Algebraic Geometry

Problem Sheet 2 – Feedback from marking

Mark scheme.

(1) 2 marks
(2) 2 marks
(3) 3 marks
(4) 3 marks
(5) 6 marks
    (a) 1 (b) $\frac{1}{2}$ (c) 1 (d) $1\frac{1}{2}$ (e) $\frac{1}{2}$ (f) $1\frac{1}{2}$
(6) 4 marks

Feedback on solutions.

- Overall, I was very impressed with the quality of the solutions.
- The most common places where people lost marks were question 5(f) (which was meant to be hard) and 5(d) (by omitting some important detail).
- In question 1, it is shorter if you interpret "$\operatorname{im}\varphi$ dense in W" as "closure of $\operatorname{im}\varphi$ is W" (as in the model solution) rather than intersecting $\operatorname{im}\varphi$ with open sets of W – but it still works in the end.
- In question 3, if $X/(1-Y) = f/g$ in $k(C)$, then $Xf - (1-Y)g$ is divisible by X^2+Y^2-1 in $k[X,Y]$. In order for this to be valid, you need to know that $k[C] = k[X,Y]/(X^2+Y^2-1)$ – and so that the ideal generated by X^2+Y^2-1 is radical. If you used this fact but didn't mention it, you lost half a mark. I only required a mention of the fact that the ideal is radical, not a proof (because we proved in sheet 1 that X^2+Y^2-1 is irreducible – in an exam I would expect this to be proved unless stated otherwise).

  Some people had the same issue about $Y^2 - X^3$ in question 6.
- A related issue: On question 5(d), several people (using a slightly different approach from the model solution) showed that $g(x, x^2) = 0$ for all x and deduced that $(Y - X^2)$ divides $G(X,Y)$. This is true, but some argument is required to justify that $Y - X^2$ divides $G(X,Y)$: you can mention the Nullstellensatz (similar reasoning to the previous point) or show it algebraically by hand.
- Well done to people who solved 5(f), or almost did! You can't just write down that $k[V]$ is not isomorphic to $k[W]$ – proving that is the hard part.