Estimating Animal Spirits: conservative risk calculation

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(Received 00 Month 20XX; in final form 00 Month 20XX)

In this paper we estimate behavioural factors—Keynes’ ‘animal spirits’—in the property market. An enhanced Hidden Markov Model is used, for both the Shiller Home Price Index and a consumer confidence index. We conclude that both house prices and consumer confidence are driven by another hidden behavioural factor, interpreted as ‘animal spirits’. Both data series imply similar paths of the hidden factor.

The estimated model is used for VaR calculation and forecasting. We introduce an intuitive method to include crisis scenarios in the model, which proves to produce much better risk estimates during the credit crunch, without substantially affecting the distribution during growth periods.

1. Introduction

When the recent credit crunch unfolded, many leading economists were trying to understand its origins. In several countries a major factor contributing to the current economic crisis was massive borrowing to fund investment projects on the basis of, in retrospect, grossly optimistic valuations. It seems that ‘behavioural’ factors—Keynes’ ‘animal spirits’ or Greenspan’s ‘irrational exuberance’—played an important role in the build-up of the bubble preceding the crisis.

In this paper we propose a way to estimate the ‘animal spirits’ in the real estate market, use the model for forecasting and to compute Value-at-Risk (VaR). We also introduce a procedure in which we add stress scenarios to the model to make the risk estimates more conservative.

A natural approach to quantifying animal spirits would be to use confidence indices. Figure 1 depicts log-returns of the Shiller house price index and the University of Michigan consumer sentiment index. Even by looking at the graph it is clear that there exists a relationship between these two quantities. There is not enough data to find evidence of causality in the sense of Granger (1969) in either direction though—which might suggest the existence of another factor, causing both of these data series.

In this paper we propose to use Hidden Markov Models (HMMs) to estimate this unobservable factor, which is assumed to be a “state of the world”, impacting both house prices and consumer confidence. Hidden Markov Models were initially developed in the field of communications, in particular for speech recognition. Recently, this approach was successfully adopted in finance, where the interpretation of the hidden state process is that it is the current market ‘regime’, and this class of models is often referred to as ‘regime-switching’. Some early discussions include Hamilton (1988) and Pagan & Schwert (1990). It was used by Hamilton (1989) to explain US GNP, Rogers & Zhang (2011) use it to reproduce several well known stylized facts about asset returns. Haidinger & Warnung (2012) analyse risk measures in the setting of Rogers & Zhang (2011). Kritzman et al. (2012) use it for stock prices to come up with a trading strategy, they also include the Matlab source code used for the estimation. Giampieri et al. (2005) analyse defaults of companies. This approach is also used in the financial computing and artificial intelligence
HMMs consist of two (possibly multidimensional) processes: observable state (property price index and consumer confidence index in our case) and a finite state Markov chain that is unobservable (hence hidden), but which impacts the distribution of the observable process. A natural interpretation of the hidden state is that it defines market regimes, or ‘animal spirits’ if we look at it from the behavioural perspective. An attractive feature of HMMs is that the Viterbi algorithm makes it possible to efficiently calculate the most likely path of the hidden state process, which gives a really intuitive way to verify the model.

Dempster et al. (2012) use a hidden factor model in the commodity market. As opposed to HMMs, the hidden factors in their model are continuous processes. The most interesting feature of the cited paper is that the authors use standard structural Vector Auto-Regressive analysis of the most likely path of the hidden factors against published economic data. This in turn might be used for prediction of the hidden factors and hence prediction of the underlying commodity prices.

In this paper we use HMMs to estimate ‘animal spirits’ from the index value itself. Instead of using the most likely path of the hidden process in a regression model however, we introduce a multidimensional model with both house price process, which is our main point of interest, as well as the consumer confidence index as observable time series. We find that a model with three hidden states captures the data well, with one of the states corresponding to the worst part of the crisis. The consumer confidence index implied very similar path of the state process as the Shiller index, however it did not improve forecasting or VaR calculation.

Finally, we use the distribution forecast implied by the model for VaR calculation. We introduce a procedure of augmenting the model with an auxiliary state that corresponds to a crisis. Later in the paper we show that this procedure provides a much more conservative risk estimate by running back-testing on the period during property crisis and that without this state the model doesn’t completely capture the negative returns on house prices observed in the crisis.

Our estimation algorithm is based on Rabiner (1989), Rabiner & Juang (1993) and improvements from Rahimi (2000). See also Zucchini & MacDonald (2009) for a comprehensive treatment of the subject. We extend the standard algorithm to cater for mean-reverting processes, show how to estimate parameters of correlated multi-dimensional continuous-time stochastic processes and provide a procedure for forecasting. Elliott et al. (1995) approach the subject using techniques from stochastic filtering. They use the filters also for maximum-likelihood parameter estimation in the expectation-maximisation (EM) iterative framework. Their approach is different from the one used in this paper, but the resulting estimation procedure is very similar.

Figure 1. Log-returns of the Shiller property index (seasonally adjusted and normalized), University of Michigan consumer sentiment index.
In the next section we introduce the concepts of ‘animal spirits’ and ‘irrational exuberance’. Section 3. discusses the models used and the link between discrete-time models and continuous-time stochastic regime-switching models. Section 4. contains discussion on hidden Markov models. Section 5. analyses forecasting procedure used and verification method. 6. lists the sources of data used for the estimation. Sections 7. and 8. contain estimation and VaR calculation results. Finally, section 9. concludes the paper.

2. Animal Spirits

The concept of ‘animal spirits’ was used in real investment modelling in our earlier paper Andruszkiewicz et al. (2013). The term was forged by John Maynard Keynes, who didn’t believe that the economic agents are necessarily driven by rational expectation calculations, in General Theory (Keynes 2007, page 161):

[A] large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits—of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.

A similar concept—irrational exuberance—was introduced by the then-Federal Reserve Board Chairman, Alan Greenspan (1996) in his speech at the Annual Dinner and Francis Boyer Lecture of The American Enterprise Institute for Public Policy Research, Washington, D.C:

Clearly, sustained low inflation implies less uncertainty about the future, and lower risk premiums imply higher prices of stocks and other earning assets. We can see that in the inverse relationship exhibited by price/earnings ratios and the rate of inflation in the past. But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade?

This term was later picked-up by other economists and journalists, including Shiller (2000), who used it as the title of his book.

3. Model

Most models in mathematical-finance are set in continuous time, mainly for easier tractability. However, real world data is only available at discrete time points (monthly in case of the Shiller index) and hence the statistics literature deals with discrete time series. To bridge these two fields easily and without introducing additional error terms, we only work with price processes that can be discretized exactly. It is hence possible to formulate a discrete-time model for the observed time series that has the same distribution as the underlying continuous-time process at the observation times.

The state process is assumed to only change value on the observation dates, so the continuous-time version has right-continuous piecewise-constant paths. For house price index and consumer confidence we use two models: geometric Brownian motion (GBM) and exponential Ornstein-Uhlenbeck process (exponential O-U). Log-returns of a GBM are normally distributed and independent of each other. In the case of the exponential O-U, the increments of the log of the process are Gaussian, conditionally on the previous observation. See Andruszkiewicz et al. (2014) for details and exact formulae.
4. Hidden Markov Models

Let $O_t$ denote the log-returns of the relevant index value at time $t$. The time index is discrete and corresponds to the market observation times. In our case we assume that $O_t$ is a continuous random variable with a density function.

The main idea behind HMMs is that there exists a corresponding time series $\{q_t\}$, which is not directly observable (hence hidden), and which denotes the ‘state’ of the system at every time $t$. In the class of models we are looking into $\{q_t\}$ is modelled as a finite-state Markov chain with initial distribution $\pi$ and transition matrix $A$. The parameters of the distribution of $O_t$ depend on the state variable $q_t$, e.g. if we assume that $O_t$ is normally distributed then the mean and variance of the distribution are functions of $q_t$.

For simplicity of exposition we assume that $q_t$ takes values in $\mathbb{N}$, the set of positive integers and the parameters of the distribution of $O_t$ given $q_t$ are in the $q_t$-th row of the matrix $\theta$. So the whole model $\lambda = (\pi, A, \theta)$ consists of the initial distribution $\pi$ of the state variable, its transition matrix $A$ and the mapping $\theta$ of the distribution parameters of the observable process given the hidden state.

To estimate the parameters of the model we shall adopt the maximum likelihood method, so we find parameter values $\hat{\lambda}$ that maximise the likelihood function of the observed series:

$$ L(\lambda; O) = \mathbb{P} \left[ O \mid \lambda \right], \quad (1) $$

where in our case the observations are continuous random variables, and hence $\mathbb{P} \left[ O \mid \lambda \right]$ denotes the density function:

$$ \mathbb{P} \left[ O \mid \lambda \right] \equiv \mathbb{P} \left[ O \in (x, x + dx) \mid \lambda \right] \quad (2) $$

Direct maximisation is in this case very difficult and hence we implement a version of Expectation-Maximisation (EM) algorithm, first introduced by Dempster et al. (1977). The algorithm starts from some user-defined initial guess for the parameter values and then improves them iteratively. It was proved that it always converges monotonically to a local maximum. A single iteration consists of two steps:

**Expectation** First, given the parameter values from previous step $\lambda^{i-1}$ we calculate the quantity:

$$ Q(\lambda, \lambda^{i-1}) = \mathbb{E} \left[ \log \mathbb{P} \left[ O, Q \mid \lambda \right] \mid O, \lambda^{i-1} \right] $$

$$ = \sum_q \log \mathbb{P} \left[ O, q \mid \lambda \right] \mathbb{P} \left[ O, q \mid \lambda^{i-1} \right] \quad (3) $$

as a function of $\lambda$.

**Maximisation** Next, we find $\lambda^i = \arg \max_\lambda Q(\lambda, \lambda^{i-1})$, which will be used in the next iteration. We iterate over these steps until we reach convergence to a local optimum. Note that the quantity that is being maximised (3) is different from a logarithm of the likelihood (1). Dempster et al. (1977) proved that the parameter values that maximise (3) also maximise (1), hence the algorithm gives the required estimate.

In this paper the (modified) observed processes are conditionally Gaussian, hence the EM method simplifies to the Baum-Welch algorithm, which effectively uses dynamic programming techniques in the Expectation step, and closed-form solution to the maximisation problem. Our implementation is based on Rabiner & Juang (1993), with a slight extension to handle the fact that (modified) observations are only conditionally Gaussian in the exponential O-U case. After estimation we use the Viterbi algorithm to calculate the most likely path of the hidden factor process. See Andruszkiewicz et al. (2014) for details.
5. Forecasting

One of the benefits of Hidden Markov Models is that they can be easily used for density forecasting. Given the history of the process up to time $T$, the distribution of the observable at time $T + 1$ is given by:

$$P[O_{T+1} = o | O_1 \ldots O_T] = \frac{P[O_1 \ldots O_T, O_{T+1} = o]}{P[O_1 \ldots O_T]}$$  (4)

Note that in our case the observable process is continuous, so we need to interpret the equation above as the density function. In our setup the forecast distribution is a mixture of Gaussians and it may be efficiently calculated using techniques from the Baum-Welch algorithm.

Of course it is impossible to verify a single forecast distribution after we get the actual realisation—a single point is not enough to do that. We can, however, verify that the model produces consistent density forecast if we look at a series of distributions and realisations. The idea is to apply the forecast cumulative distribution function at every time $t$ to the actual realisation at that time. The resulting random variable is uniformly distributed, provided that the model is correct. Moreover, these uniform random variables will be independent of each other. This approach was first introduced by Diebold et al. (1998), and can be summarised in the following proposition:

**Proposition 5.1:** Assume that at each point in time $T_0 \leq t \leq T$ we are given a distribution forecast for the value of process $X$:

$$F_t(x) = P[X_t < x | \mathcal{G}_{t-1}],$$  (5)

where for simplicity we assume that $F_t$ is a continuous and strictly increasing function (as it is the case for mixture Gaussian distribution) and $\mathcal{G}_{t-1}$ is the sigma algebra representing the information available at time $t - 1$. Let $Y_t = F_t(X_t)$ for every $t$. Then the random variables $Y_t$ are uniformly distributed on $(0, 1)$ and are mutually independent.

**Proof:** For proof see Diebold et al. (1998) or Andruszkiewicz et al. (2014). □

6. Data

For estimation we use the Standard & Poor’s (S&P)/Case-Shiller 10-City Composite Home Price Index (ticker SPCS10) as the indicator of property prices in the United States. Announcements of index levels are made at 09:00 AM Eastern Time, on the last Tuesday of each month. Historic index values are available on the S&P website: http://eu.spindices.com/indices/real-estate/sp-case-shiller-10-city-composite-home-price-index. Before the estimation we calculate the seasonally adjusted version of the index using X-12-ARIMA Seasonal Adjustment Program (see http://www.census.gov/srd/www/x12a/). Specifically we use the implementation provided as part of the open-source Gretl package (http://gretl.sourceforge.net/).

To capture consumer confidence we use the University of Michigan Consumer Sentiment Index. The preliminary index releases are scheduled around the middle of the relevant month, and the final release is around the end of the month. Exact release dates are available on the index web page. The historic press releases are available from the index page: http://thomsonreuters.com/products_services/financial/financial_products/a-z/umichigan_surveys_of_consumers/. The historical values can also be downloaded in a more convenient format from FRED: http://research.stlouisfed.org/fred2/series/UMCSENT.

We also tried the Conference Board consumer confidence index, but the estimation results were inferior to the University of Michigan Consumer Sentiment Index, and hence are not presented.
in the paper.

7. Estimation results

The Shiller index is characterised by a significantly lower volatility compared to stock indices, and most of the volatility has strongly seasonal characteristics. This is mostly due to low liquidity, long transaction times and high transaction costs. The distribution of log-returns of the

![Figure 2. A histogram of log-returns of seasonally-adjusted Shiller index and a fitted normal distribution density.](image)

seasonally-adjusted Shiller index is highly non-Gaussian and multi-modal, as can be seen in Figure 2. The fitted normal distribution does not capture the underlying data well. Moreover, unlike the stock markets, using a fat-tailed distribution is not going to help in this case either. However, employing a hidden Markov model proves to be fruitful.

![Figure 3. Log-returns of the Shiller property index (seasonally adjusted and normalized) and an estimated most likely path of the hidden state process with two states.](image)

First, we estimate a HMM with an underlying geometric Brownian motion model and with two hidden states The most likely path of the hidden state process, obtained using the Viterbi
<table>
<thead>
<tr>
<th>State</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.0304282</td>
<td>0.0210861</td>
<td>Decline/Stagnation</td>
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<td>1</td>
<td>0.112926</td>
<td>0.0114603</td>
<td>Growth</td>
</tr>
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</table>

Table 1. Estimated parameters of the Shiller index (seasonally adjusted) for every value of the hidden state process. The model used is a geometric Brownian motion with two regimes.

algorithm as indicated in Section 4., is in Figure 3. The estimated parameters of the process are in Table 1 and the transition matrix between states is given by:

$$A = \begin{bmatrix} 0.9854 & 0.0146 \\ 0.0175 & 0.9825 \end{bmatrix}$$

(6)

The estimation procedure is able to identify different regimes surprisingly well. Figure 4 contains

Figure 4. Histograms of log-returns of seasonally-adjusted Shiller index corresponding to most likely states and a fitted normal distribution density.

the histograms of log-returns corresponding to the two states. Whereas the observations corresponding to the growth regime are matching the normal distribution reasonably well, given the limited number of data points, the histogram corresponding to the weak market conditions is clearly skewed by the extreme losses observed during the crisis. We decided to include an extra

Figure 5. Log-returns of the Shiller property index (seasonally adjusted and normalized) and an estimated most likely path of the hidden state process with three states.
Table 2. Estimated parameters of the Shiller index (seasonally adjusted) for every value of the hidden state process. The model used is a geometric Brownian motion with three regimes.

<table>
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<th>State</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Interpretation</th>
</tr>
</thead>
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<td>0.0139859</td>
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</tr>
<tr>
<td>1</td>
<td>-0.00798775</td>
<td>0.00962791</td>
<td>Stagnation</td>
</tr>
<tr>
<td>2</td>
<td>0.110411</td>
<td>0.0116515</td>
<td>Growth</td>
</tr>
</tbody>
</table>

state and as seen in Figure 5 the estimation procedure used it for the observations corresponding to the market crash. Figure 6 contains the relevant histograms. Note that the first "crash"

and the third state is estimated as the initial state.\(^1\) Log-likelihood for this model is 1318.23, which is higher than 1212.1 and 1065.47 for a model with two hidden states and a simple geometric Brownian motion respectively. Following Zucchini & MacDonald (2009) we use two

\(^1\)It is always the case that the initial distribution degenerates to a single state. Intuitively this is caused by the fact that the observation data includes only one realisation of the initial state. Hence we can only estimate the most likely realisation of that state, as opposed to the actual distribution.
measures of ‘lack of fit’ of the model: the Akaike information criterion (AIC) and Bayesian information criterion (BIC). AIC has the value of $-2608.45$, as opposed to $-2410.2$ and $-2126.95$ respectively. BIC with value $-2556.09$ also confirms the model choice, as opposed to the values of $-2384.02$ and $-2119.47$ for the other models respectively. The most likely path of the hidden process had 26 observations in the first state, 133 in the second state and 151 in the third state.

<table>
<thead>
<tr>
<th>Number of states</th>
<th>State number</th>
<th>Number of observations</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>Jarque-Bera statistic</th>
</tr>
</thead>
<tbody>
<tr>
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<td>310</td>
<td>-0.7569</td>
<td>0.6344</td>
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<tr>
<td></td>
<td>1</td>
<td>170</td>
<td>-1.354</td>
<td>1.2474</td>
<td>62.96567</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>26</td>
<td>0.5674</td>
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<td></td>
<td>1</td>
<td>133</td>
<td>-0.5046</td>
<td>-0.3809</td>
<td>6.448114</td>
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<tr>
<td></td>
<td>2</td>
<td>151</td>
<td>0.5101</td>
<td>-0.0129</td>
<td>6.549464</td>
</tr>
</tbody>
</table>

Table 3. Statistics for log-returns of the Shiller index in models with varying number of states.

Table 3 summarises the basic statistics for the most likely observations corresponding to all the cases described above, including the Jarque-Bera test statistic (see Jarque & Bera 1987). Even though the null hypothesis that the values are normally distributed is rejected in every case, the value of the statistic is much lower in the model with three states. We believe it is unlikely that standard distributions will fit this data well, because the Shiller index is artificially constructed and then manipulated further to remove seasonality. Despite these shortcomings, we get promising results in the next section.

We also estimated a geometric Ornstein-Uhlenbeck model (known as the Longstaff-Schwartz model in the commodity pricing literature), but the mean-reversion coefficient in the growth state was negative—suggesting that the Shiller index doesn’t have any mean-reversion properties and hence the model is inadequate. The model was also estimated for joint Shiller property price index and consumer confidence index, using a geometric Brownian motion model for the former data series, and a geometric Ornstein-Uhlenbeck model for the latter. Obtained results were very similar to the single-dimensional case, which suggests that the consumer confidence index doesn’t add more information over what is already contained in property prices. The most likely path is presented in Figure 7. The estimated parameters of the regime-switching geometric Brownian

![Figure 7. Log-returns of the Shiller property index (seasonally adjusted and normalized), University of Michigan consumer sentiment index and an estimated most likely path of the hidden state process with three states.](image-url)
Table 4. Estimated parameters of the seasonally adjusted Shiller index ($\mu$ and $\sigma$) and University of Michigan consumer confidence index ($\alpha$, $\beta$ and $\sigma_{cons}$) for every value of the hidden state process. The model used is a geometric Brownian motion and exponential Ornstein-Uhlenbeck with three regimes.

<table>
<thead>
<tr>
<th>State</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\sigma_{cons}$</th>
<th>Interpretation</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.14999</td>
<td>0.0180475</td>
<td>2.29584</td>
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<td>1</td>
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<td>0.71729</td>
<td>4.44809</td>
<td>0.164324</td>
<td>Stagnation</td>
</tr>
<tr>
<td>2</td>
<td>0.11024</td>
<td>0.0116677</td>
<td>1.00023</td>
<td>4.52742</td>
<td>0.149742</td>
<td>Growth</td>
</tr>
</tbody>
</table>

motion process corresponding to the Shiller index and the parameters of exponential Ornstein-Uhlenbeck process for consumer confidence are given in Table 4. The transition matrix of the hidden factor process is given by:

$$A = \begin{bmatrix} 0.9406 & 0.0318 & 0.0276 \\ 0.0163 & 0.9671 & 0.0166 \\ 0 & 0.0201 & 0.9799 \end{bmatrix}$$

The joint model didn’t improve value at risk estimation.

8. Risk management and animal spirits

One of the main purposes of this paper is to propose a model for value at risk for house prices. Back-testing is done for the period from July 2006 to November 2012, starting just before the credit crunch. First we employ the forecasting method described in Section 5. We estimate the model for every period in the back-testing range, and use the parameters for one-period ahead forecast. Using the forecast distribution, we calculate the 95% VaR level\(^1\). For all the relevant periods we calculate the proportion of observations that were below the corresponding VaR level. For a perfect forecast we would expect this ratio to be 5%. Unfortunately, just by looking at the data, it is clear that purely statistical methods are bound to fail, because the series before the crisis is in no way representative for the crash period. Indeed, more than 22% of all the observations fall in the VaR region, which shows that our risk was grossly underestimated. The results would be much better if we knew the true distribution of the data series before the crisis happened. If, instead of estimating the parameters at every period, we use the parameters estimated for the whole series (of course in reality they would not be known at that time), we get the proportion of observations down to a much more reasonable 7.8%.

We can achieve similar results without ‘cheating’—it is enough to introduce stress scenarios to the data set. To get conservative risk estimates, one should assume that the house prices might fall—even though they haven’t in the historic data series. To illustrate that this simple idea works in our context, we proceed as follows. We estimate parameters of a hidden Markov model with two hidden states in June 2006. The resulting model is given in Table 5, with the transition matrix of the hidden Markov chain given by:

$$A = \begin{bmatrix} 0.9900 & 0.0100 \\ 0.0154 & 0.9846 \end{bmatrix}$$

\(^1\)The 95% level was chosen because of the limited number of observations.
Next we introduce an arbitrary ‘crisis’ state using the following heuristics:

- The mean of the log-returns in the new state is the $(-2)$ times the mean of the existing growth state, with variance the same as the existing decline state.
- The crisis starts only when the system is in the decline state, with probability $0.03$.
- The crisis ends with probability $0.05$, and the system moves directly into the growth state to capture the ‘bounce-back’ effect.

<table>
<thead>
<tr>
<th>State</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.2351763</td>
<td>0.0094117</td>
<td>New Crisis state</td>
</tr>
<tr>
<td>2</td>
<td>0.0002445</td>
<td>0.0094117</td>
<td>Stagnation</td>
</tr>
<tr>
<td>3</td>
<td>0.11767</td>
<td>0.0109299</td>
<td>Growth</td>
</tr>
</tbody>
</table>

Table 6. Estimated parameters of the Shiller index (seasonally adjusted) with an auxiliary state added for a house price crash.

The model parameters after applying the simple heuristics described above are given in Table 6, and the transition matrix is given by:

$$A = \begin{bmatrix} 0.95 & 0 & 0.05 \\ 0.03 & 0.9600 & 0.0100 \\ 0 & 0.0154 & 0.9846 \end{bmatrix}, \quad (10)$$

For simplicity of calculations, we keep these parameters throughout the benchmarking period. This simple inclusion of a ‘crisis’ state allows us to get the proportion of observations in the VaR region down to $7.8\%$. The need of an auxiliary state is even more apparent in Figure 8, which shows the $5\%$ to $95\%$ range of possible future paths of the Shiller index starting from July 2006. The left graph was generated using the two state model estimated in June 2006, whereas the right graph was generated using the model with a crisis state added using the procedure described above. The most striking feature is that the original model doesn’t account for a fall in house prices at all, which caused the risk estimates to be grossly overoptimistic. This situation exemplifies Greenspan’s ‘irrational exuberance’.

Of course, there is no reason to believe that this new model with a crisis state reflects the true distribution of the house prices, it provides a way to obtain a conservative risk estimates though. The key idea is to identify the stress scenario that could happen, but is not reflected in historic data, and assign a positive probability to it. This is a very intuitive procedure in hidden Markov models, because the stress scenario corresponds in a natural way to a new state of the
hidden factor process. Note also that if the system is in a growth state, then the probability of a crash is very low—hence the model is not over-conservative in the periods of high returns.

9. Summary

In this paper we summarised the Baum-Welch algorithm for estimating parameters of Hidden Markov Models. We extended the standard algorithm to cater for two most popular models in finance: geometric Brownian motion and Ornstein-Uhlenbeck type, both in single and multiple dimensions. Moreover, we introduced formulas for density forecasts and risk calculation. We estimated the model for Shiller Home Price Index and consumer confidence index, and calculated the most likely paths of the ‘animal-spirits’ hidden state process. We used the model for VaR calculation.

Finally, using the example of property price crash, we highlighted the importance of including auxiliary stress scenarios in risk calculations, especially in markets driven by ‘animal spirits’ or ‘irrational exuberance’. Conservative risk managers should always account for negative sentiment and bubble burst. HMMs prove to be perfect models for inclusion of stress scenarios. The procedure is intuitive, the extended model captures the downside risk, and yet is not overly pessimistic during good times.

References


