_i / O_i , M_j(P_j+1)/I_j\}$

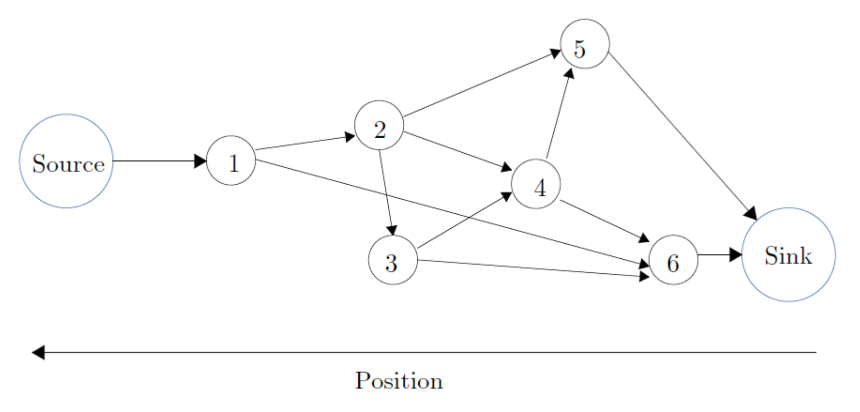


Figure 1: Schematic of network, showing the direction of links.

3. Cash update of company i:

$$(c_i)^{t+1} = (C_i)^t - \sum_j a_{ij} T_{ij} + \sum_j a_{ji} T_{ji} - \text{PAYMENT}$$

Cash spent buying materials

Cash gained selling materials

4. Materials update:

$$(M_i)^{t+1} = (M_i)^t + \sum_j a_{ij} T_{ij}/(P_j+1) - \sum_j a_{ji} T_{ji}/(P_i+1)$$

Source and sink

Before each exchange/trading round
a source and a sink is added.
They are linked to the m=4 nodes of most extreme position.

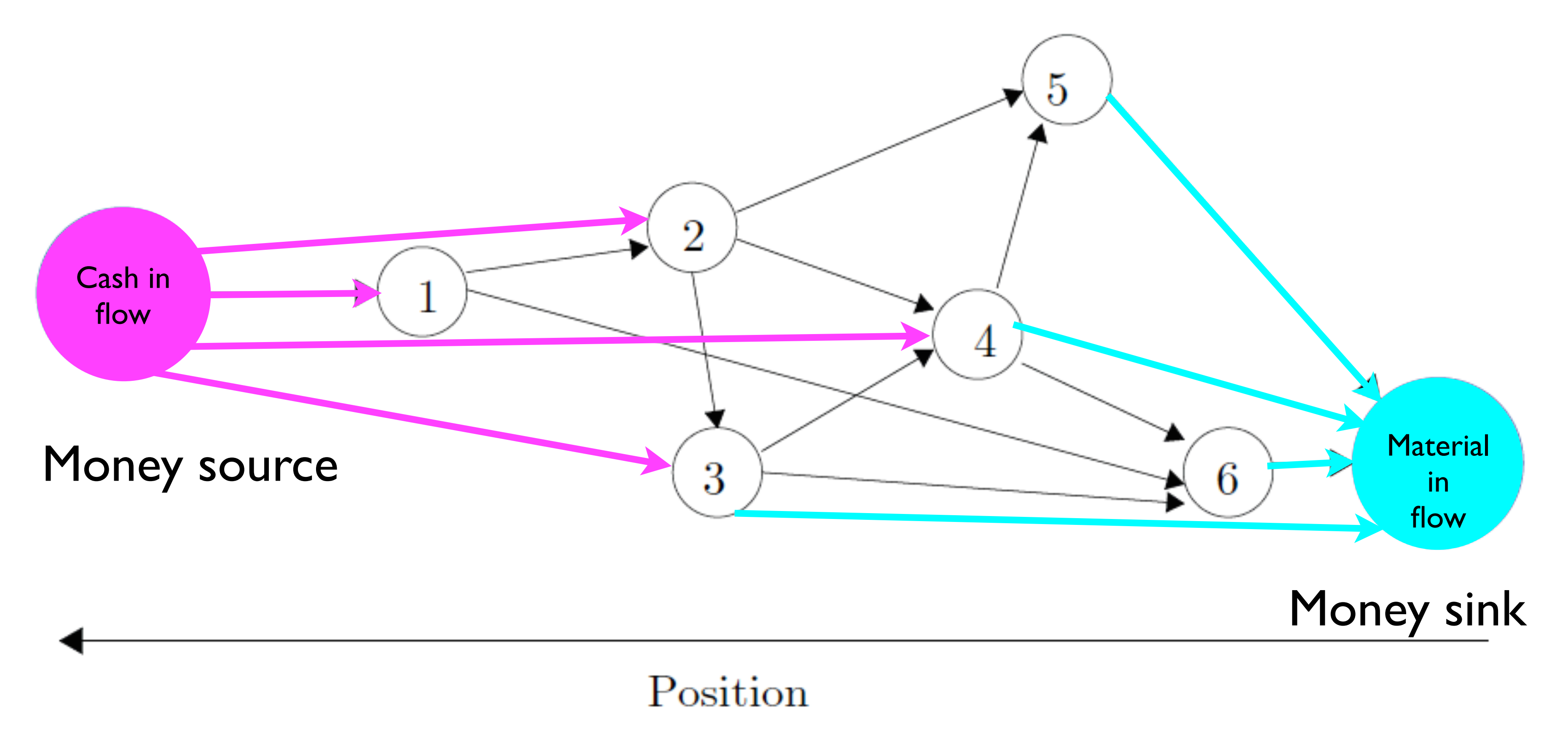
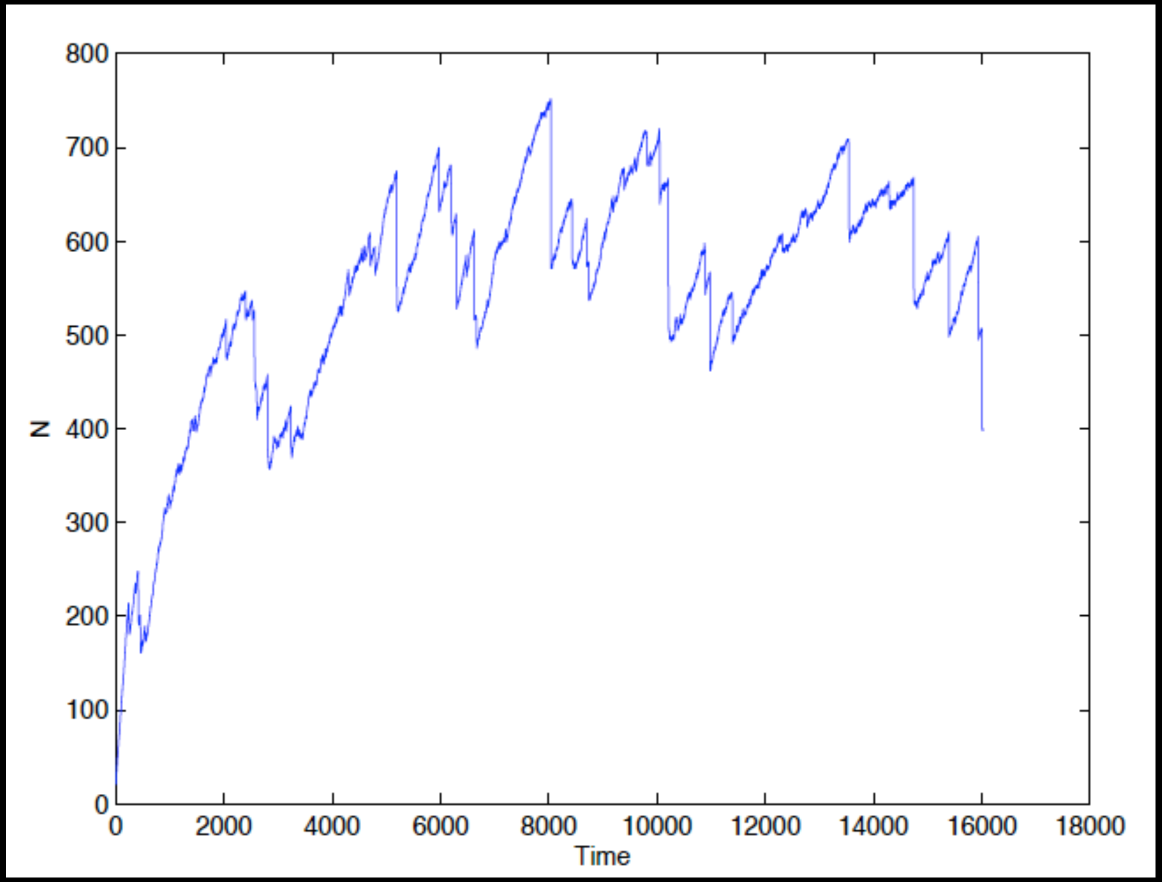
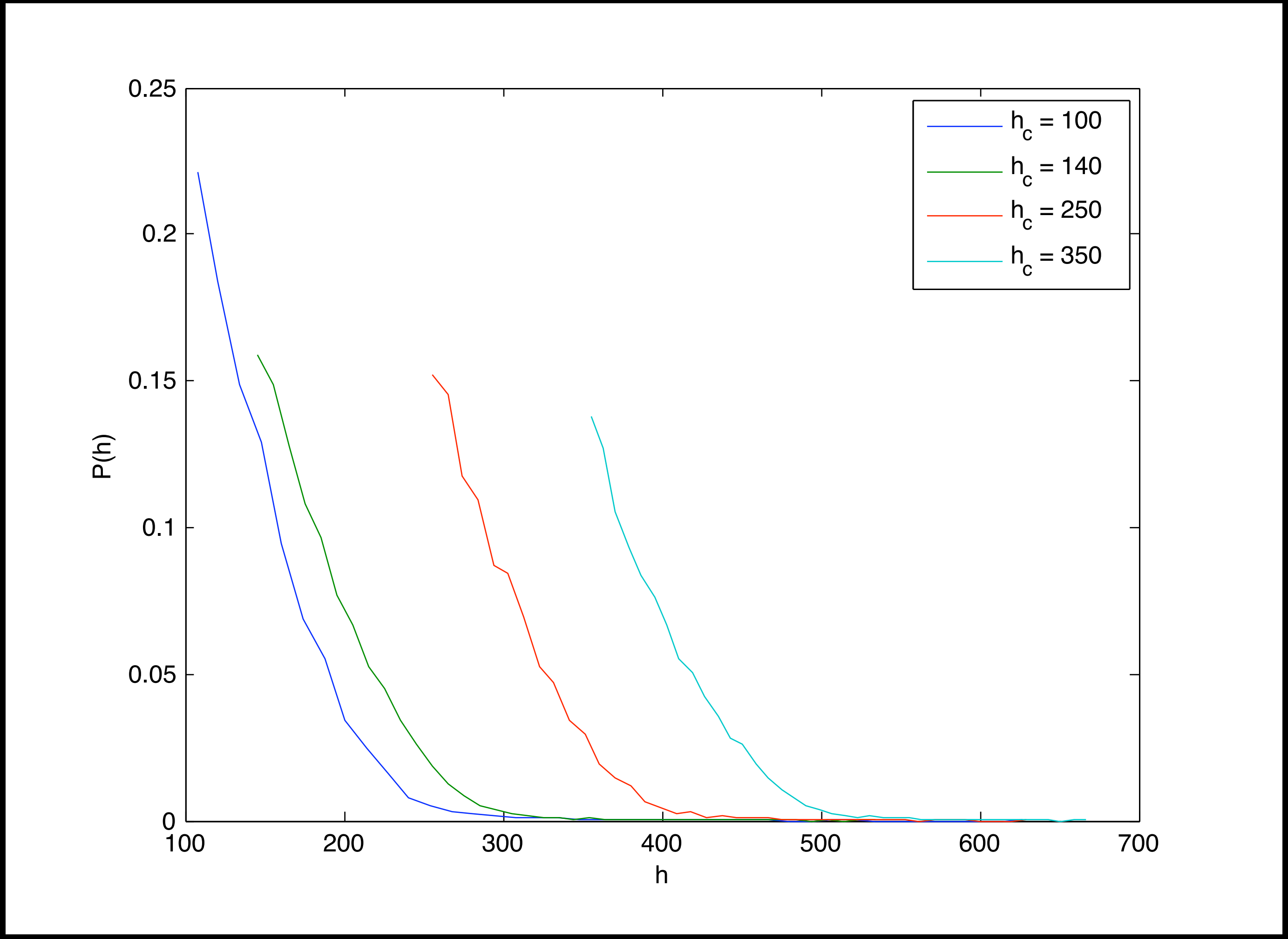


Figure 1: Schematic of network, showing the direction of links.

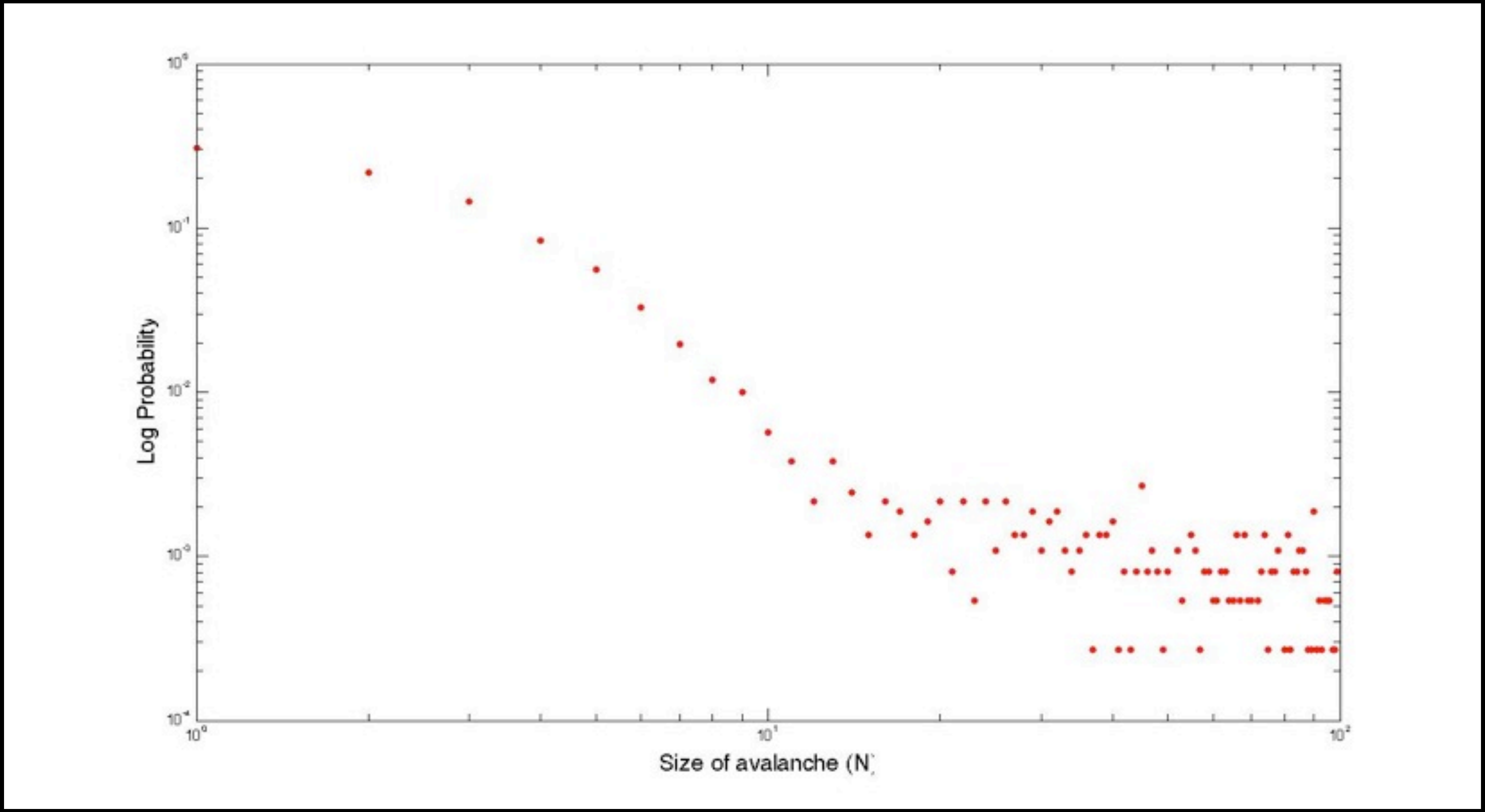
These nodes are removed again after each exchange round.

Fluctuations and perturbation





Size distribution of spontaneous "crashes"



Spread in degreed decreases robustness

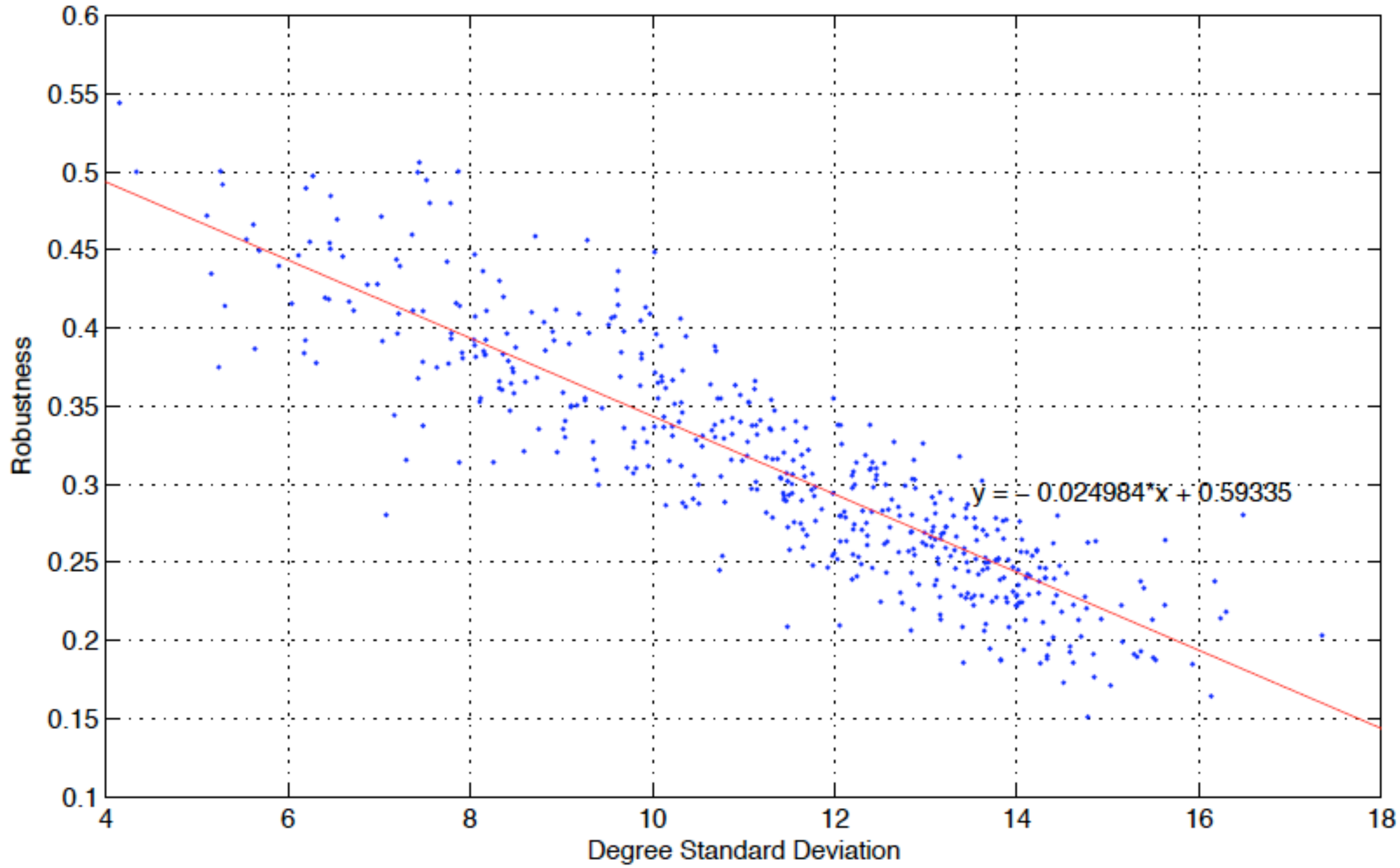


Figure 4: Plot of Robustness against standard deviation of Degree of 500 runs. Shows that increase in diversity of degree reduce robustness.

Spread in risk willingness increases robustness

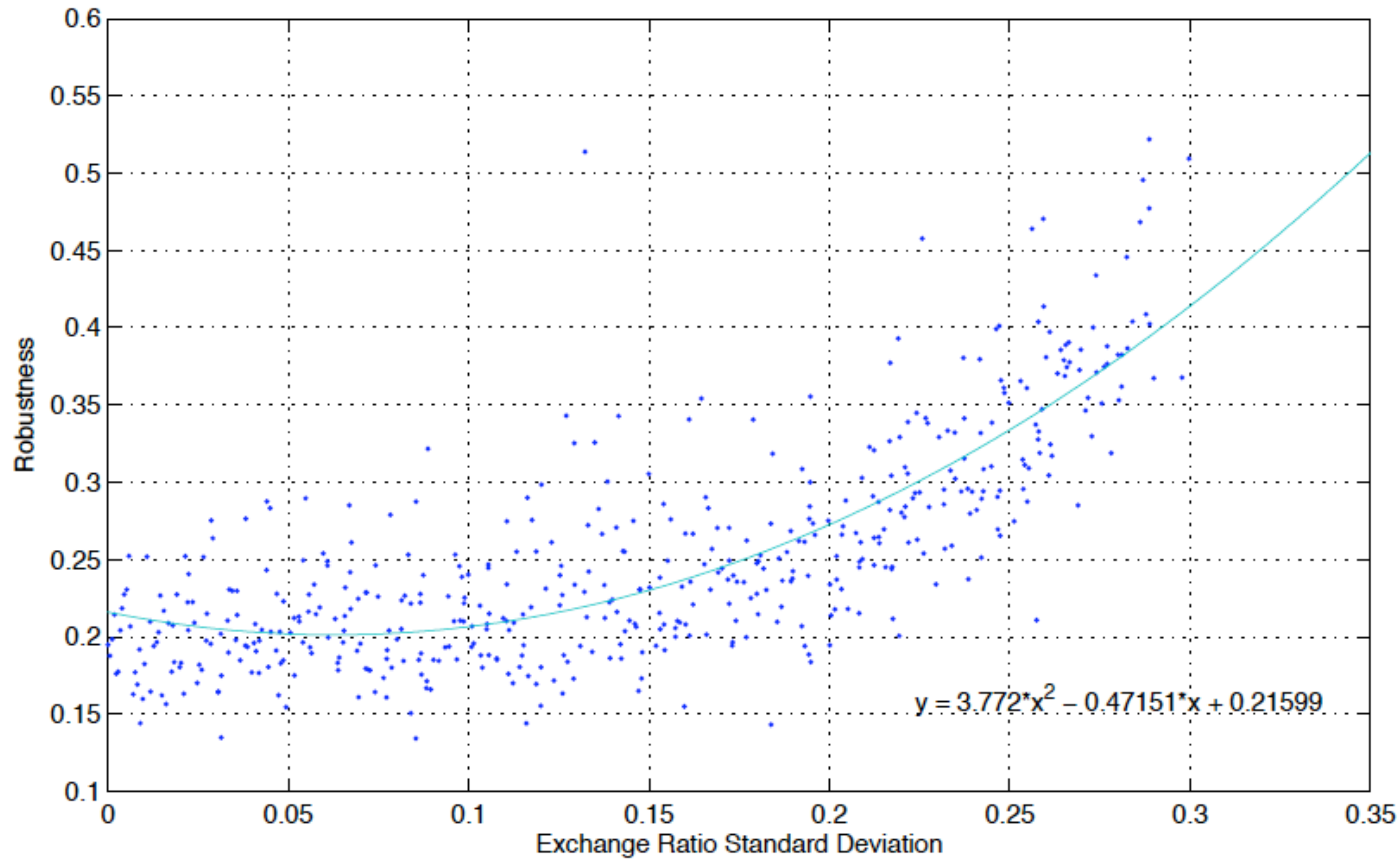


Figure 5: Plot of Robustness against standard deviation of Exchange ratio of 500 runs. Shows that increase in diversity of exchange ratio *increases* robustness.

Summary + Conclusion

- Multi-level dynamics
  intermittency at macro-level
- Non-stationary
  collective adaptation
- Transition times found to follow record statistics
  not always but often

Thank you

Web sites: www.ma.ic.ac.uk/~hjjens
www3.imperial.ac.uk/complexityandnetworks