Stochastic reconstruction of a dynamical system for early prediction of critical transitions

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THE PROBLEM:

Early prediction of critical transitions in natural systems

THE WAY OF SOLUTION:

1. Construction of *parameterized non-autonomous* model of an evolution operator by virtue of *a direct distillation of the observed time series*

2. Analysis of the model behavior outside observed time interval

OUTLINE:

• Global reconstruction of non-autonomous dynamical systems from time series: very brief introduction
(Takens theorems, evolution operator form, non-stationarity & long-term behaviour prediction, necessity of Bayesian approach)

• Damnation of the dimensionality

• Low-dimensional stochastic reconstruction: description and demonstration of predictive abilities

• Optimal low-dimensional stochastic models: Bayesian evidence as a cost function for selection of structural parameter values

Distillation of the model operator from observed time series

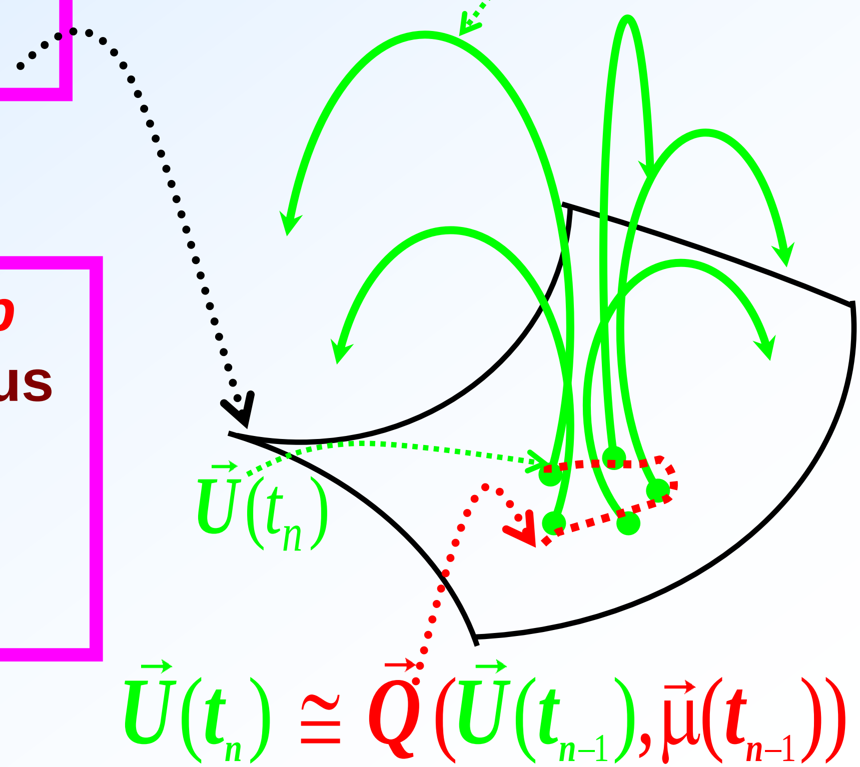
1. Reconstruction *of phase trajectory (Takens, 1981)*

$$\vec{Y}(t_k)=\{y(t_k),\ y(t_k+\Delta t),\ \ldots,\ y(t_k+(d_E-1)\Delta t)\},\qquad d_E \geq 2d_s+1$$

2. Choice of *Poincare* section

3. Approximation of *Poincare map* by a parameterized non-autonomous model

$$\vec{U}(t_n)\cong\vec{Q}(\vec{U}(t_{n-1}),\vec{\mu}(t_{n-1}))$$



Distillation of the model operator from observed time series

4. Unavoidable measurement noise $\vec{\xi}_t$:

$$\vec{x}_t = \vec{u}_t + \vec{\xi}_t$$

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Statistical description of the model parameters via posterior Probability Density Function (*PDF*): Bayesian approach

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Probabilistic analysis of future model behaviour: Markov Chain Monte-Carlo (*MCMC*) technique

Distillation of the model operator from observed time series

5. General configuration of the evolution operator model:

$$\vec{u}_{t+1} = \vec{Q}(\vec{u}_t, \vec{\mu}(t)) + \vec{\eta}_t\ , \quad \vec{x}_t = \vec{u}_t + \vec{\xi}_t$$

Bayes theorem:

$$p(\vec{\mu}|\vec{x}) \propto p(\vec{x}|\vec{\mu}) \times p(\vec{\mu})$$

Here $p(\vec{\mu}|\vec{x})$ **is posterior *conditional* PDF of model parameters,**
$p(\vec{x}|\vec{\mu})$ **is *likelihood* (prior *conditional* PDF), and**
$p(\vec{\mu})$ **reflects prior information about reconstructed system.**

Approximation of "good" model: $\vec{\eta}_t \to 0$,

$$p(\vec{x}|\vec{\mu}) = \int p(\vec{x}|\vec{u}, \vec{\mu})\, d\vec{u}$$

$$p(\vec{x}\,|\,\vec{u}, \vec{\mu}) = \prod_{l=0}^{[T/w]-1} \prod_{j=0}^{w-1} w_{\xi}(\vec{x}_{l\times w+j} - \vec{f}^{\,j}(\vec{u}_{l\times w}, \vec{\mu}))$$

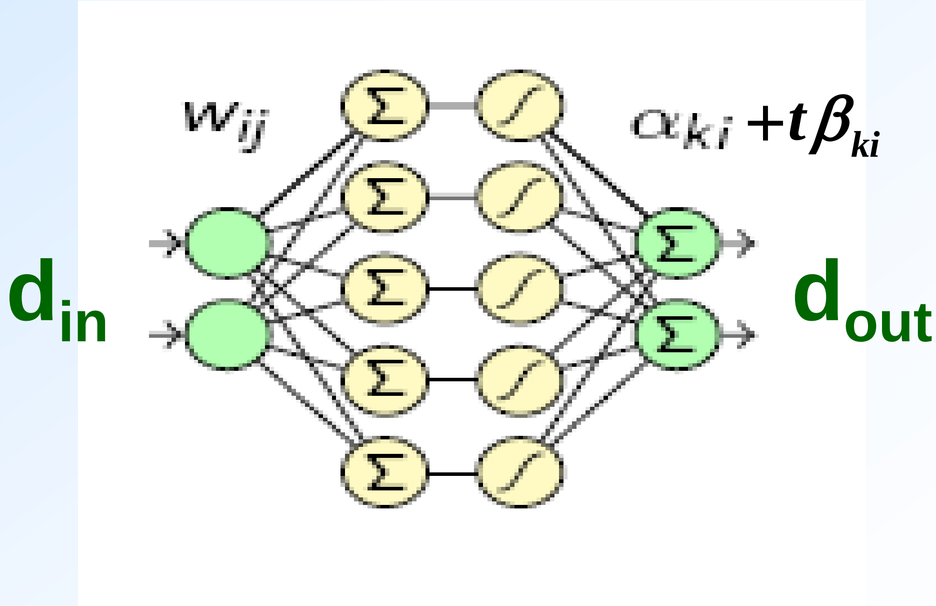
Above T is the *time series* (TS) duration, t is the number of time instants, l numbers *segments* of the TS, w is duration of the segment, and j numbers *time instants within a separate segment*

Distillation of the model operator from observed time series

6. Functional form of the model:

Artificial Neural Networks (ANN)

$$ANN_{d_{in}}^{d_{out}}(U) = \left\{ \sum_{i=1}^{m} (\alpha_{ki} + t\beta_{ki}) \tanh\left(\sum_{j=1}^{d_{in}} w_{ij} U_j + \gamma_i \right) \right\}_{k=1}^{d_{out}}$$



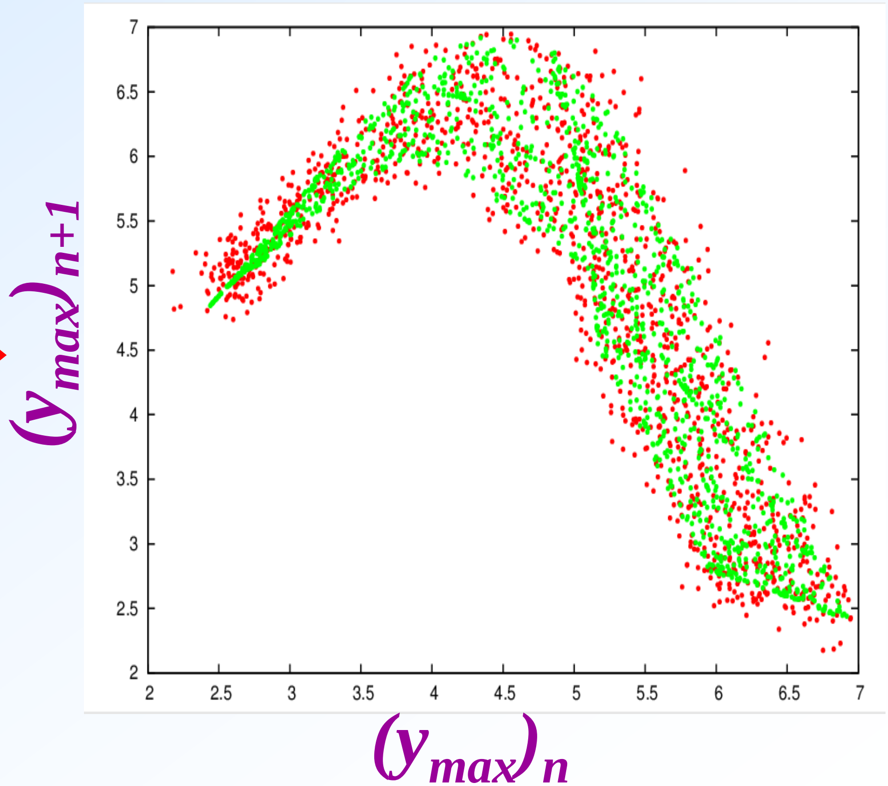
Example #1 *Prognosis of bifurcations by the noisy chaotic TS*

Rössler system: $\dot{x} = -y - z,\ \dot{y} = x + e\cdot y,\ \dot{z} = f - \mu\cdot z + x\cdot z$

Observer is *nonstationary noisy* time series $y_n = u_n + \xi_n$.

Measurement noise ξ_n **is Gaussian**

"Experimental" *nonstationary* evolution operator, obtained via reconstruction of phase trajectory, for *noiseless* (*green*) and *noisy* TS (*red*). Noise to signal ratio is 0.1



Example#1: *Prognosis of bifurcations by the noisy chaotic TS*

Rössler system

Prognosis of the bifurcations

Top Figure:

Correct BD
$(\mu \in [5.13; 2]; \ e = f = 0.2)$
with marked *(lilac)* part corresponding to *noiseless* TS.
The diagram corresponding to the "observed" *noisy* TS is shown by *light blue* points

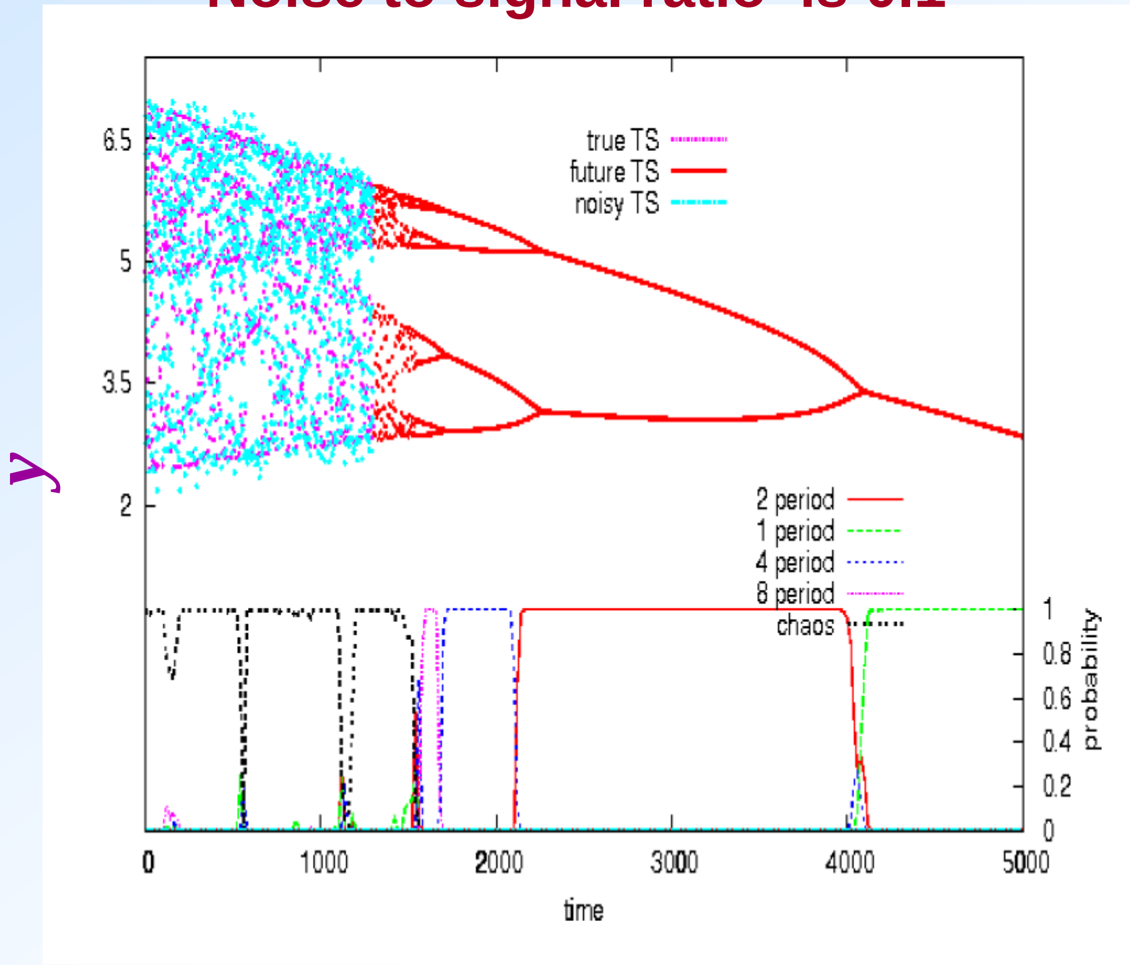
$(\mu \in [5.13; 4.41])$

Bottom Figure:

Probability of the predicted behaviour regimes calculated by the Algorithm applied to the "observed" TS.

Model dimension is *N* =1

Noise to signal ratio is 0.1



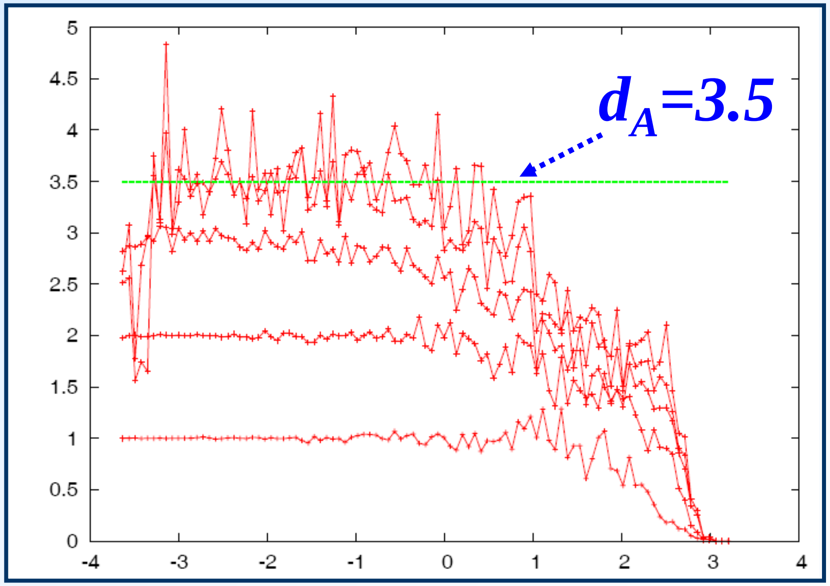
The segment length w=4

EXAMPLE OF HIGH-DIMENTIONAL DETERMINISTIC SYSTEM RECONSTRUCTION

TWO COUPLED RŐSSLER'S SYSTEMS:

$$
\begin{cases}
dx_1/dt = -y_1 - z_1 + 0.03x_2^2 \\
dy_1/dt = x_1 + 0.2y_1 \\
dz_1/dt = 0.2 - cz_1 + x_1 z_1 \\
dx_2/dt = -y_2 - z_2 + 0.02x_1 \\
dy_2/dt = x_2 + 0.2y_2 \\
dz_2/dt = 0.2 - cz_2 + x_2 z_2
\end{cases}
$$

Correlation dimension estimated from y_2 variable time-series (for $c=6$)



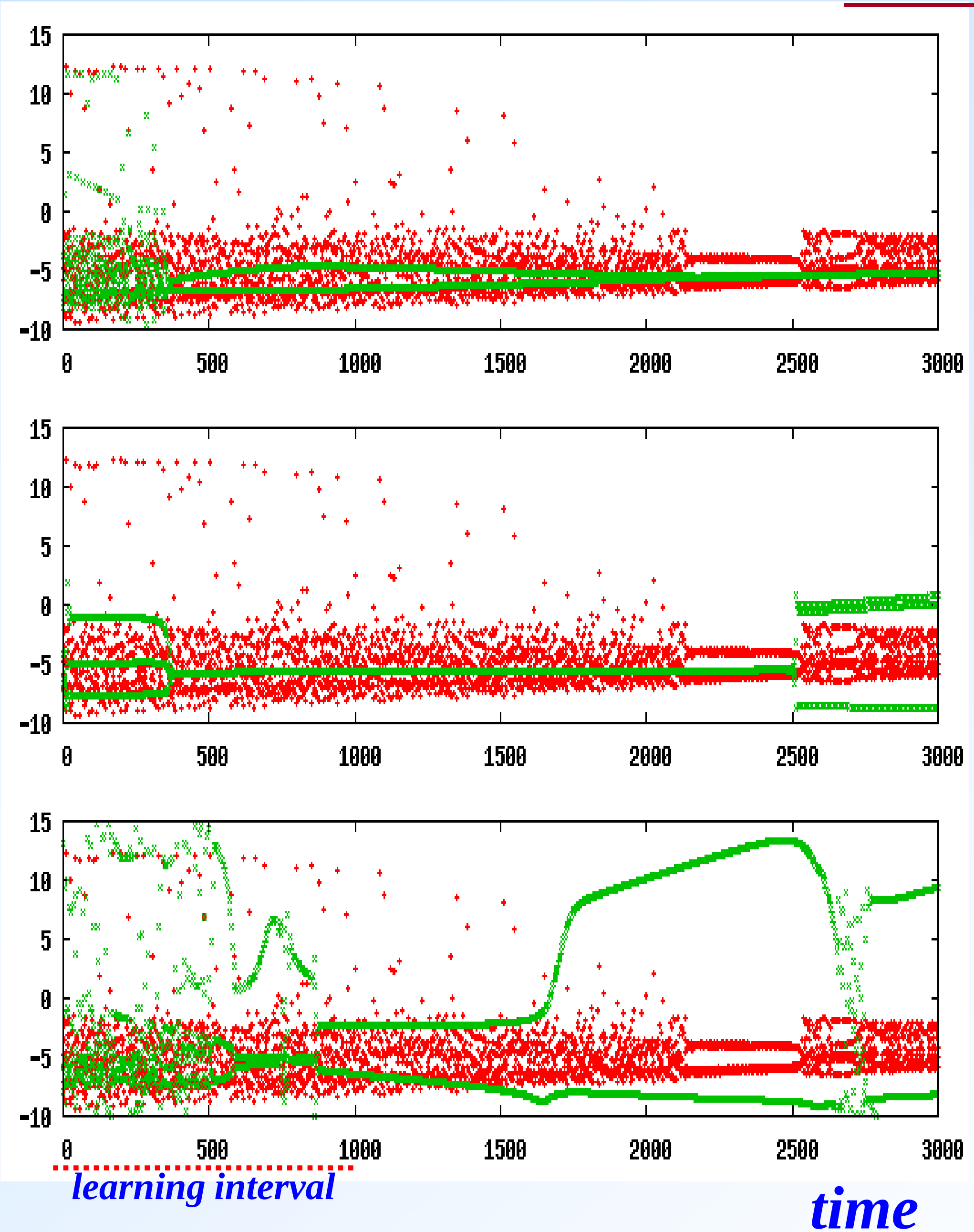
INABILITY OF DETERMINISTIC MODELING:

From Taken's theorem:

$$d_E \geq 2d_S + 1 > 2d_A + 1$$

Overembedding problem:
we are forced to construct high-dimensional model having in the hands (reconstructed from time series) significantly lower dimensional attractor

ATTEMPT OF PROGNOSIS VIA CONSTRUCTION OF HIGH-DIMENSIONAL DETERMINISTIC MODELS



RED DOTS:

Discrete time series of y_2 variable generated by two coupled Rőssler's systems under slow trend of parameter *c* value (changed from *6* to *3* during time interval *[0, 3000]*).

GREEN DOTS:

Time series generated by three different *deterministic* models that were learned by piece *[0, 1000]* of red dots' time series. All models are six-dimensional and equally probable (correspond to close vicinity of posterior distribution function $P_{ps}(\vec{\mu}/\vec{U})$ maximum).

RESULT:

All models exhibit qualitatively different behaviour even within time interval of the learning!

CONCLUSION:

Overembedding leads to non-robust models that couldn't to use for prognosis of qualitative changes of the underlaying system behaviour!

Fundamental limitations for *deterministic* reconstruction of *complex* systems by time series (*TS*)

Limitation on system complexity:

"Damnation" of overembedding:
(Takens theorem)

$$d_E \geq 2d_a+1$$

d_E *is embedding dimension,*
d_a *is attractor dimension*

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Impossibility of high-dimensional system reconstruction

Limitation on prognosis direction:

TS contains information exclusively about phase sub-space that is determined by underlying attractor

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Impossibility to predict bifurcations from complex to simpler behavior

Limitation on prior information:

We need to know before hand, that TS is generated by deterministic dynamical system!?

THE WAY OF DECISION

Idea:

To construct by time series
***non-autonomous low-dimensional stochastic* models.**
The robust dynamic properties of the system evolution can be described by a low dimensional deterministic operator, while other features are considered as a stochastic disturbance.

The way of realization:

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Choice of configuration of a *stochastic* model of the evolution operator

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Implementation of the operator structure by the "universal" functional form

⇩

Applying Bayesian approach to learning of the model

⇩

Prognosis of qualitative behavior of the system

Let we have a time series

$$\{\mathbf{U}(t_n) = \mathbf{U}_n\}_{n=1}^{N}, \mathbf{U}(t) \in \Re^d$$

Suppose these data are coupled by random evolution operator

$$\begin{cases} \mathbf{U}_{n+1} = \boldsymbol{\varphi}(\omega_n) \circ \mathbf{U}_n, \ \boldsymbol{\varphi} : \Omega \times \Re^d \to \Re^d \\ \omega_{n+1} = \theta \circ \omega_n, \theta : \Omega \to \Omega \end{cases}$$

$$\mathbf{U}_{n+1} = \mathbf{f}(\mathbf{U}_n) + \boldsymbol{\eta}(\omega, \mathbf{U}_n)$$

$$\mathbf{f}(\mathbf{U}) = \mathbf{E}(\boldsymbol{\varphi}(\omega, \mathbf{U})) \qquad \boldsymbol{\eta}(\omega, \mathbf{U}) = \boldsymbol{\varphi}(\omega, \mathbf{U}) - \mathbf{f}(\mathbf{U})$$

Suppose that deterministic component f is defined by long-correlated processes, while stochastic component has short time scale and takes the form:

$$\eta(\omega, U) = \hat{g}(U) \cdot \xi(\omega), \ \hat{g} : \Re^d \to \Re^{d \times M}, \xi : \Omega \to \Re^M$$

We approximate distribution of ξ by Gaussian form and consider it as white noise.

Finally, we have a model in following form:

$$U_{n+1} = f(U_n) + \hat{g}(U_n) \cdot \xi_n$$

Stochastic model

$$\mathbf{x} = \mathbf{f}(\bar{\mathbf{x}}, t, \boldsymbol{\mu}_1) + \mathbf{g}(\bar{\mathbf{x}}, t, \boldsymbol{\mu}_2)\boldsymbol{\xi}, \qquad \mathbf{x} \in \Re^{d_1}, \boldsymbol{\xi} \in \Re^{d_2}$$

Dynamical properties

Nonuniformity of stochastic component

$$\mathbf{g} = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1d_2} \\ 0 & g_{22} & \cdots & g_{2d_2} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & g_{d_1 d_2} \end{pmatrix}$$

$$P_\xi(\xi_1, \ldots \xi_N) \propto \prod_i \exp\left(-\frac{|\xi_i|^2}{2}\right)$$

$$\mathbf{C} = \mathbf{g}\mathbf{g}^{\mathrm{T}}$$ *- covariance matrix of noise*

Parameterization of f and g by artificial neural networks:

$$\mathrm{ANN}_{d_{in}}^{d_{out}}(\mathbf{x}, t) = \left\{ \sum_{i=1}^{m} (\alpha_{ki} + t\beta_{ki}) \tanh\left(\sum_{j=1}^{d_{in}} w_{ij} x_j + \gamma_i \right) \right\}_{k=1}^{d_{out}}$$

$$\mathbf{f}(\mathbf{x}, t, \boldsymbol{\mu}_1) = \mathrm{ANN}_{d_1}^{d_2}(\mathbf{x}, t) \qquad \mathbf{g}(\mathbf{x}, t, \boldsymbol{\mu}) = \mathrm{ANN}_{d_1}^{d_2(d_2+1)/2}(\mathbf{x}, t)$$

The cost function for estimation of parameters

We have a time series: $\{\mathbf{x}_i \equiv \mathbf{x}(t_i)\}_{i=1}^{N}, \mathbf{x} \in \Re^d$

and we have a model:

$$\mathbf{x} = \mathbf{f}(\bar{\mathbf{x}}, t, \boldsymbol{\mu}_1) + \mathbf{g}(\bar{\mathbf{x}}, t, \boldsymbol{\mu}_2)\boldsymbol{\xi}, \quad P_\xi(\xi_1, \ldots \xi_N) \propto \prod_i \exp\left(-\frac{|\xi_i|^2}{2}\right)$$

$$P(\mathbf{x}_1, \ldots \mathbf{x}_N \mid \boldsymbol{\mu}) \propto \prod_i \left\| \mathbf{g}(\mathbf{x}_i, \boldsymbol{\mu}_2)\mathbf{g}^{\mathrm{T}}(\mathbf{x}_i, \boldsymbol{\mu}_2) \right\|^{-\frac{1}{2}} \times$$

$$\times \exp\left(-\frac{1}{2}\left(\mathbf{x}_{i+1} - \mathbf{f}(\mathbf{x}_i, t_i, \boldsymbol{\mu}_1)\right)^{\mathrm{T}} \left(\mathbf{g}(\mathbf{x}_i, \boldsymbol{\mu}_2)\mathbf{g}^{\mathrm{T}}(\mathbf{x}_i, \boldsymbol{\mu}_2)\right)^{-1} \left(\mathbf{x}_{i+1} - \mathbf{f}(\mathbf{x}_i, t_i, \boldsymbol{\mu}_1)\right)\right)$$

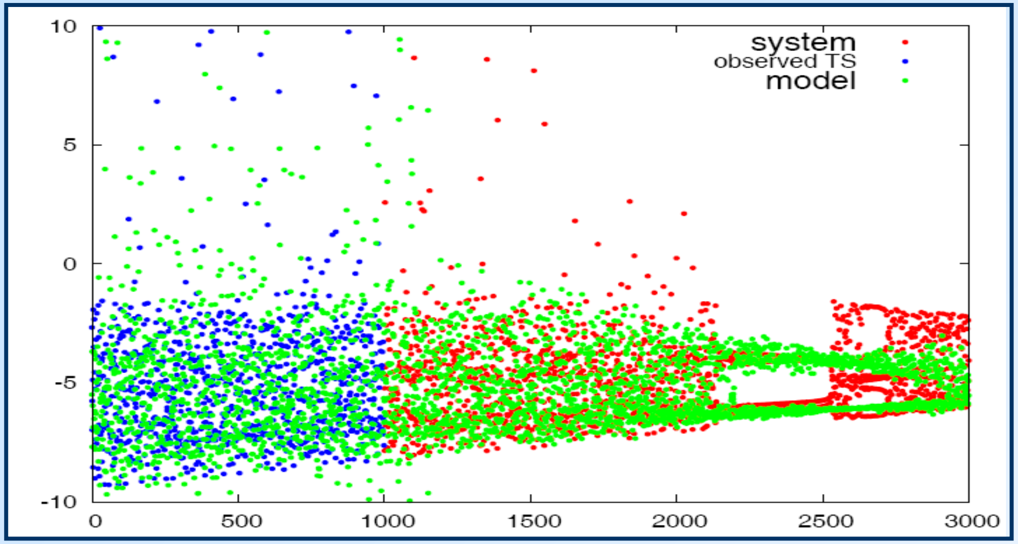
Bayes theorem:

$$P(\boldsymbol{\mu} \mid \mathbf{x}_1, \ldots \mathbf{x}_N) \propto P(\mathbf{x}_1, \ldots \mathbf{x}_N \mid \boldsymbol{\mu}) P(\boldsymbol{\mu}) \quad (*)$$

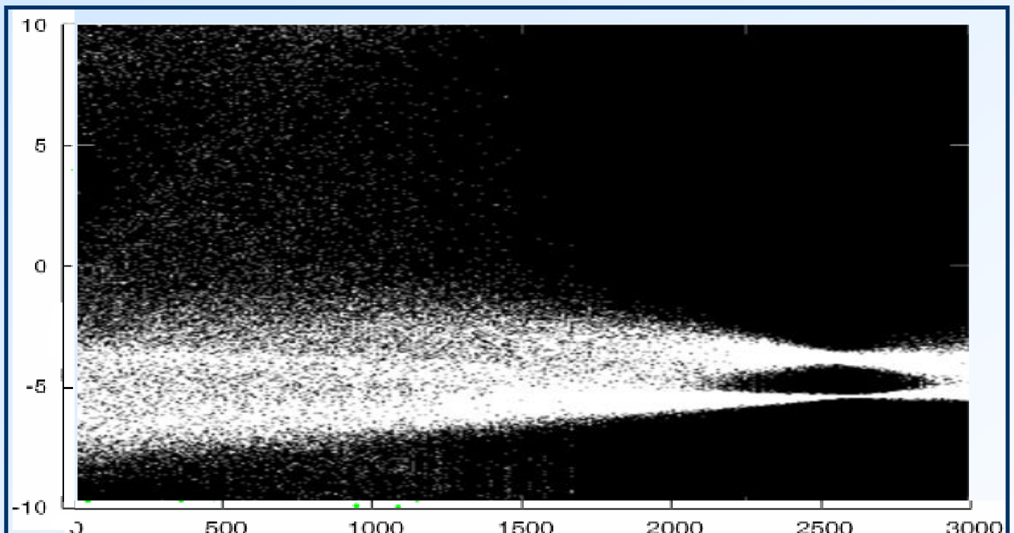
The cost function

TWO COUPLED RŐSSLER'S SYSTEMS:

PROGNOSIS VIA CONSTRUCTION OF LOW-DIMENSIONAL STOCHASTIC MODEL



learning interval *interval of prognosis*



learning interval *interval of prognosis*

TOP FIGURE:

BLUE and RED dots:

Discrete time series of y_2 variable generated by two coupled Rőssler's systems under slow trend of parameter *c* value (changed from *6* to *5* during time interval *[0, 1000]* and from *5* to *3* during time interval *[1000, 3000]*).

GREEN DOTS:

Time series generated by *one-dimensional stochastic* model that were learned by blue dots' time series.

The model corresponds to maximum of posterior distribution function

$$P_{ps}(\vec{v},\vec{\omega}/\vec{U}) \quad .$$

BOTTOM FIGURE:

Invariant measure (i.e. probability density of states in phase space) generated by *one-dimensional stochastic* model that were learned by blue dots' time series.

Prognosis of ENSO dynamics

Example #1

The periodically forced, nonlinear DDE model of ENSO *(Tziperman et al., 1994):* description

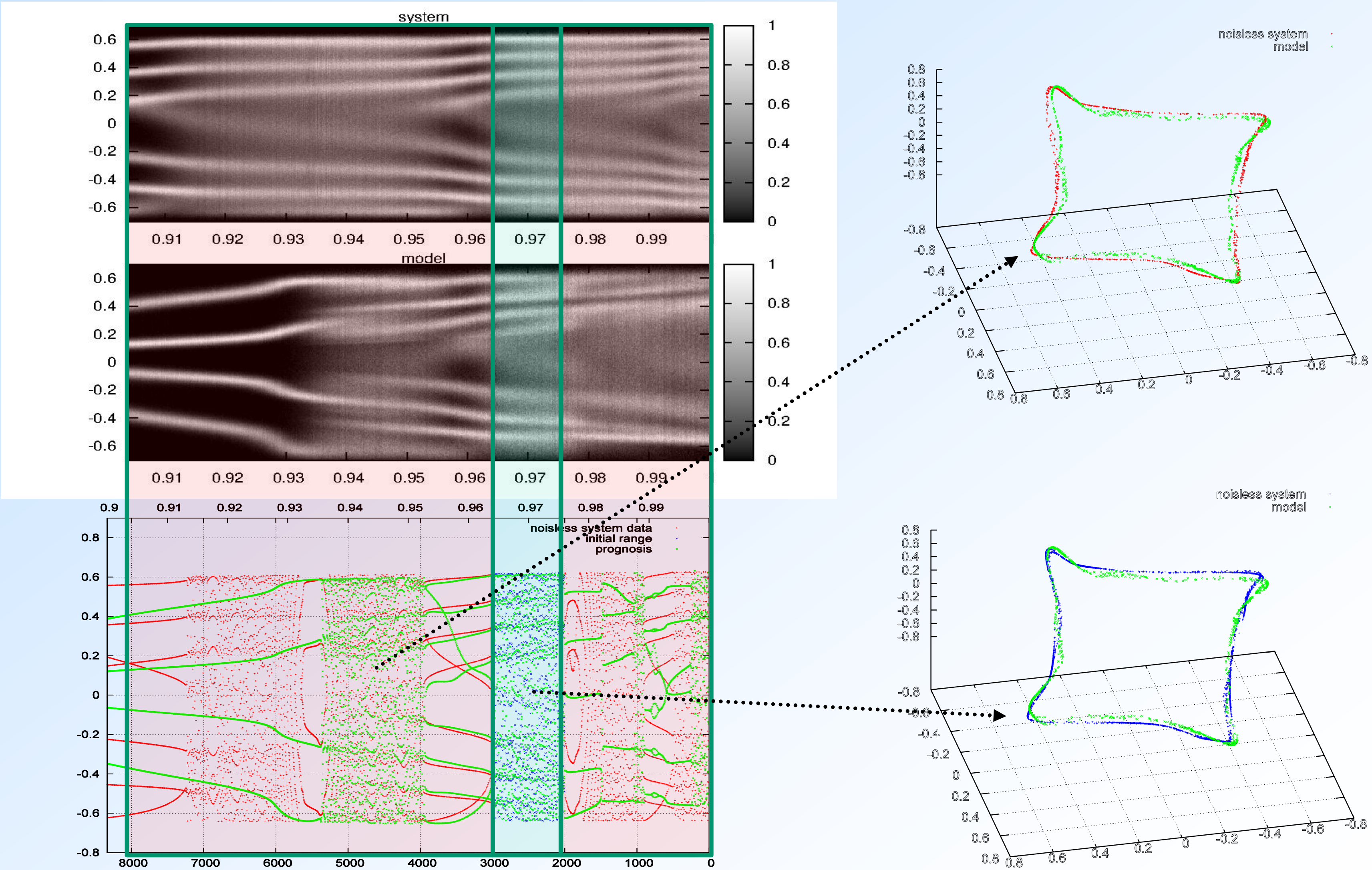
$$dh/dt = -\alpha \cdot \tanh[kh(t-\tau_1)] + \beta \cdot \tanh[kh(t-\tau_2)] + \gamma \cos(2\pi \cdot t)$$

add noise

$$\frac{dh}{dt} = -\alpha \cdot \tanh[k(1+\sigma \cdot y(t))h(t-\tau_1)] + \beta \cdot \tanh[k(1+\sigma \cdot y(t))h(t-\tau_2)] + \gamma \cos(2\pi \cdot t)$$

$\tau_1 = 0.6,\ \tau_2 = 0.2,\ k = 7,\ \sigma = 0.15$

The periodically forced, nonlinear stochastic DDE model of ENSO: prediction of behaviour



Prognosis of ENSO dynamics

Example #2

The Galanti-Tziperman (GT) model (*JAS*, 1999)

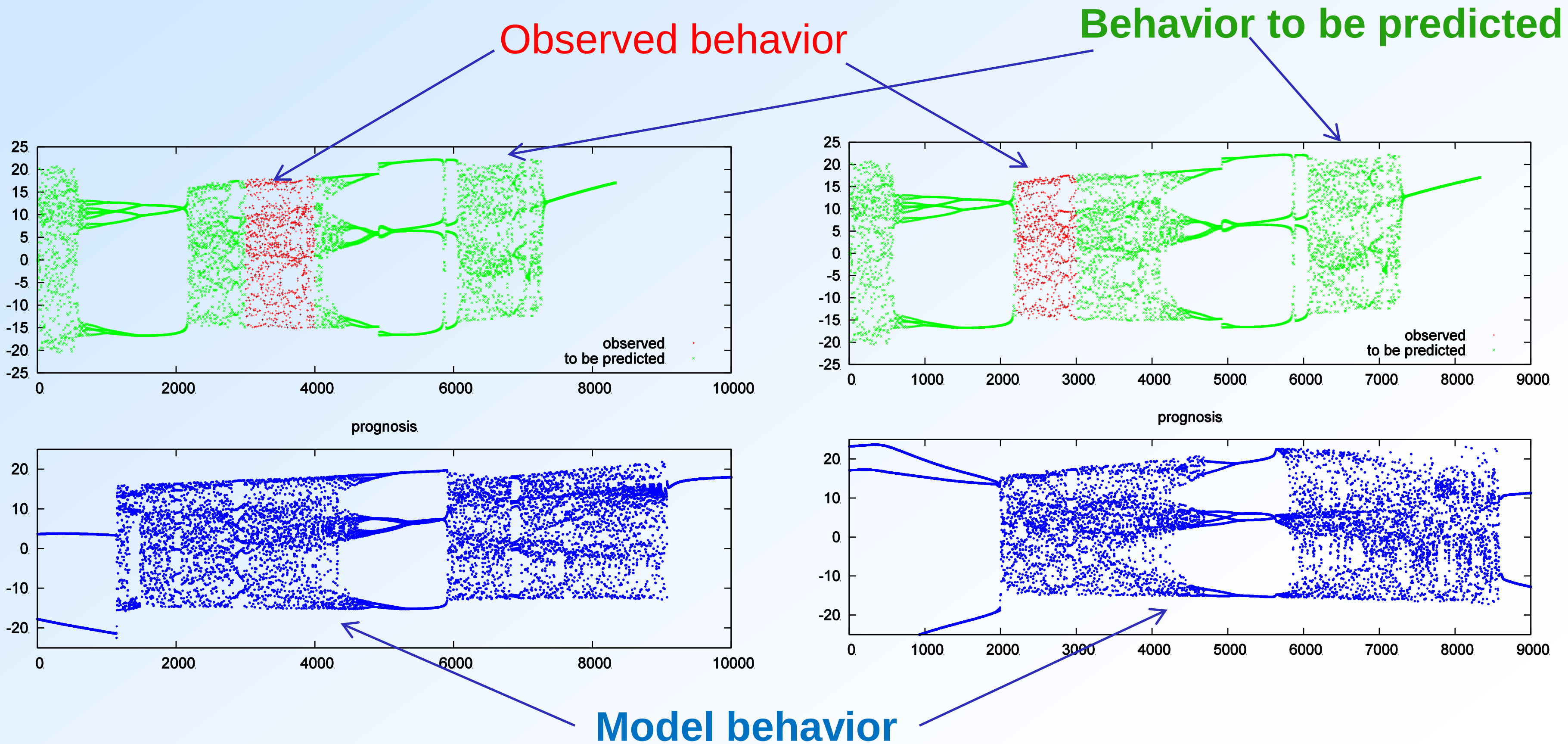
$$\frac{dT}{dt} = -\epsilon_T T(t) - M_0(T(t) - T_{sub}(h(t))),$$

$$h(t) = M_1 e^{-\epsilon_m(\tau_1+\tau_2)} h(t - \tau_1 - \tau_2)$$
$$- M_2 \tau_1 e^{-\epsilon_m(\frac{\tau_1}{2}+\tau_2)} \mu(t - \tau_2 - \frac{\tau_1}{2}) T(t - \tau_2 - \frac{\tau_1}{2})$$
$$+ M_3 \tau_2 e^{-\epsilon_m \frac{\tau_2}{2}} \mu(t - \frac{\tau_2}{2}) T(t - \frac{\tau_2}{2}).$$

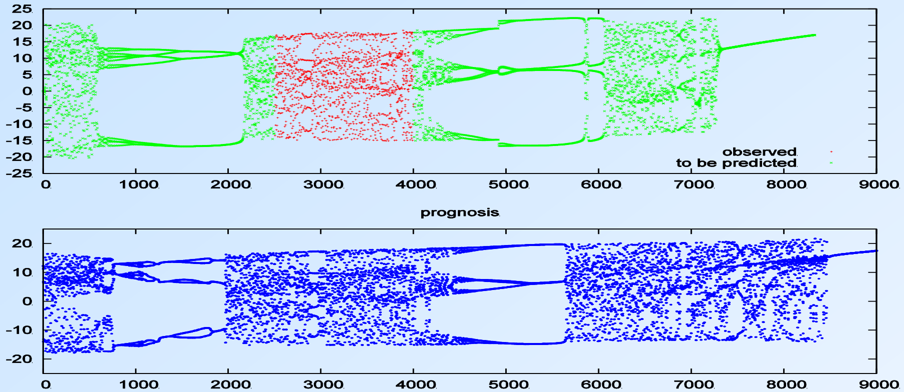
Neutral delay-differential equation (NDDE), derived from Cane-Zebiak and Jin-Neelin models for ENSO: *T* is East-basin SST and *h* is thermocline depth.

Seasonal forcing given by $\mu(t) = 1 + \epsilon \cos(\omega t + \phi)$.

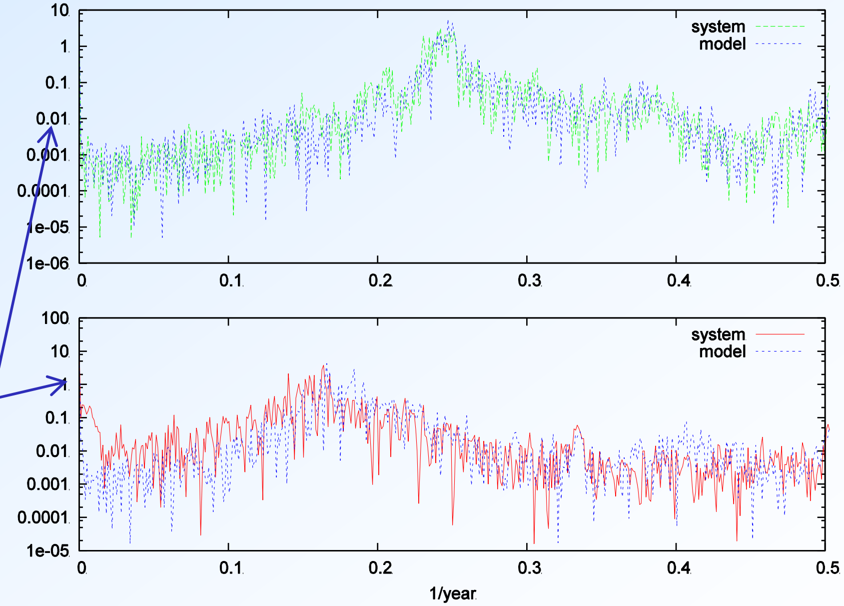
The DDE deterministic model of ENSO (*Galanti-Tziperman model, JAS , 1999*): prediction of behaviour



The DDE deterministic model of ENSO (*Galanti-Tziperman model, JAS , 1999*): prediction of behaviour

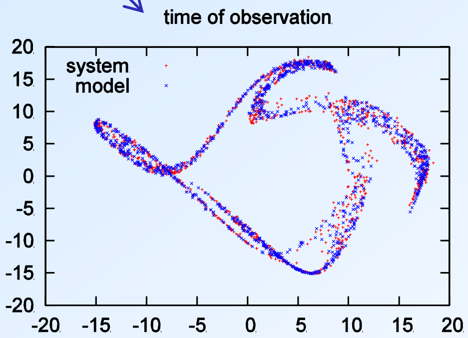


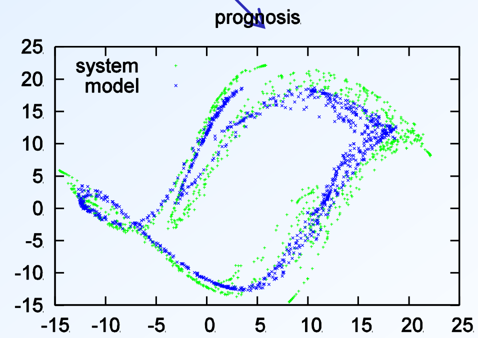
Power spectrums



Time of observation

Prognosis



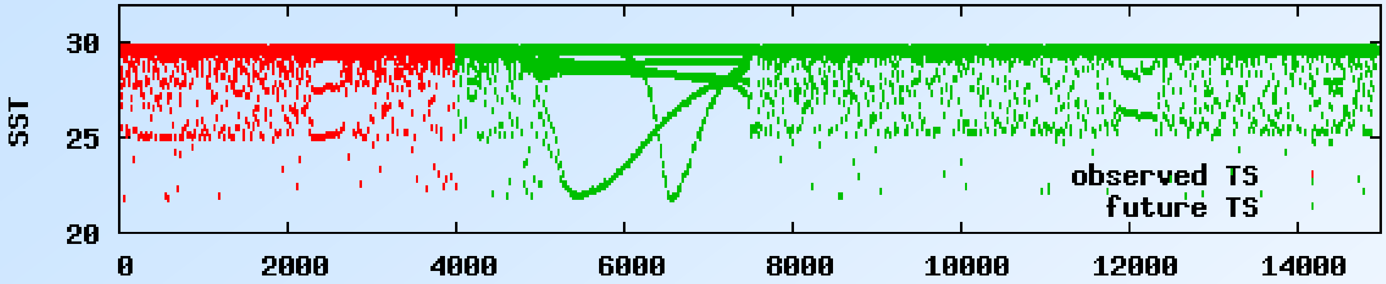


Phase space projections

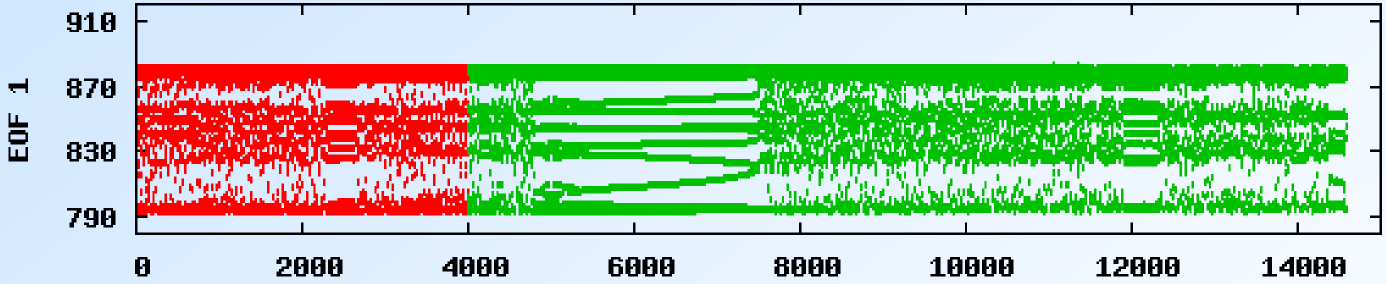
Prognosis of ENSO dynamics: Example #3

Coupled ocean-atmosphere intermediate PDE model

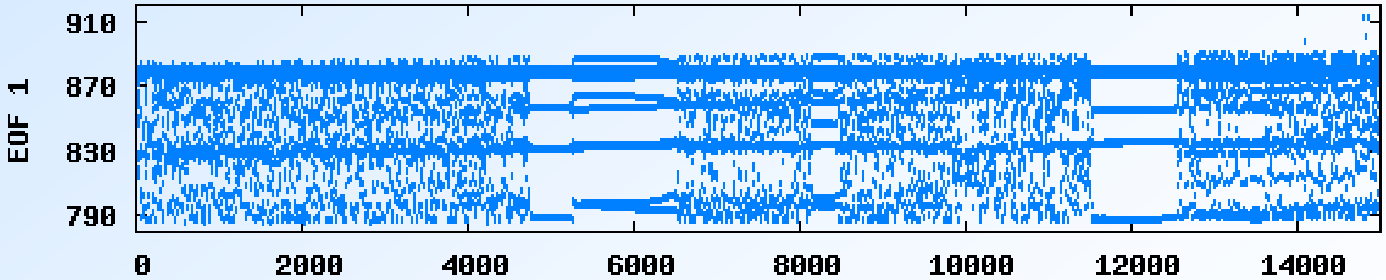
(Jin-Neeling model, JAS , 1993)



SST in one spatial point



First MSSA EOF



Prognosis (model behavior)

2-D Model:

$$X_{n+1} = F(x_n, x_{n-1}) + g(x_n, x_{n-1}) \cdot \xi_n$$

$$\xi_n \sim N(0,1)$$

Example #3

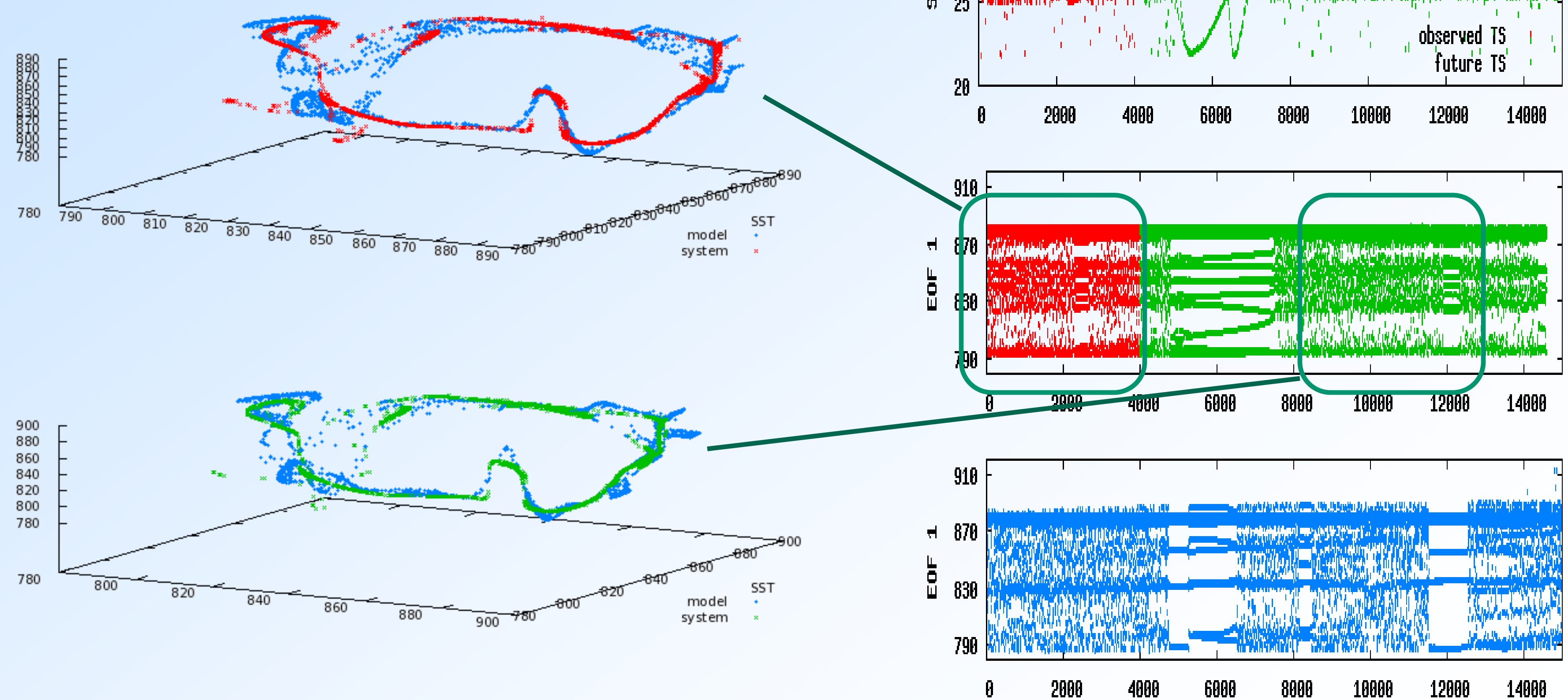
Coupled ocean-atmosphere intermediate PDE model

(Jin-Neeling model, JAS , 1993)

2-D Model:

$$X_{n+1} = F(x_n, x_{n-1}) + g(x_n, x_{n-1}) \cdot \xi_n$$

$$\xi_n \sim N(0,1)$$

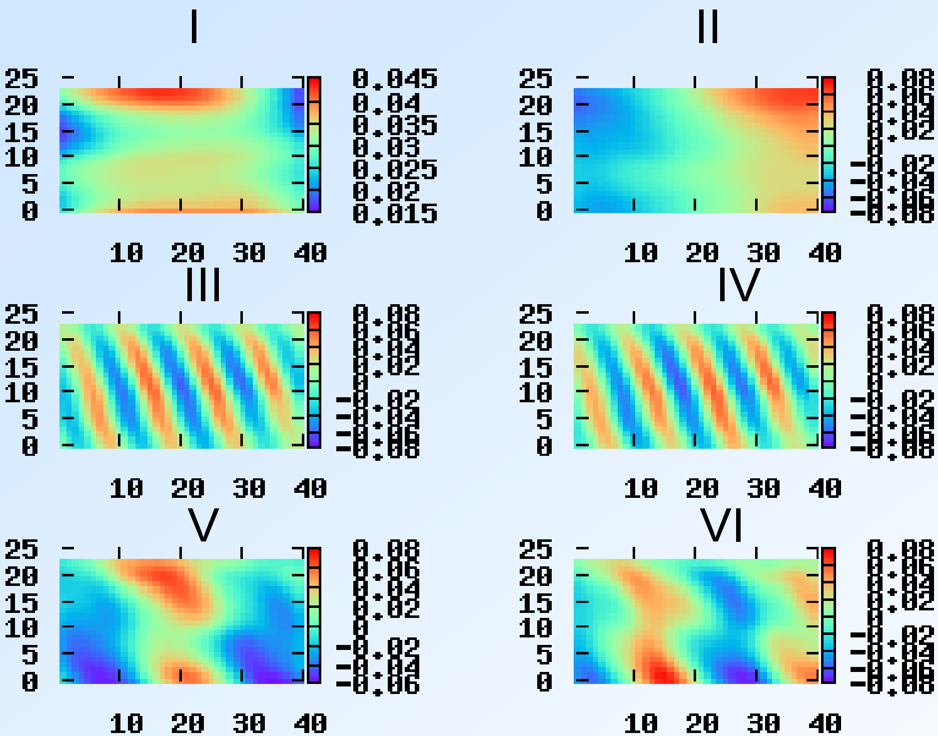


Example #3

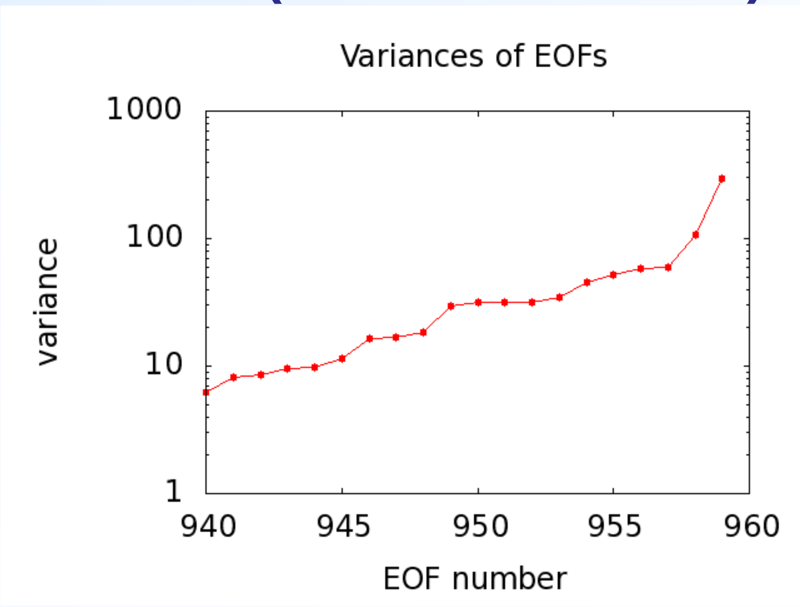
Coupled ocean-atmosphere intermediate PDE model

(Jin-Neeling model, JAS , 1993)

MSSA analysis: spatial-temporal EOF basis:



Eigenvalues of covariance matrix (variances of EOFs)



Conclusion # 1

Dynamics of the high-dimensional systems (deterministic or stochastic) can be predicted by low-dimensional stochastic models.

Optimal model selection criterion: Bayesian evidence

$$U_{n+1} = f\left(x_n^d, \mu\right) + g\left(x_n^d, v\right)\xi_n$$

We define structural parameters as parameters which are responsible for the complexity of the model, and also prior distribution parameters:

$s = \left(m_f, m_g, \sigma_f, \sigma_g, d\right)$ ***- vector of structural parameters in the case of using artificial neural networks***

m_f, m_g ***- amounts of neurons in f and g ,***

σ_f, σ_g ***- prior distribution parameters for f and g***

Bayesian evidence is a posterior distribution density of structural parameters space (i. e. of "model space"):

$$E(s) = P_{posterior}\left(s \middle| U\right) = \iint P\left(U \middle| \mu, v, s\right) \cdot P_{prior}\left(\mu, v, s\right) \cdot d\mu \cdot dv$$

Likelihood

The maximum of E(s) corresponds to the compromise between quality of the data fit and predictiveness of the model. There must be the optimum.

Evidence estimations for stochastic model

To approximate the integral in the case of stochastic model we introduce the function F as follows:

$$E(s)=\iint exp\left(-F(\mu,v,s)\right)\cdot d\mu\cdot dv$$

$$F(\mu,v,s)=\frac{1}{2}\sum_{n=1}^{N}\left[\left(U_{n+1}-f\left(x_n^d,\mu\right)\right)^T\hat{G}^{-1}\left(x_n^d,v\right)\left(U_{n+1}-f\left(x_n^d,\mu\right)\right)+ln\left|\hat{G}\left(x_n^d,v\right)\right|\right]-ln\,P_{prior}(\mu,v,s)$$

The approximation is that we consider the posterior distribution on model parameter space ("nonstructural" parameter space) to be quasi-gaussian in the vicinity of point of its maximum μ_0,v_0

$$F(\mu,v,s)\approx F(\mu_0,v_0,s)+\frac{1}{2}\begin{bmatrix}\mu\\ v\end{bmatrix}^T Q(\mu_0,v_0,s)\begin{bmatrix}\mu\\ v\end{bmatrix}$$

Q is the matrix of the second derivatives of F with respect to μ,v

It can be shown that within this approximation the optimal model corresponds to the minimum of the function:

$$\Phi(s)=-ln\left(E(s)\right)=\left(F+\frac{1}{2}ln\left|det\,Q\right|-\frac{M}{2}ln(2\pi)\right)\Bigg|_{(\mu_0,v_0,s)}$$

M is the dimension of model parameter space μ,v

Evidence estimations for stochastic model

In the further demonstration of the method the following functions will be considered :

$$\Phi\left(m_f, m_g, d\right) = \min_{\sigma_f, \sigma_g} \Phi\left(m_f, m_g, d, \sigma_f, \sigma_g\right)$$ *- dependencies on the ANN complexity*

$$\Phi\left(d\right) = \min_{m_f, m_g, \sigma_f, \sigma_g} \Phi\left(m_f, m_g, d, \sigma_f, \sigma_g\right)$$ *- dependencies on the model dimension*

Example: Stochastic Lorentz system with classical parameters

$$\dot{x} = 10\left(y - x\right)$$

$$\dot{y} = rx - y - xz$$

$$\dot{z} = -\frac{8}{3}z + xy + \sigma\xi\left(t\right)$$

noise with intensity σ

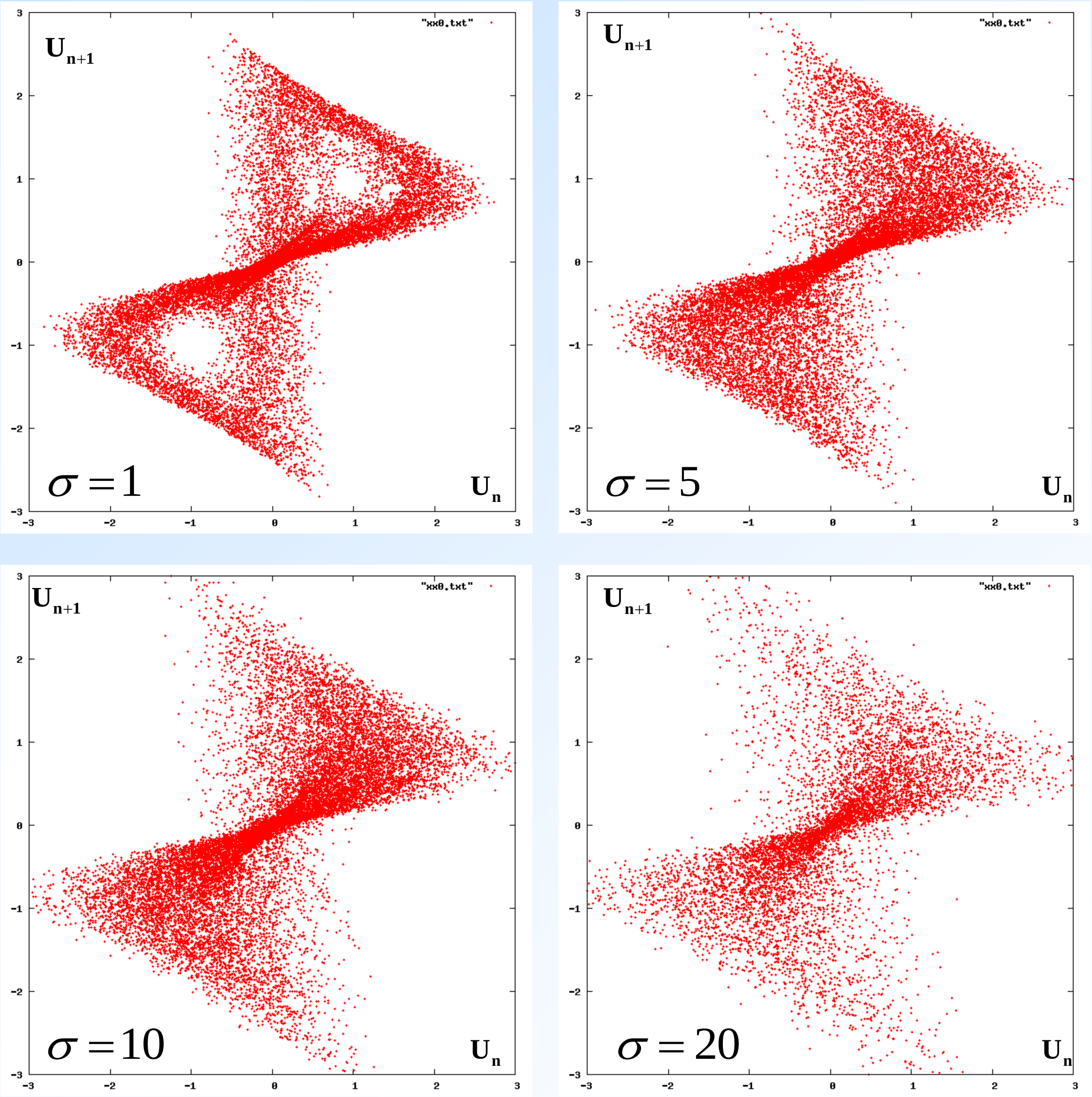
"Observed" time series (at r=25 corresponding to chaotic behavior of the "deterministic" system):

$$U_n = y\left(t_0 + n\tau\right),\ \tau = 0{,}17$$

Construction of d-dimensional state vectors from observations:

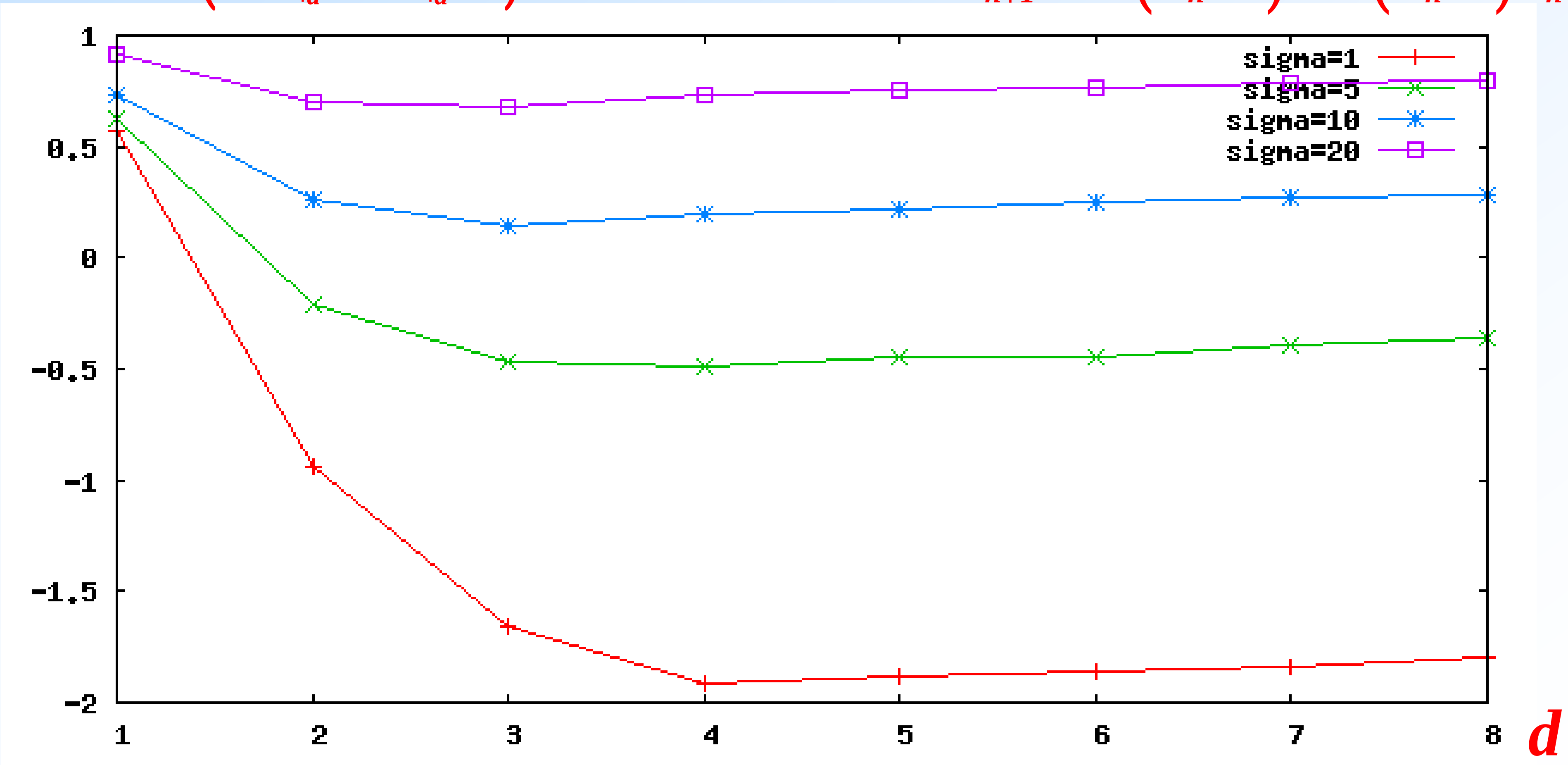
$$U_n^d = \left(U_n, U_{n-1}, \ldots, U_{n-(d-1)}\right)$$

Observable evolution plane at different values of σ



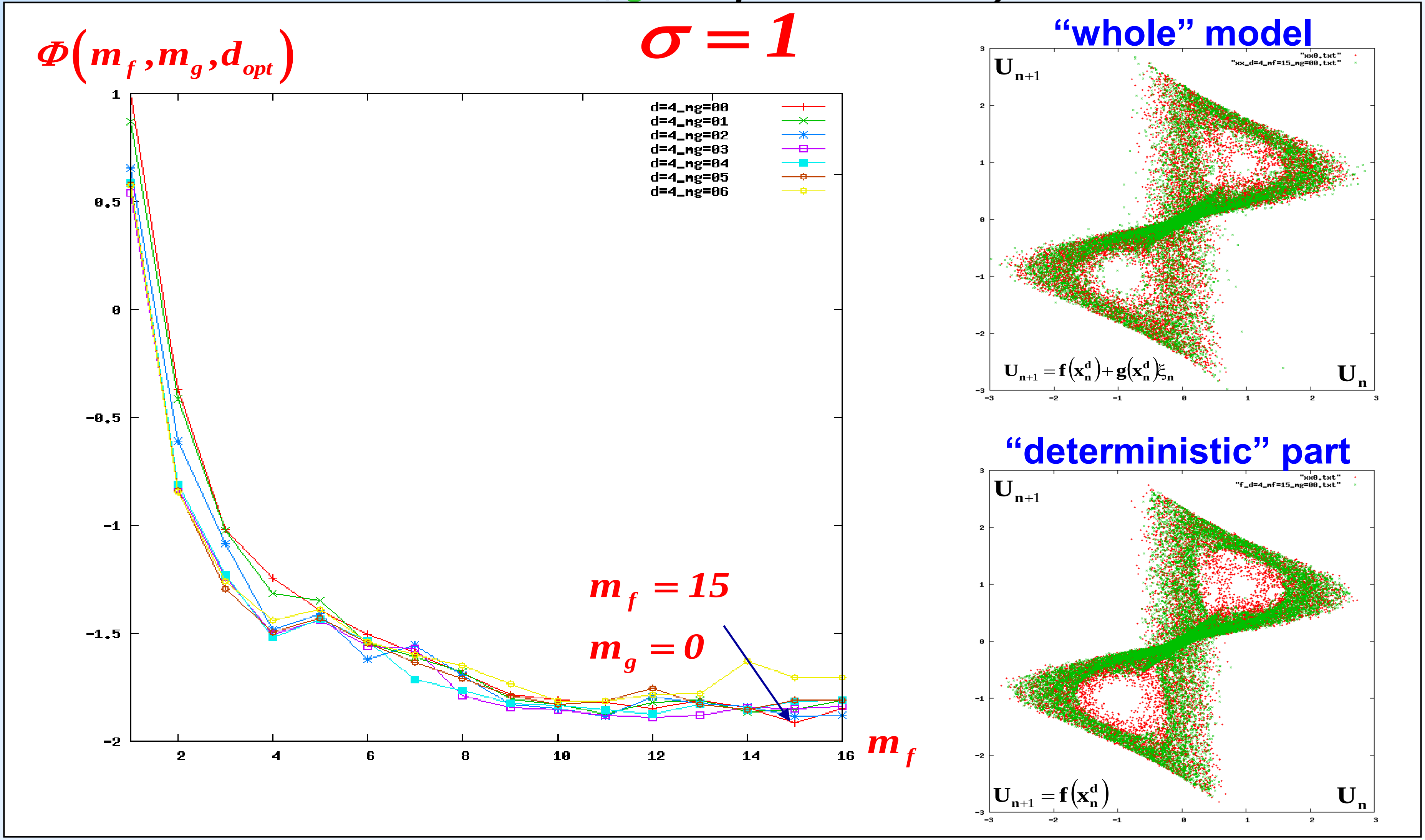
Evidence behavior with respect to the dimension (different colors correspond to systems with different noise intensities)

$$\Phi(d) = \Phi\left(m_f^{opt}\Big|_d, m_g^{opt}\Big|_d, d\right) \qquad U_{n+1} = f\left(x_n^d, \mu\right) + g\left(x_n^d, \nu\right)\xi_n$$

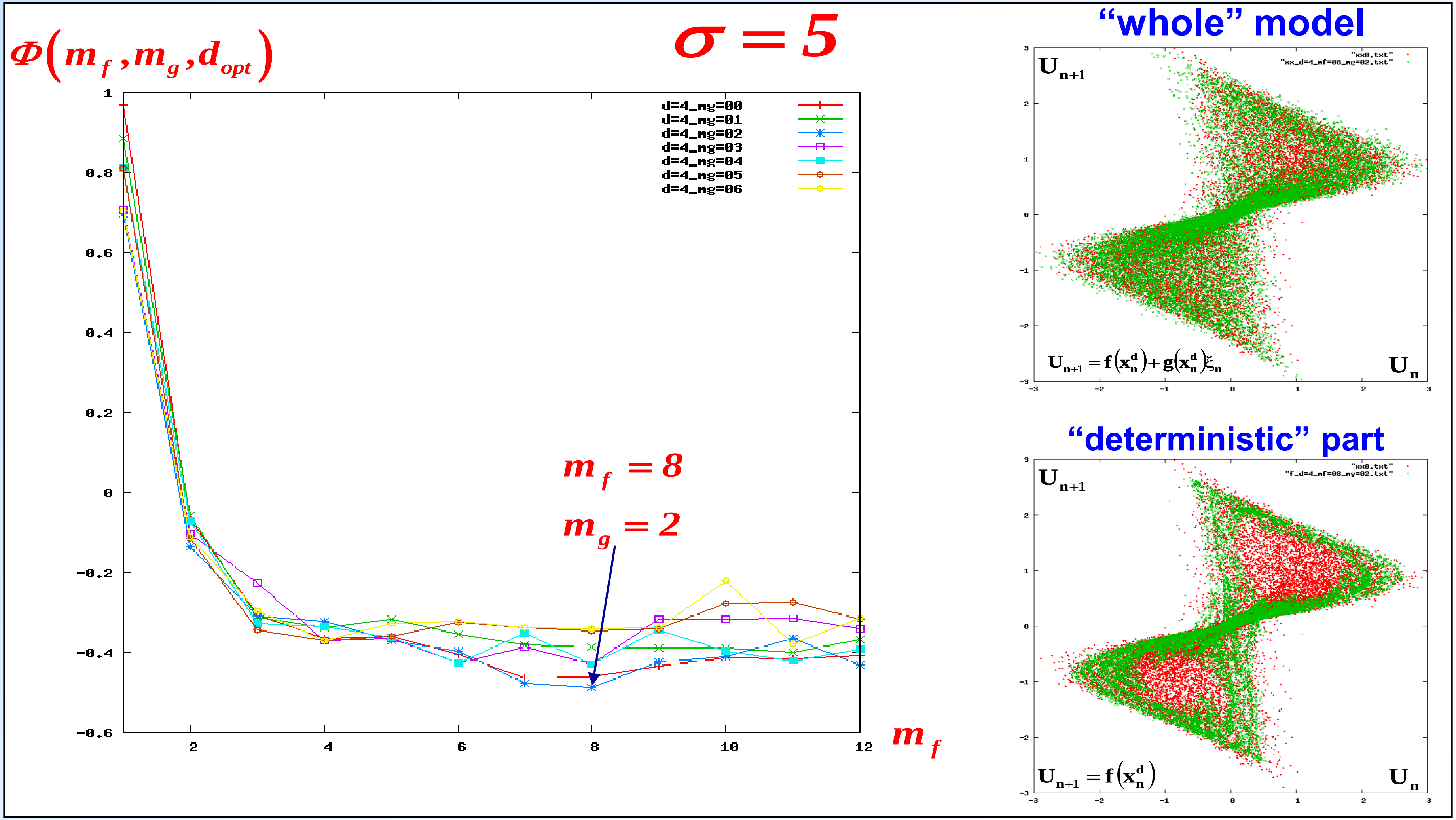


The more the noise intensity, the weaker the connection between neighbor data points, and the less the dimension of the model

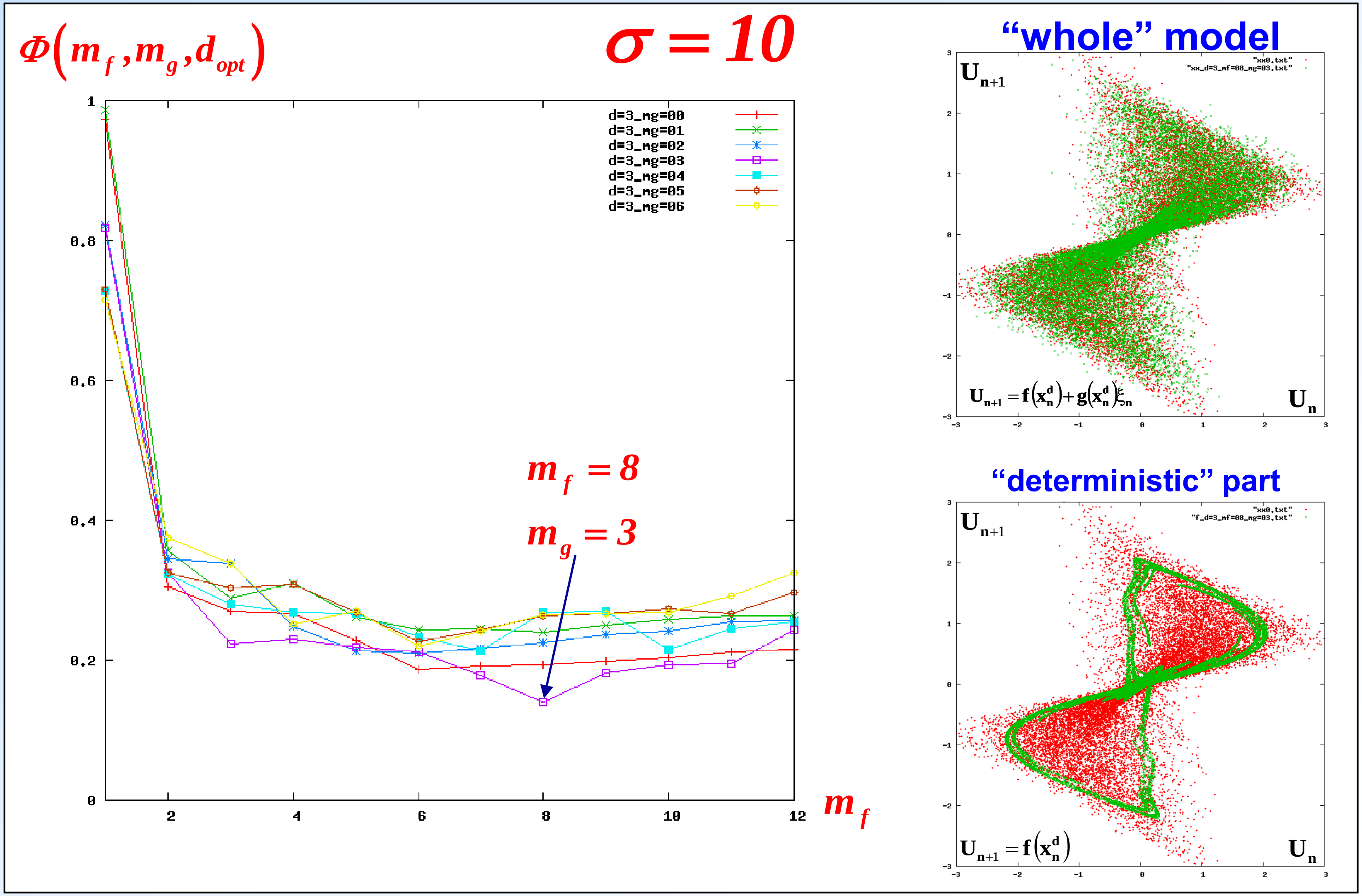
Evidence behavior with respect to the ANN complexity (different lines correspond to different m_g values) and evolution planes (red points – observation, green points – model)



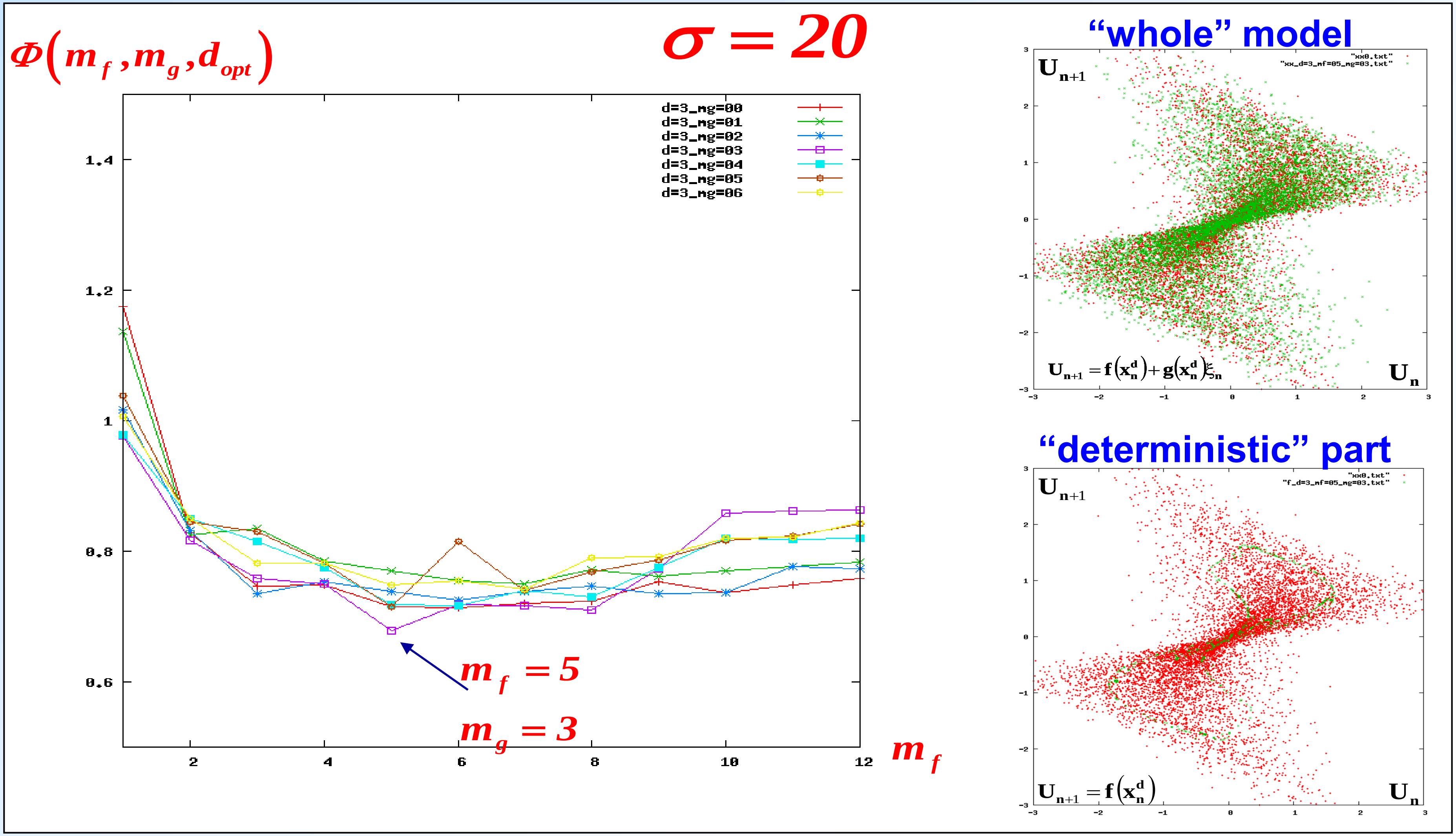
Evidence behavior with respect to the ANN complexity (different lines correspond to different m_g values) and evolution planes (red points – observation, green points – model)



Evidence behavior with respect to the ANN complexity (different lines correspond to different m_g values) and evolution planes (red points – observation, green points – model)



Evidence behavior with respect to the ANN complexity (different lines correspond to different m_g values) and evolution planes (red points – observation, green points – model)



Conclusions # 2&3

(about optimal stochastic model complexity):

- ***In the general case the model with the non-uniform stochastic part is optimal***

- ***The optimal dimension of the model and the optimal complexity of deterministic and stochastic parts are closely connected with the noise intensity ("stochasticity" of the system):***
the higher noise intensity, the more the non-uniformity of the optimal stochastic part, and the less the optimal dimension and the complexity of the deterministic part

NEXT STEPS:

- ***Reconstruction by real climatic data***
- ***Algorithm of optimal variables' choice***
- ***Study of teleconnection phenomena***
- ***Separation of the climatic sub-systems***
- ***Applications in other fields***

Thank you!

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