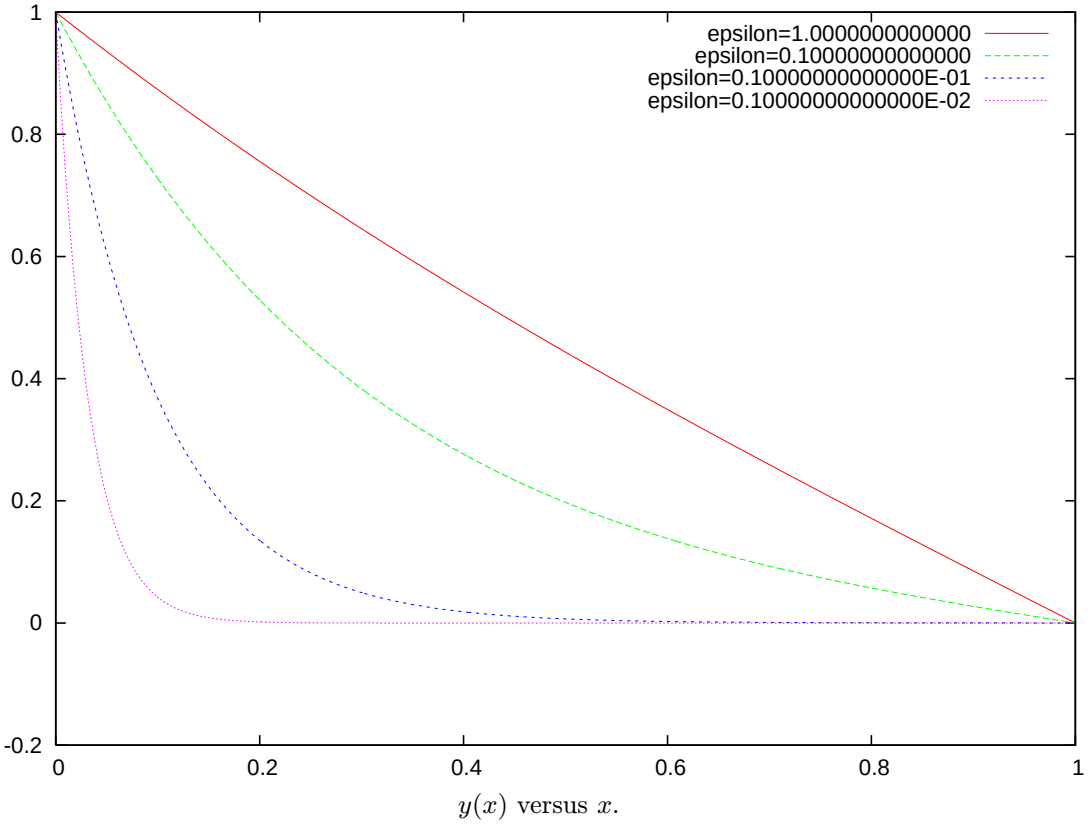
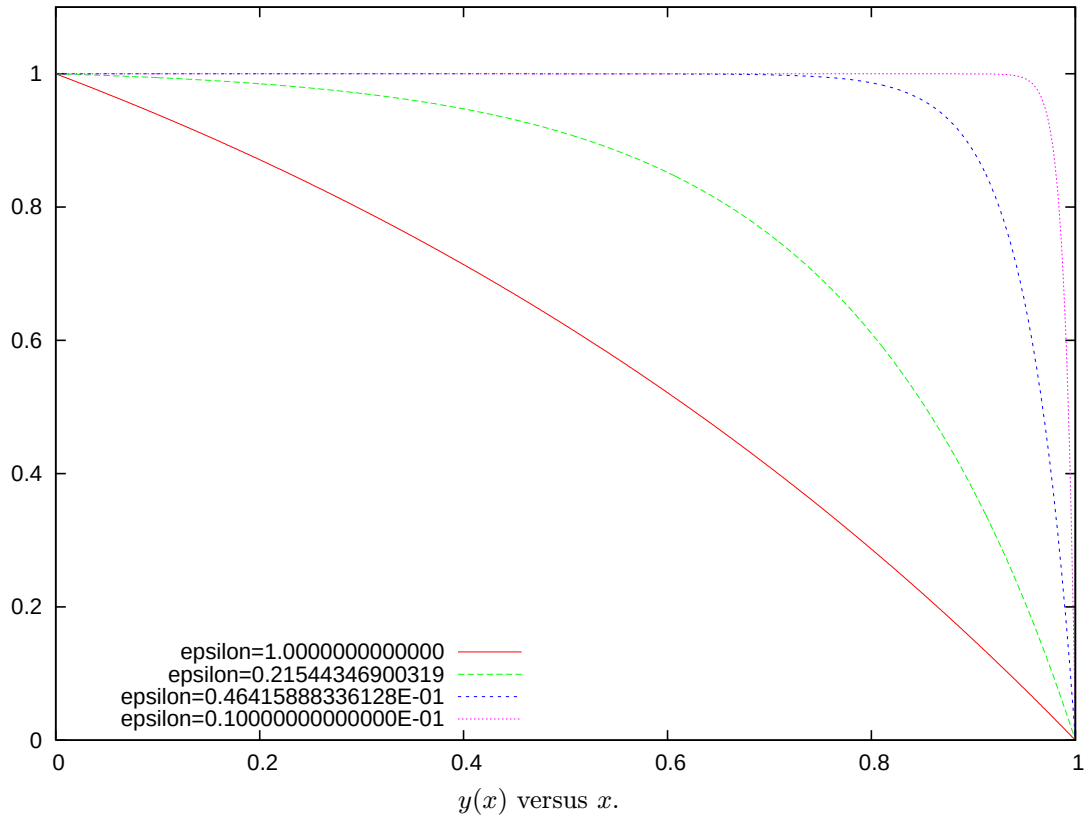
Test Problem 01: $\epsilon y'' - y = 0, \quad y(0) = 1, \;\; y(1) = 0.$
Analytic solution: $y(x) = (\exp(-x/\sqrt{\epsilon}) - \exp((x-2)/\sqrt{\epsilon}))/(1 - \exp(-2/\sqrt{\epsilon}))$.



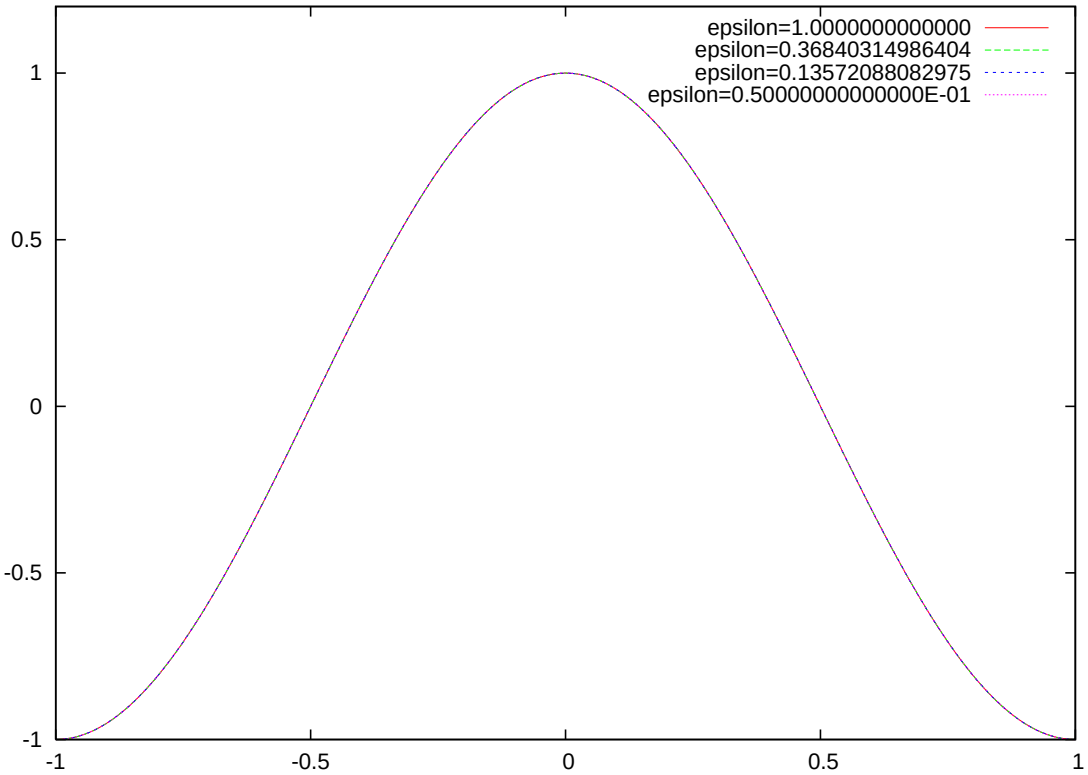
$y(x)$ versus x.

Test Problem 02: $\epsilon y'' - y' = 0, \quad y(0) = 1, \;\; y(1) = 0.$
Analytic solution: $y(x) = (1 - \exp((x-1)/\epsilon))/(1 - \exp(-1/\epsilon))$.



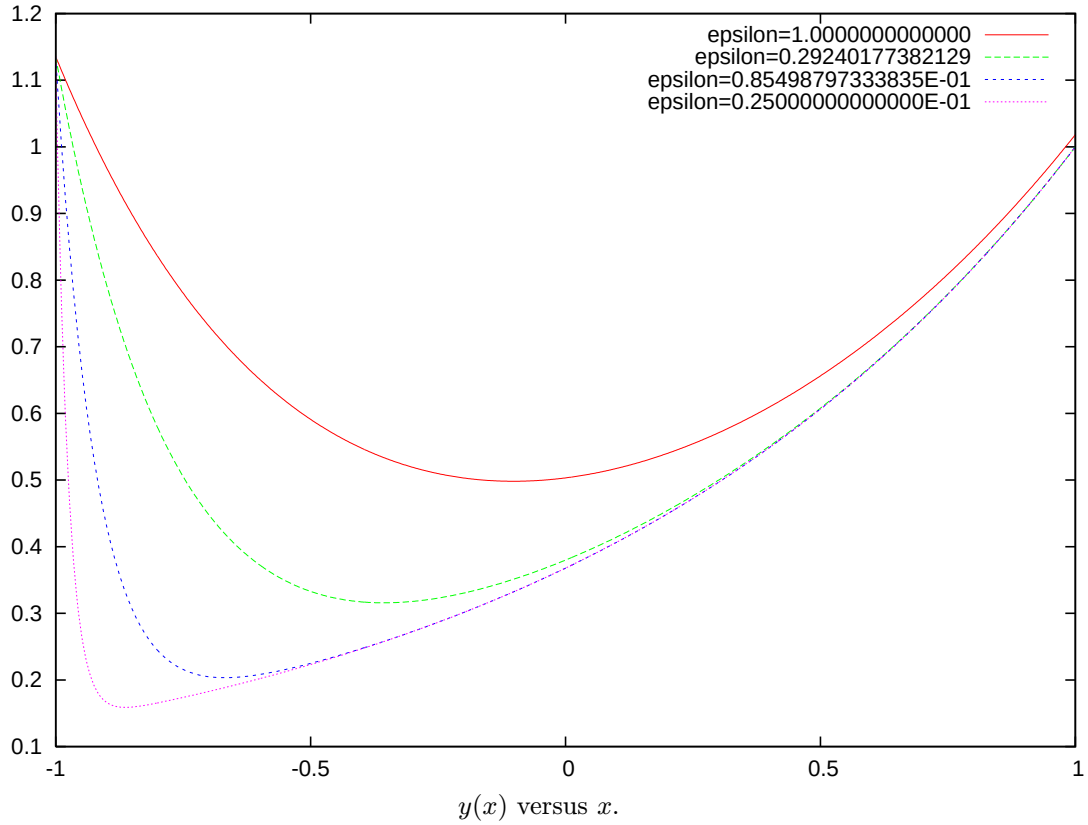
$y(x)$ versus x.

Test Problem 03: $\epsilon y'' + (2+\cos(\pi x))y' - y = -(1+\epsilon\pi^2)\cos(\pi x) - (2+\cos(\pi x))\pi\sin(\pi x)$, $y(-1) = y(1) = -1$
Analytic solution: $y(x) = \cos(\pi x)$.



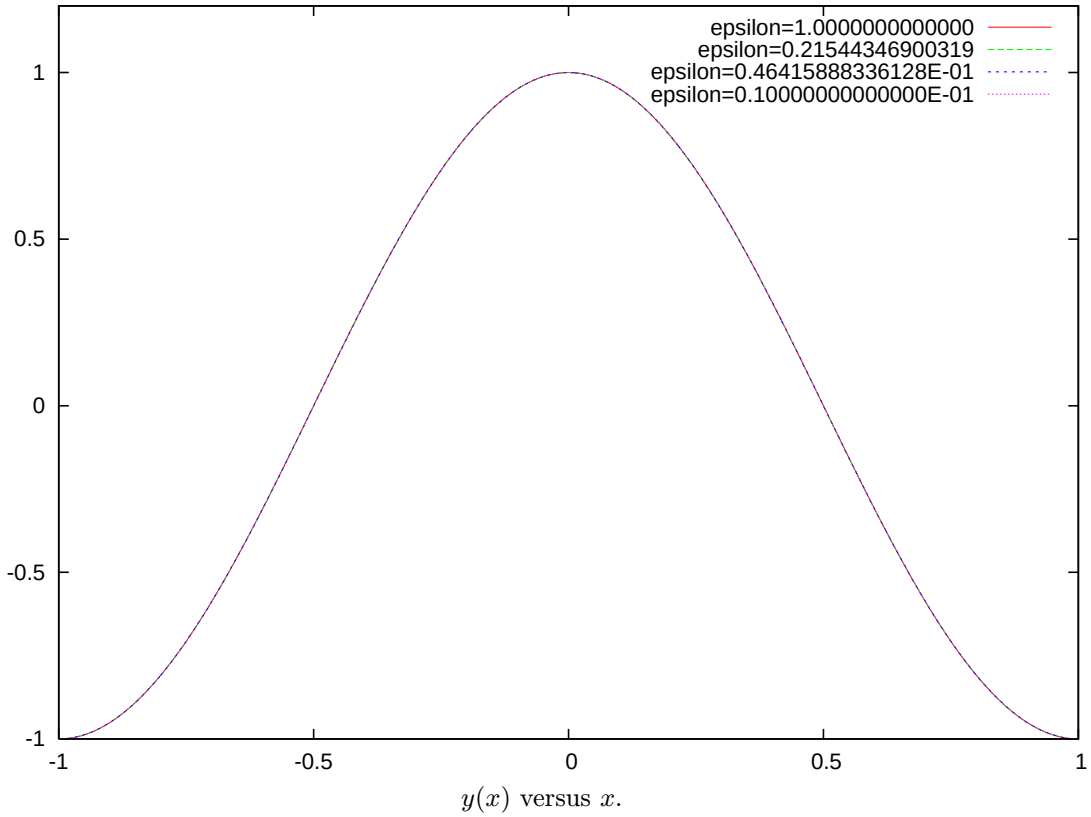
$y(x)$ versus x.

Test Problem 04: $\epsilon y'' + y' - (1+\epsilon)y = 0$, $y(-1) = 1+\exp(-2)$, $y(1) = 1+\exp(-2(1+\epsilon)/\epsilon)$,
Analytic solution: $y(x) = \exp(x-1) + \exp(-(1+\epsilon)(1+x)/\epsilon)$.



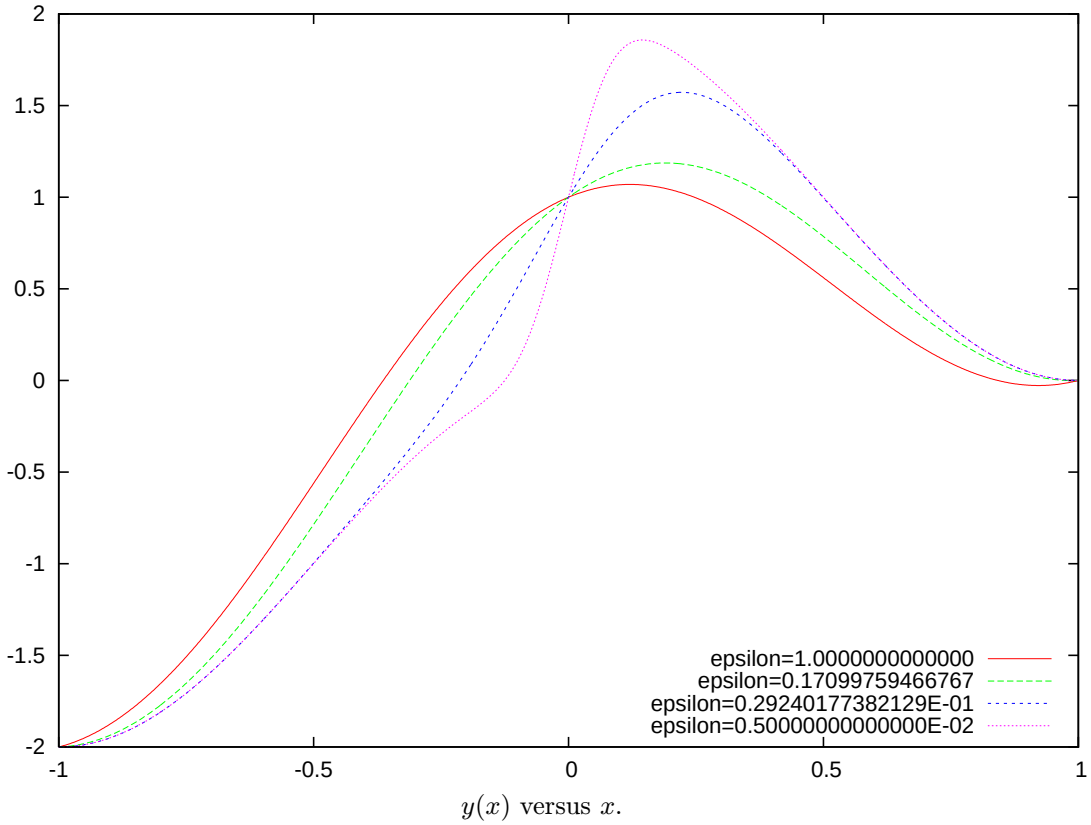
$y(x)$ versus x.

Test Problem 05: $\epsilon y'' - xy' - y = -(1+\epsilon\pi^2)\cos(\pi x) + \pi x \sin(\pi x), \quad y(-1) = y(1) = -1.$
Analytic solution: $y(x) = \cos(\pi x)$.



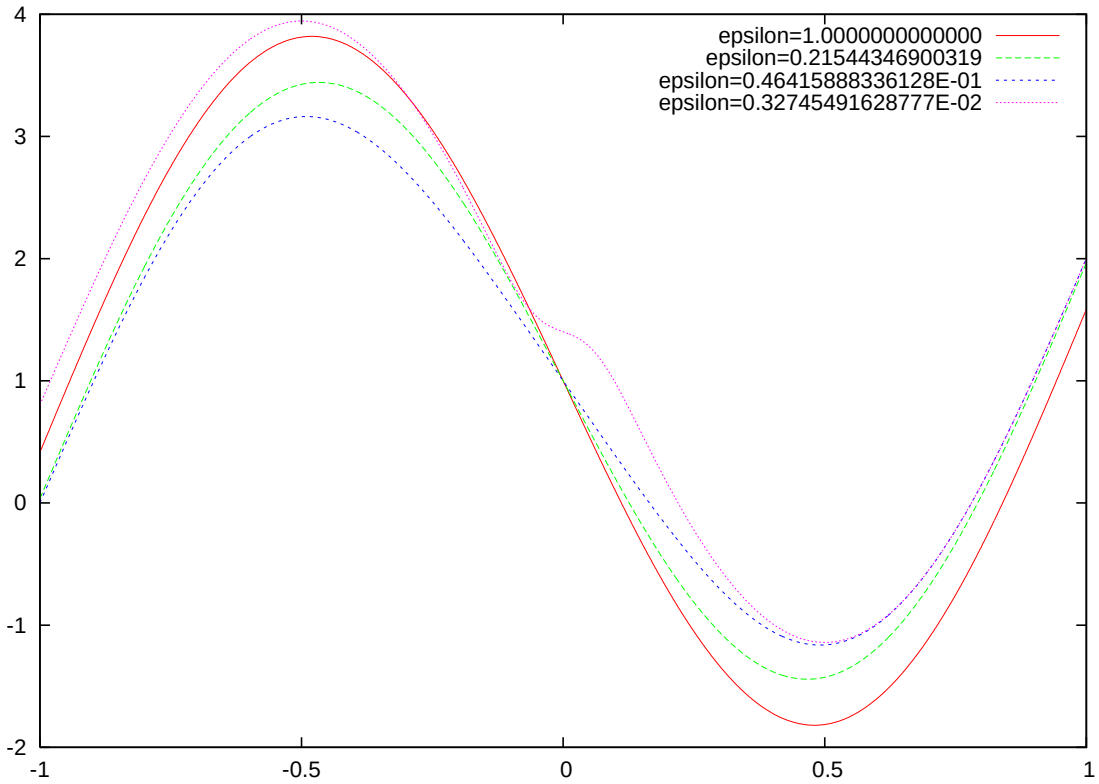
$y(x)$ versus x.

Test Problem 06: $\epsilon y'' + xy' = -\epsilon\pi^2 \cos(\pi x) - \pi x \sin(\pi x), \quad y(-1) = -2, \;\; y(1) = 0.$
Analytic solution: $y(x) = \cos(\pi x) + \mathrm{erf}(x/\sqrt{2\epsilon})/\mathrm{erf}(1/\sqrt{2\epsilon})$.



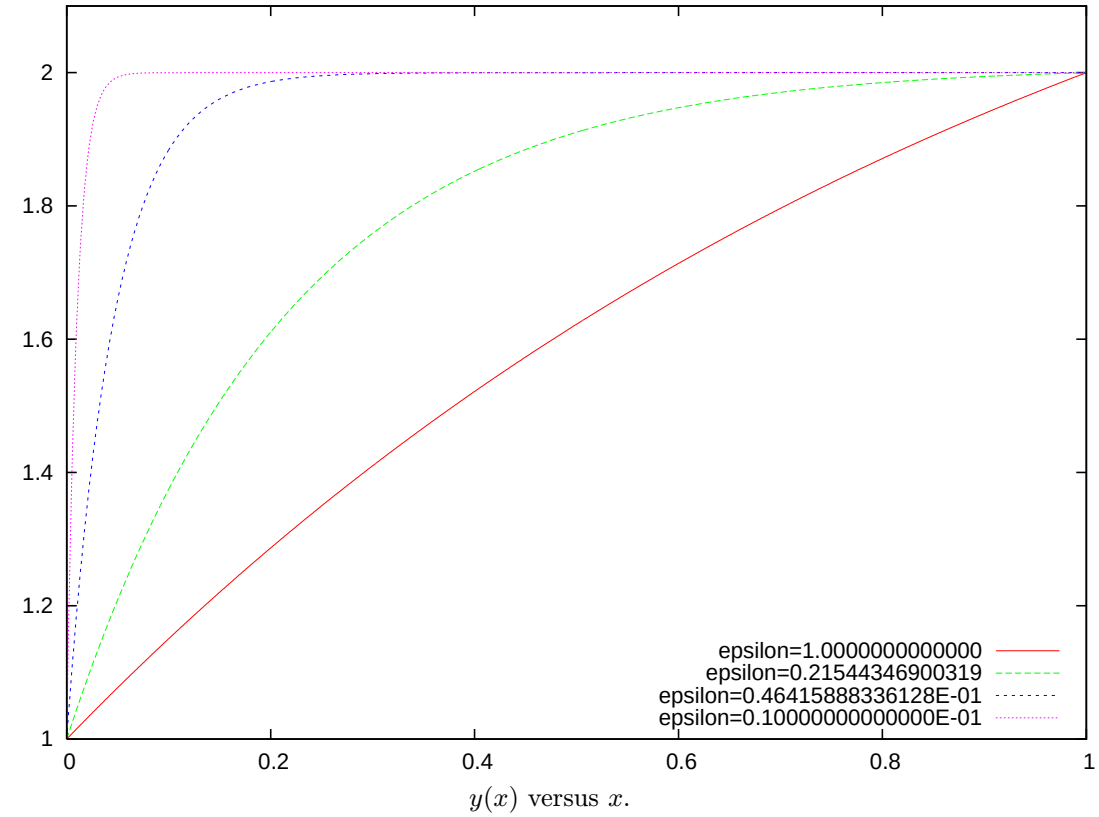
$y(x)$ versus x.

Test Problem 07: $\epsilon y'' + xy' - y = -(1+\epsilon\pi^2)\cos(\pi x) - \pi x \sin(\pi x)$, $y(-1) = -1, y(1) = 1$,
Analytic solution: $y(x) = \cos(\pi x) + x + \frac{x\,\mathrm{erf}(x/\sqrt{2\epsilon}) + \sqrt{2\epsilon/\pi}\exp(-x^2/2\epsilon)}{\mathrm{erf}(1/\sqrt{2\epsilon}) + \sqrt{2\epsilon/\pi}\exp(-1/2\epsilon)}$.



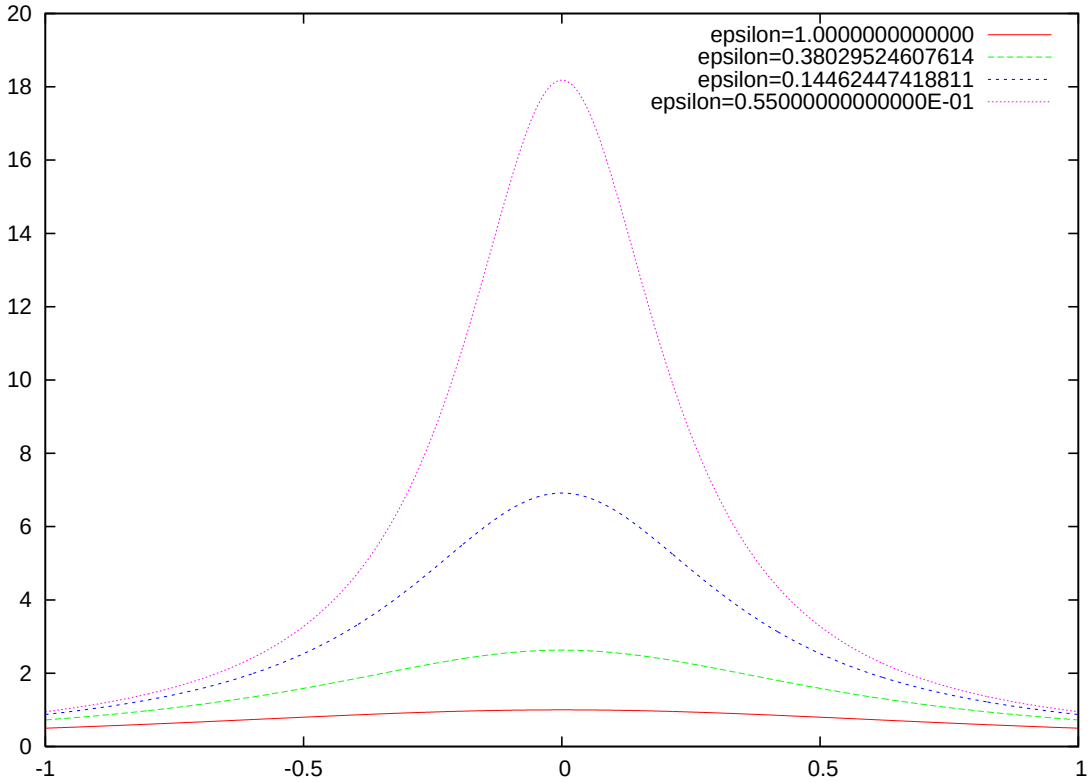
$y'(x)$ versus x.

Test Problem 08: $\epsilon y'' + y' = 0, \quad y(0) = 1, \; y(1) = 2$,
Analytic solution: $y(x) = (2 - \exp(-1/\epsilon) - \exp(-x/\epsilon))/(1 - \exp(-1/\epsilon))$.



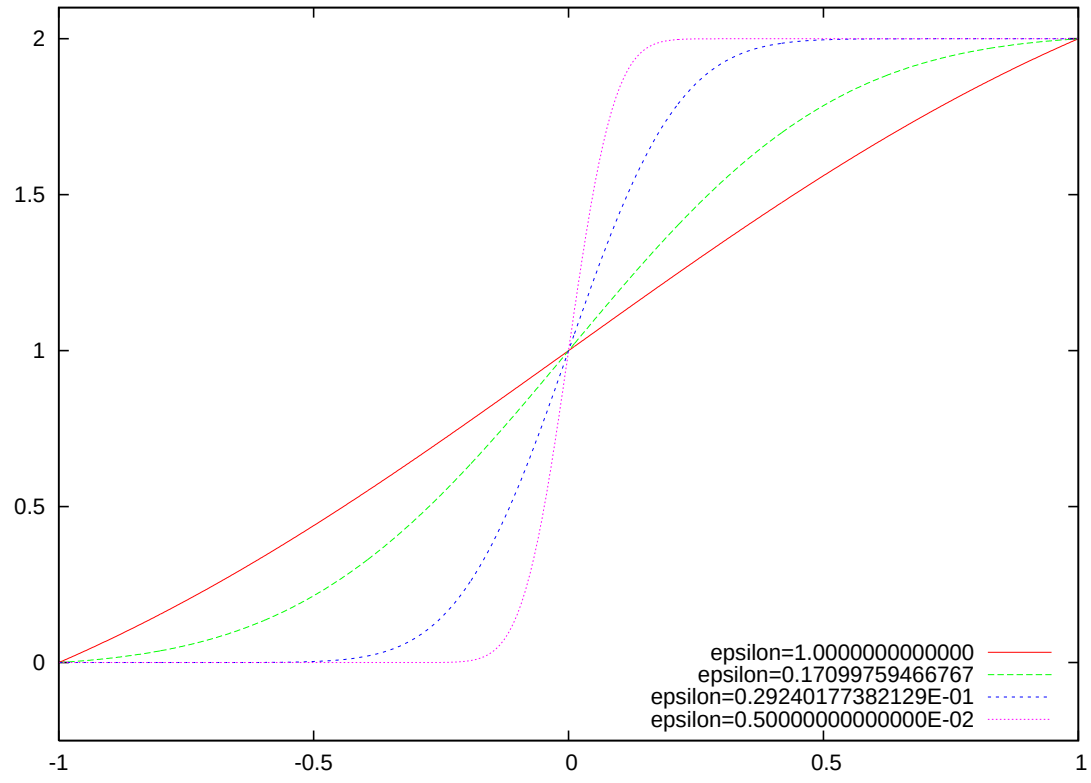
$y(x)$ versus x.

Test Problem 09: $(\epsilon + x^2)y'' + 4xy' + 2y = 0, \quad y(-1) = y(1) = 1/(1+\epsilon)$,
Analytic solution: $y(x) = 1/(\epsilon + x^2)$.



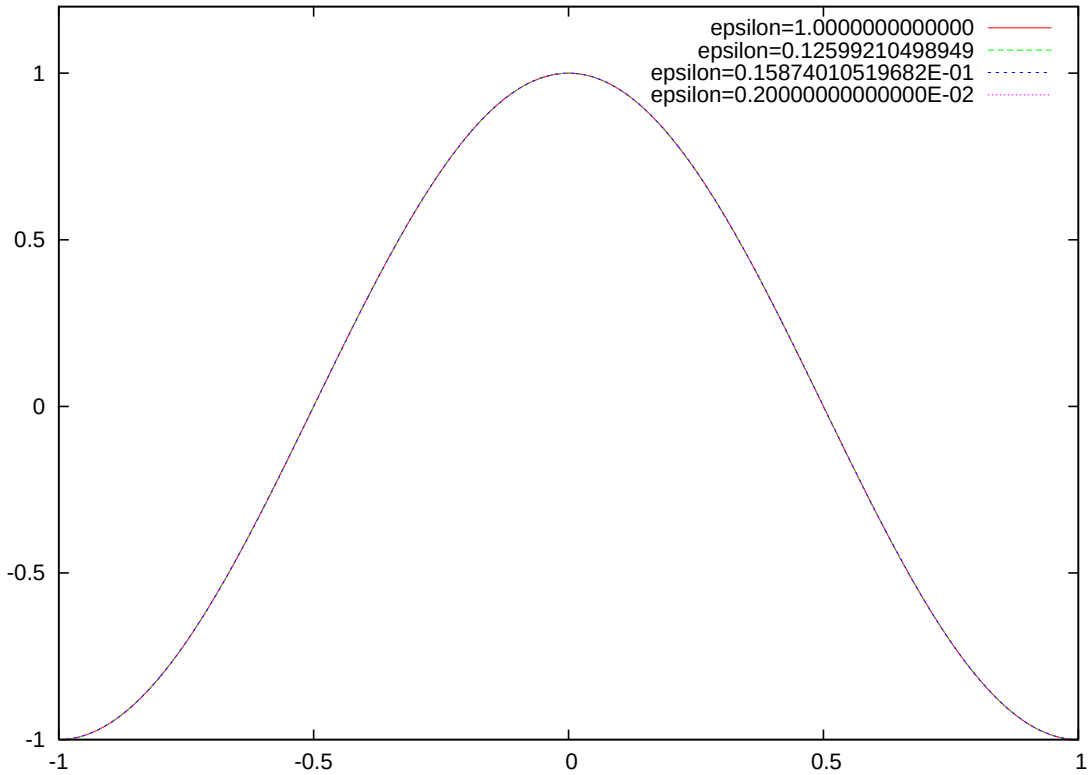
$y(x)$ versus x.

Test Problem 10: $\epsilon y'' + xy' = 0, \quad y(-1) = 0, \;\; y(1) = 2$,
Analytic solution: $y(x) = 1 + \mathrm{erf}(x/\sqrt{2\epsilon})/\mathrm{erf}(1/\sqrt{2\epsilon})$,



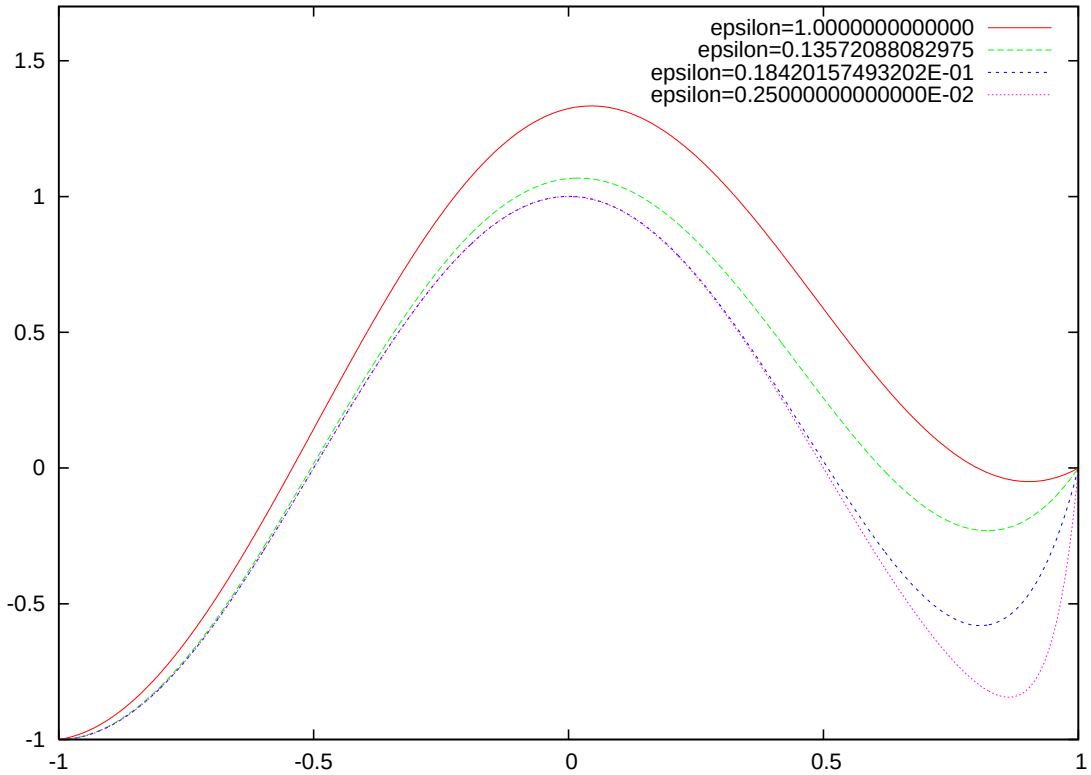
$y(x)$ versus x.

Test Problem 11: $\epsilon y'' - y = -(\epsilon\pi^2 + 1)\cos(\pi x), \quad y(-1) = y(1) = -1,$
Analytic solution: $y(x) = \cos(\pi x),$



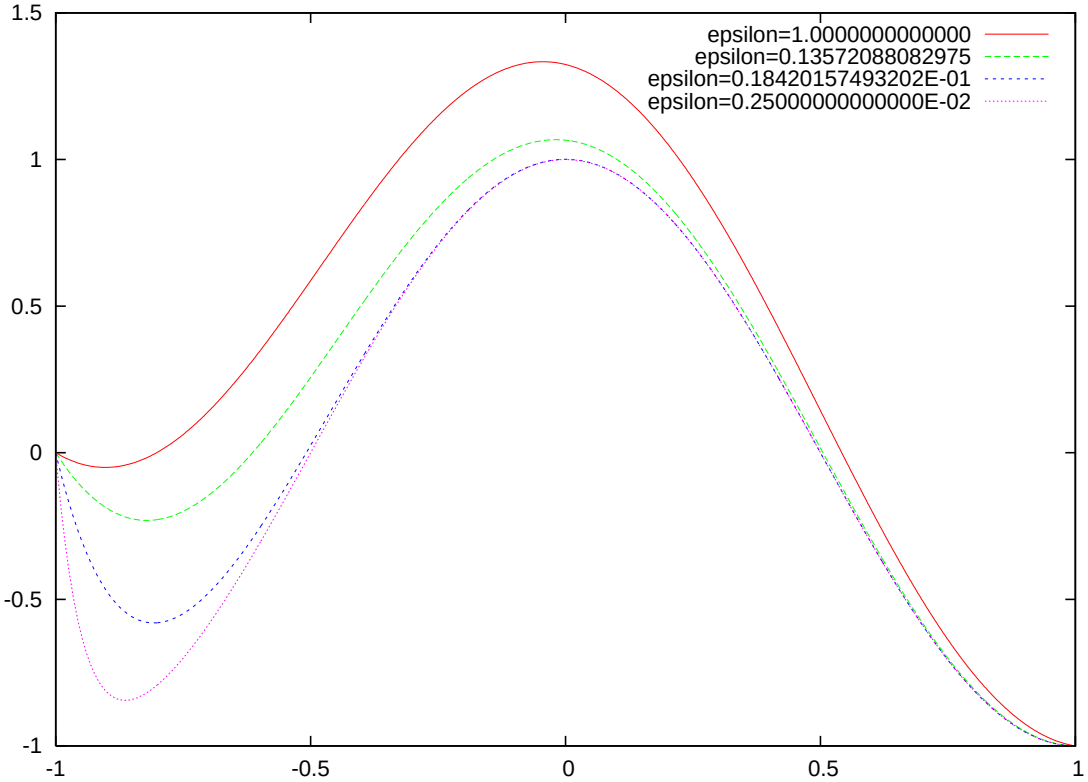
$y(x)$ versus x.

Test Problem 12: $\epsilon y'' - y = -(\epsilon\pi^2 + 1)\cos(\pi x), \quad y(-1) = -1, \;\; y(1) = 0,$
Analytic solution: $y(x) = \cos(\pi x) + \frac{\exp((x+1)/\sqrt{\epsilon}) - \exp((-x-1)/\sqrt{\epsilon})}{\exp(2/\sqrt{\epsilon}) - \exp(-2/\sqrt{\epsilon})},$



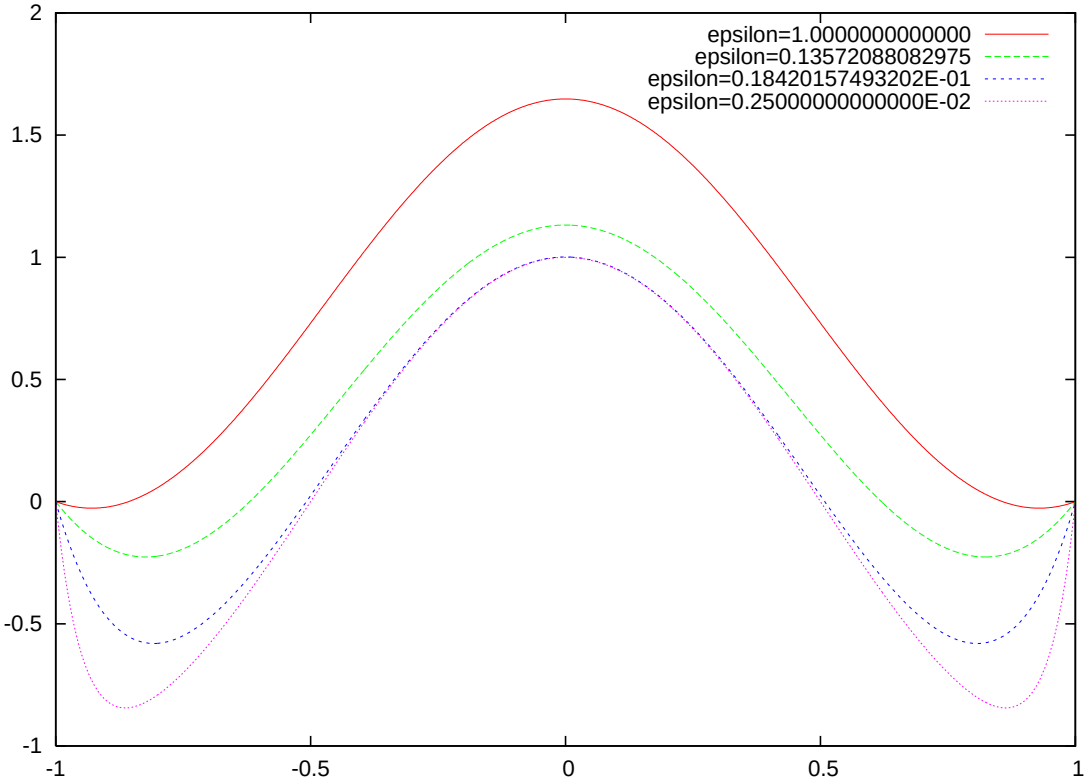
$y(x)$ versus x.

Test Problem 13: $\epsilon y'' - y = -(\epsilon\pi^2 + 1)\cos(\pi x), \quad y(-1) = 0, \;\; y(1) = -1.$
Analytic solution: $y(x) = \cos(\pi x) + \exp(-(x+1)/\sqrt{\epsilon})$.



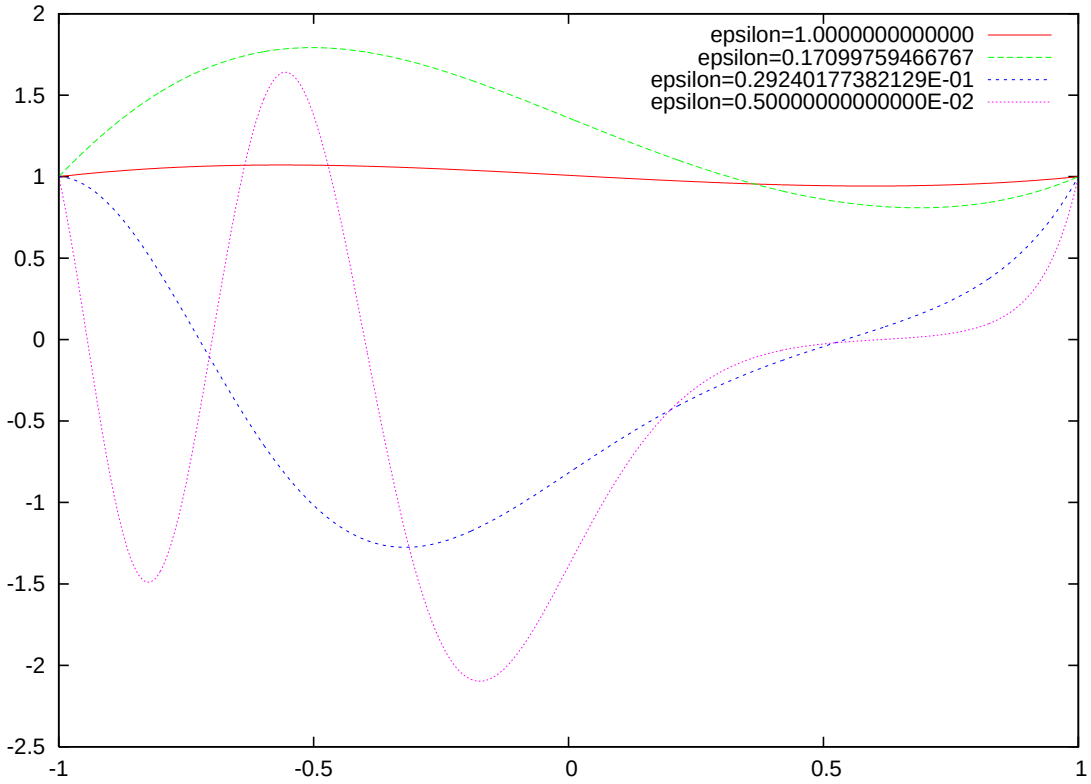
$y(x)$ versus x.

Test Problem 14: $\epsilon y'' - y = -(\epsilon\pi^2 + 1)\cos(\pi x), \quad y(-1) = y(1) = 0,$
Analytic solution: $y(x) = \cos(\pi x) + \exp((x-1)/\sqrt{\epsilon}) + \exp(-(x+1)/\sqrt{\epsilon})$.



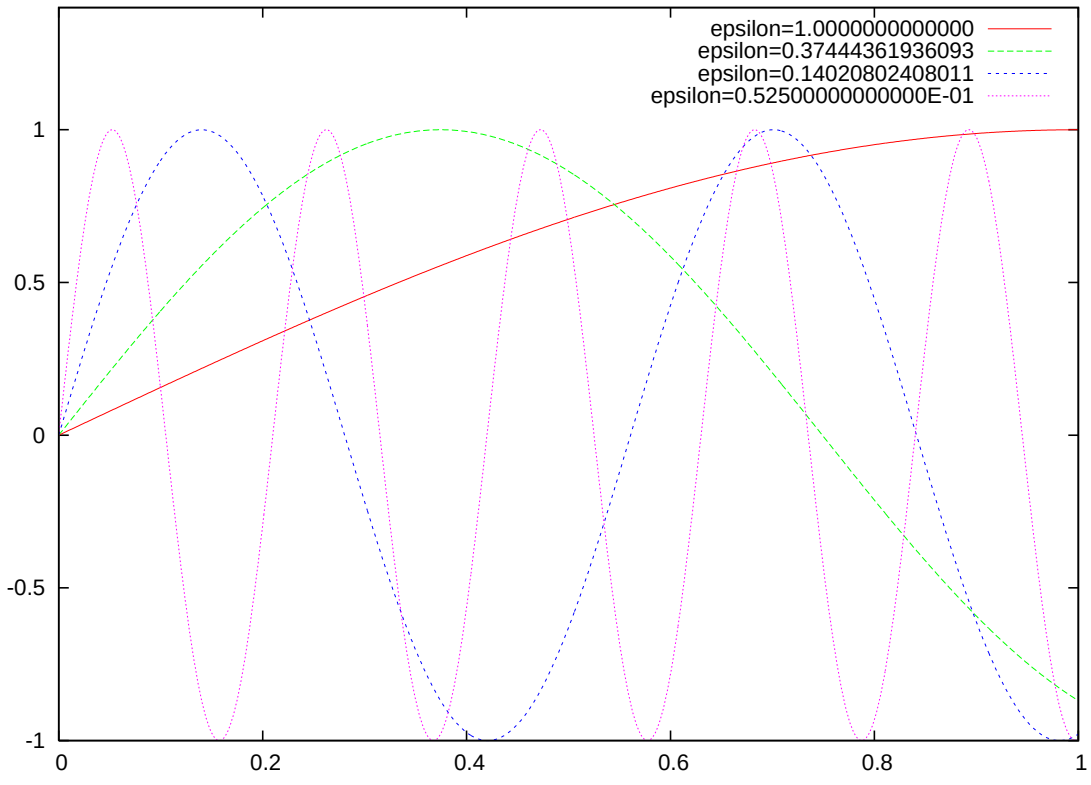
$y(x)$ versus x.

Test Problem 15: $\epsilon y'' - xy = 0, \quad y(-1) = y(1) = 1.$
General solution: $y(x) = A\mathrm{A}_i(x\epsilon^{-1/3}) + B\mathrm{B}_i(x\epsilon^{-1/3}).$



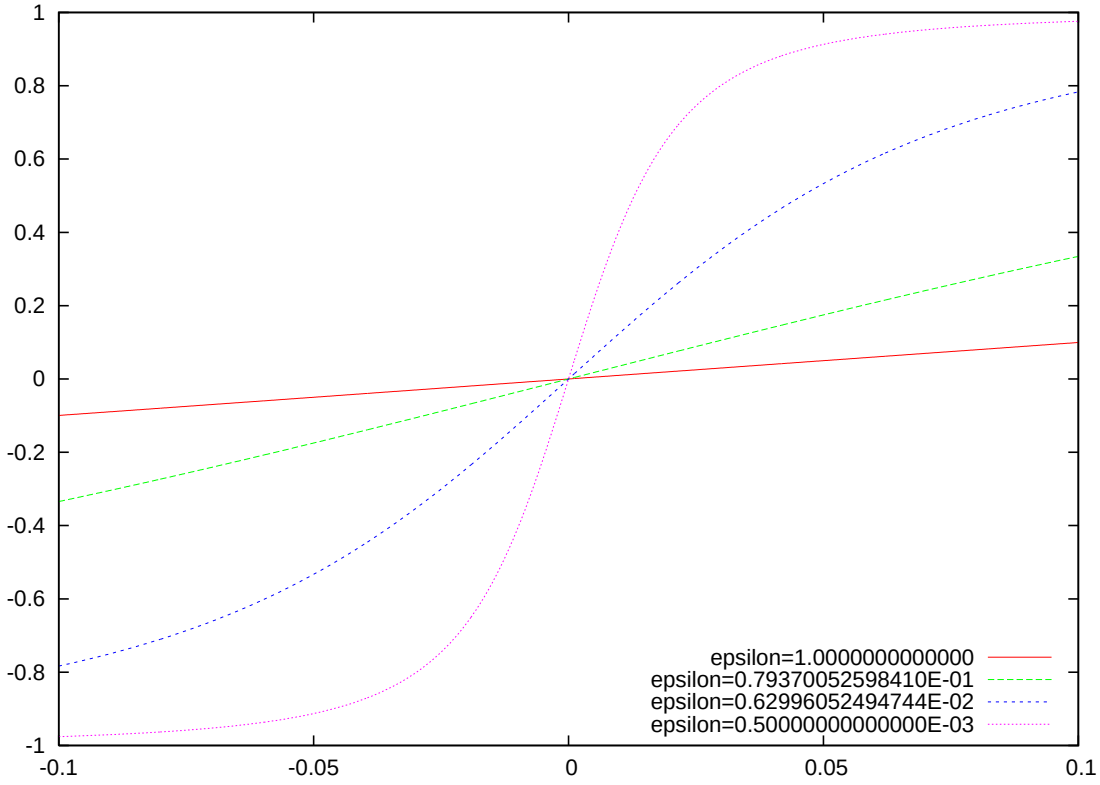
$y(x)$ versus x.

Test Problem 16: $\epsilon^2 y'' + \pi^2 y/4 = 0, \quad y(0) = 0, \;\; y(1) = \sin(\pi/2\epsilon),$
Analytic solution: $y(x) = \sin(\pi x/2\epsilon)$ (when $1/\epsilon$ is odd).



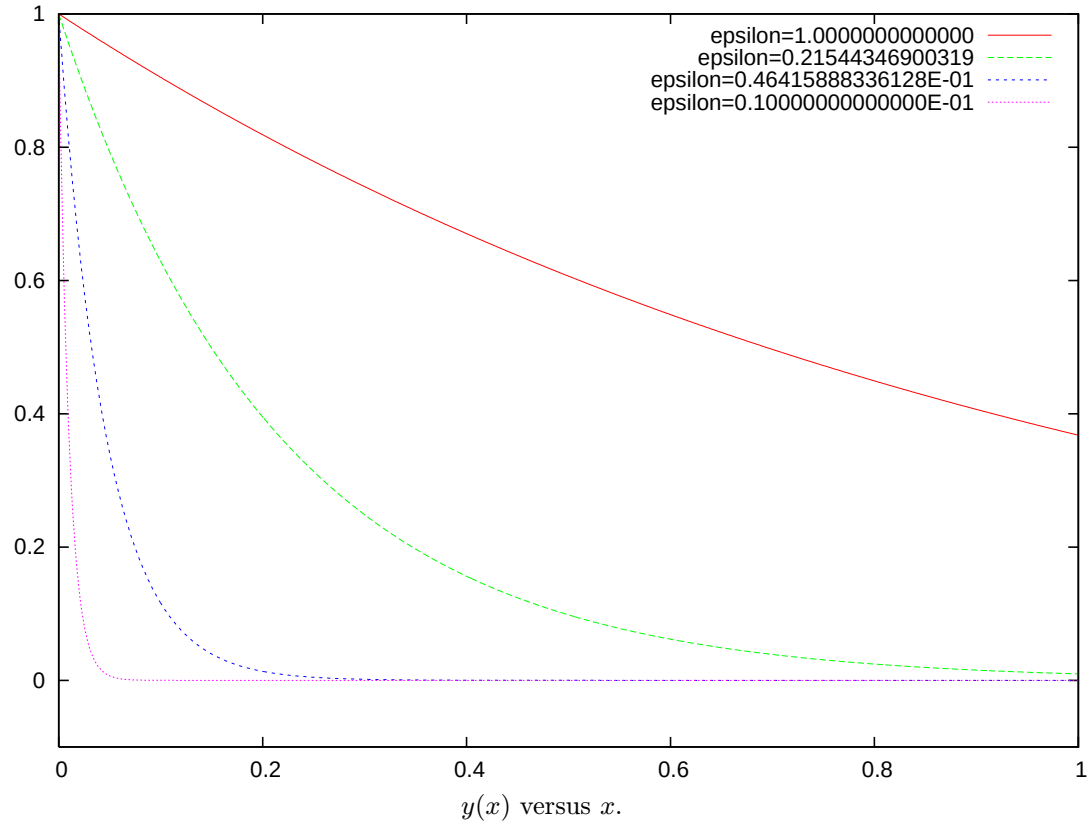
$y(x)$ versus x.

Test Problem 17: $y'' = -3\epsilon y/(\epsilon + x^2)^2, \quad y(0.1) = -y(-0.1) = 0.1/\sqrt{\epsilon + 0.01}$,
Analytic solution: $y(x) = x/\sqrt{\epsilon + x^2}$.



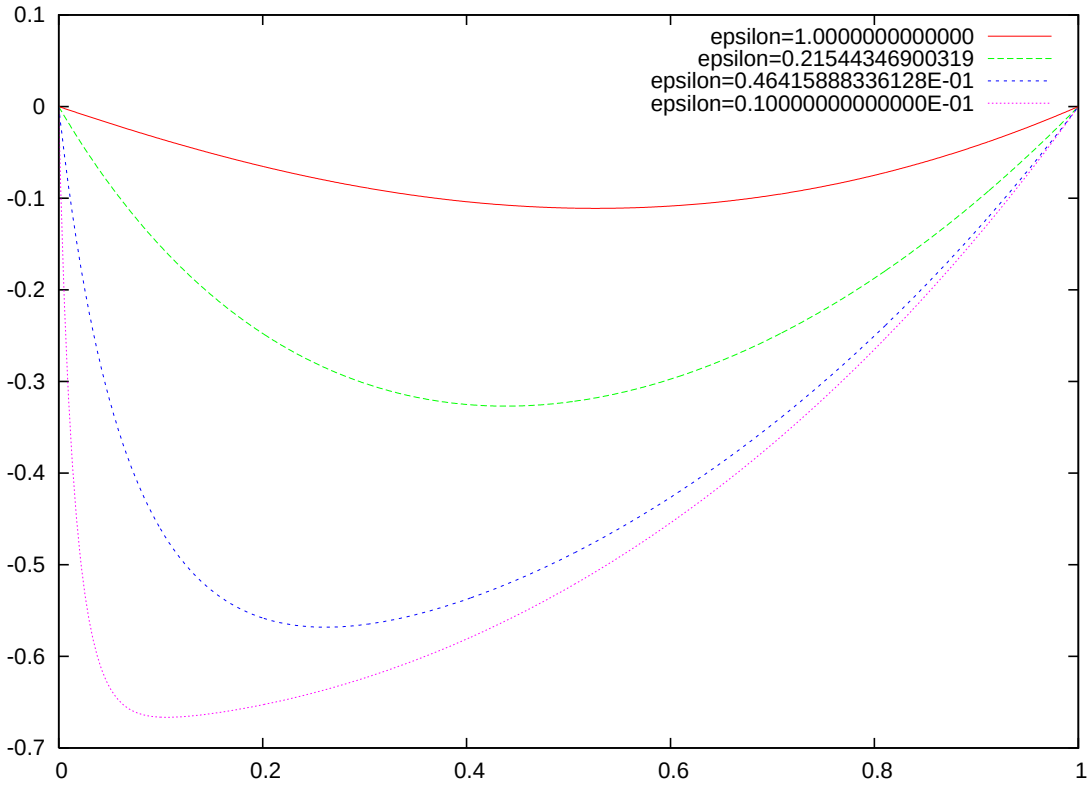
$y(x)$ versus x.

Test Problem 18: $\epsilon y'' = -y' \quad y(0) = 1, \;\; y(1) = \exp(-1/\epsilon)$,
Analytic solution: $y(x) = \exp(-x/\epsilon)$.



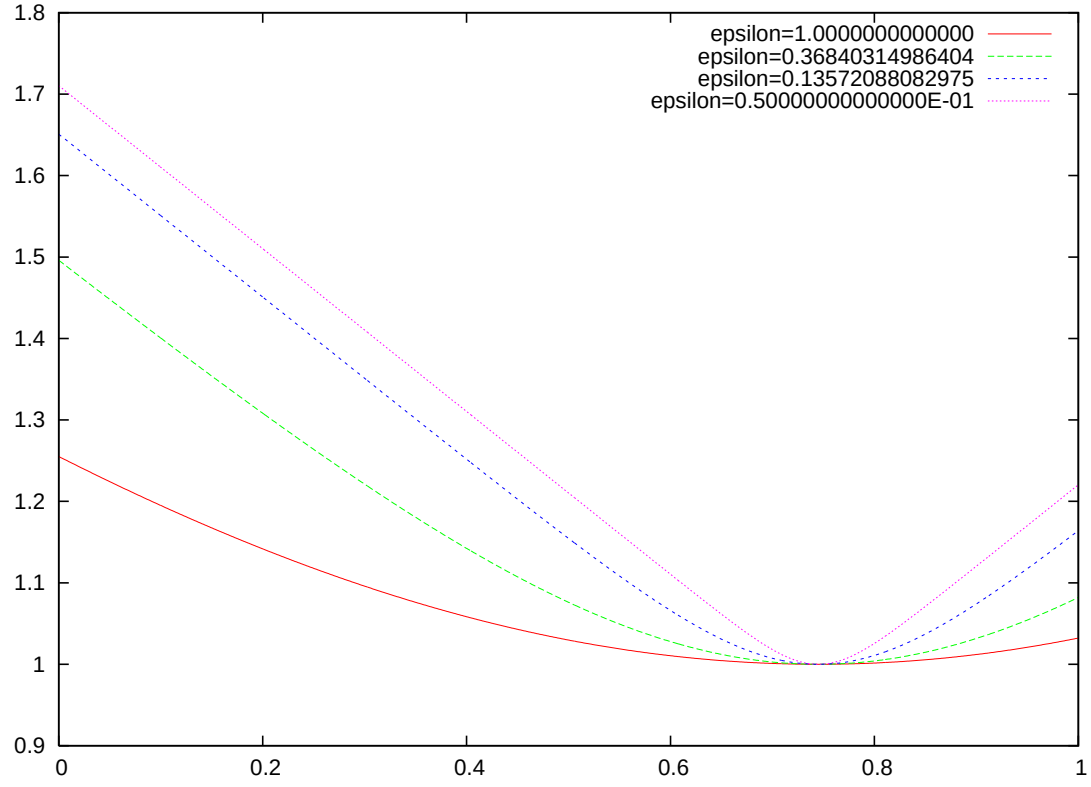
$y(x)$ versus x.

Test Problem 19: $\epsilon y'' + \exp(y)y' - \frac{\pi}{2}\sin(\pi x/2)\exp(2y) = 0, \quad y(0) = y(1) = 0.$



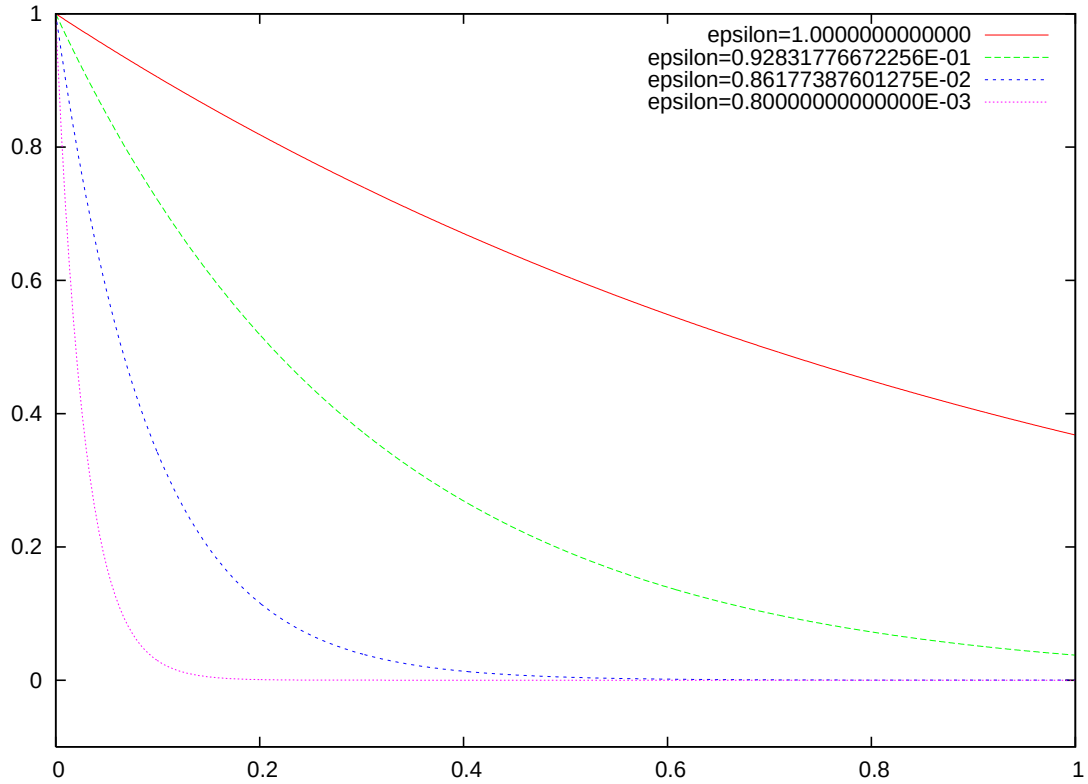
$y(x)$ versus x.

Test Problem 20: $\epsilon y'' + (y')^2 = 1, \;\; y(0) = 1 + \epsilon \ln\cosh(-0.745/\epsilon), \;\; y(1) = 1 + \epsilon \ln\cosh(0.255/\epsilon),$
Analytic solution: $y(x) = 1 + \epsilon \ln\cosh((x - 0.745)/\epsilon)$.



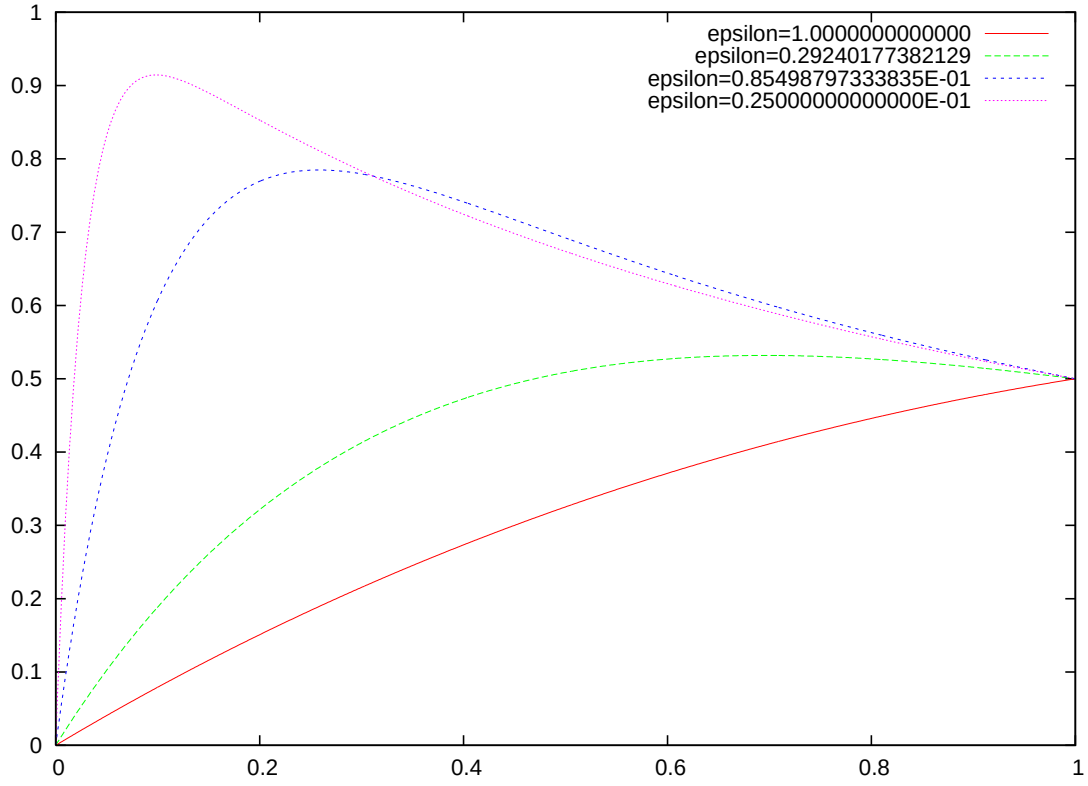
$y(x)$ versus x.

Test Problem 21: $\epsilon y'' = y + y^2 - \exp(-2x/\sqrt{\epsilon}), \quad y(0) = 1, \;\; y(1) = \exp(-1/\sqrt{\epsilon})$.
Analytic solution: $y(x) = \exp(-x/\sqrt{\epsilon})$.



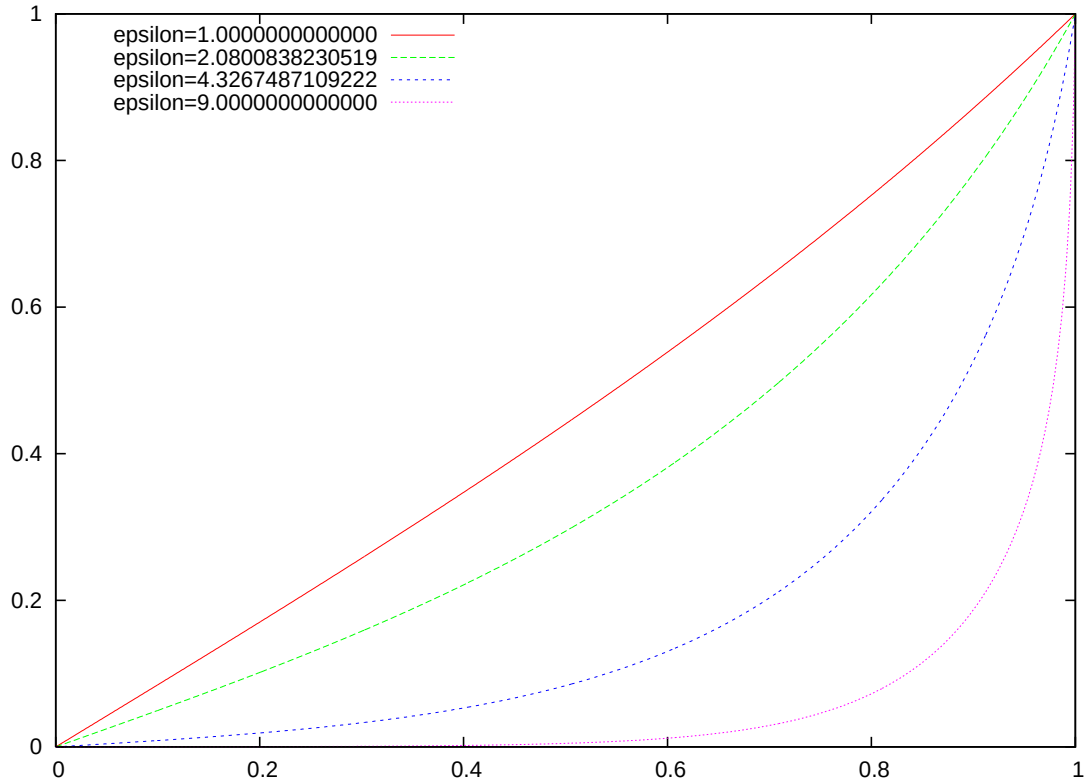
$y(x)$ versus x.

Test Problem 22: $\epsilon y'' + y' + y^2 = 0, \quad y(0) = 0, \;\; y(1) = \frac{1}{2}$.



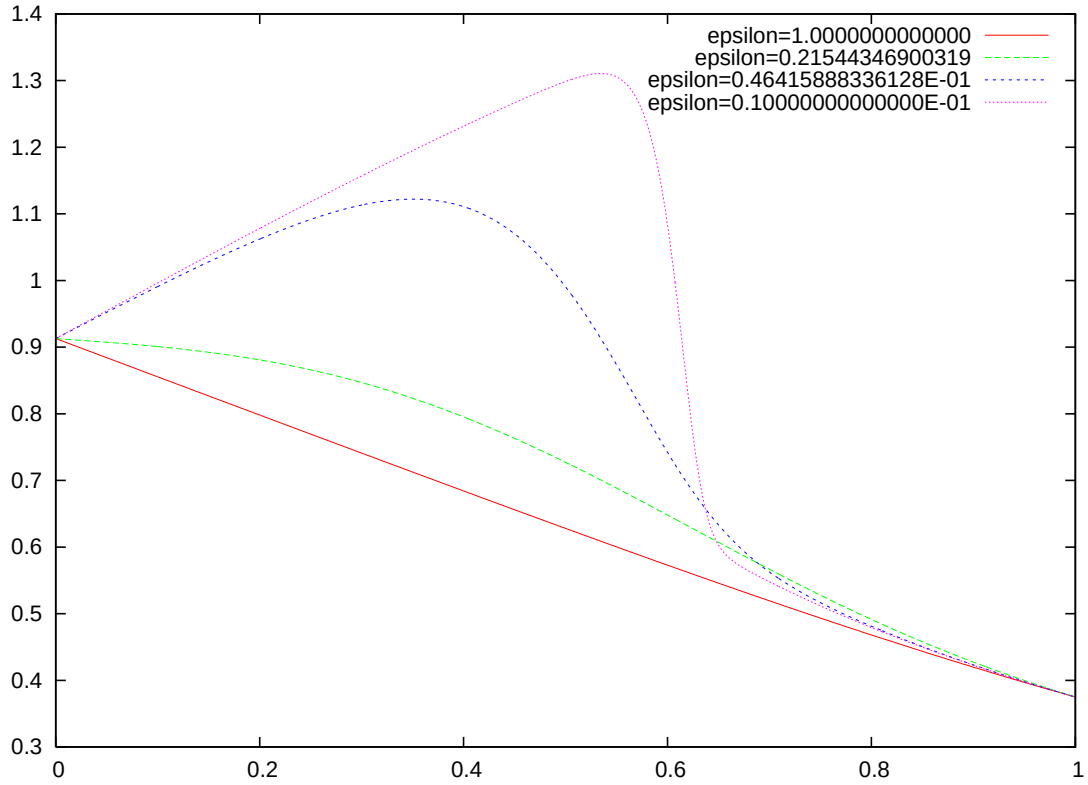
$y(x)$ versus x.

Test Problem 23: $y'' = \epsilon \sinh(\epsilon y), \quad y(0) = 0, \;\; y(1) = 1.$



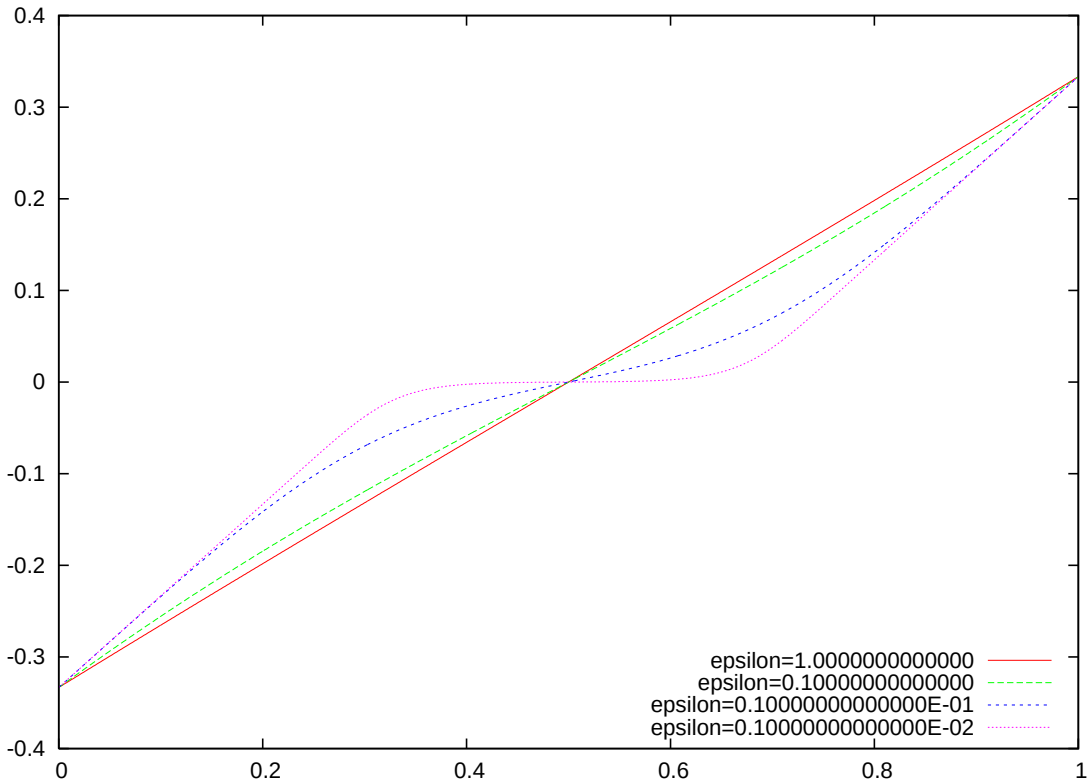
$y(x)$ versus x.

Test Problem 24: $\epsilon A(x) y y'' - \left(\frac{1+\gamma}{2} - \epsilon A'(x)\right) y y' + \frac{y'}{y} + \frac{A'(x)}{A(x)} \left(1 - \left(\frac{\gamma-1}{2}\right) y^2\right) = 0,$
$A(x) = 1 + x^2, \;\; \gamma = 1.4, \quad y(0) = 0.9129, \;\; y(1) = 0.375.$



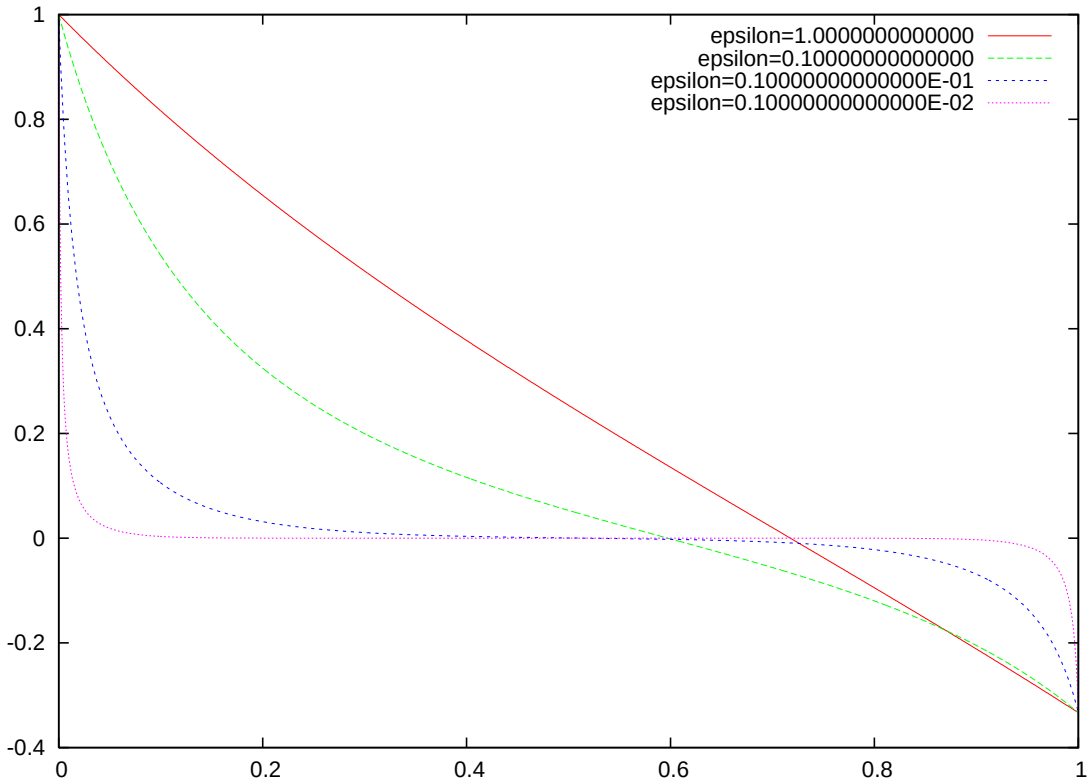
$y(x)$ versus x.

Test Problem 25: $\epsilon y'' + yy' - y = 0, \quad y(0) = -\frac{1}{3}, \;\; y(1) = \frac{1}{3}.$



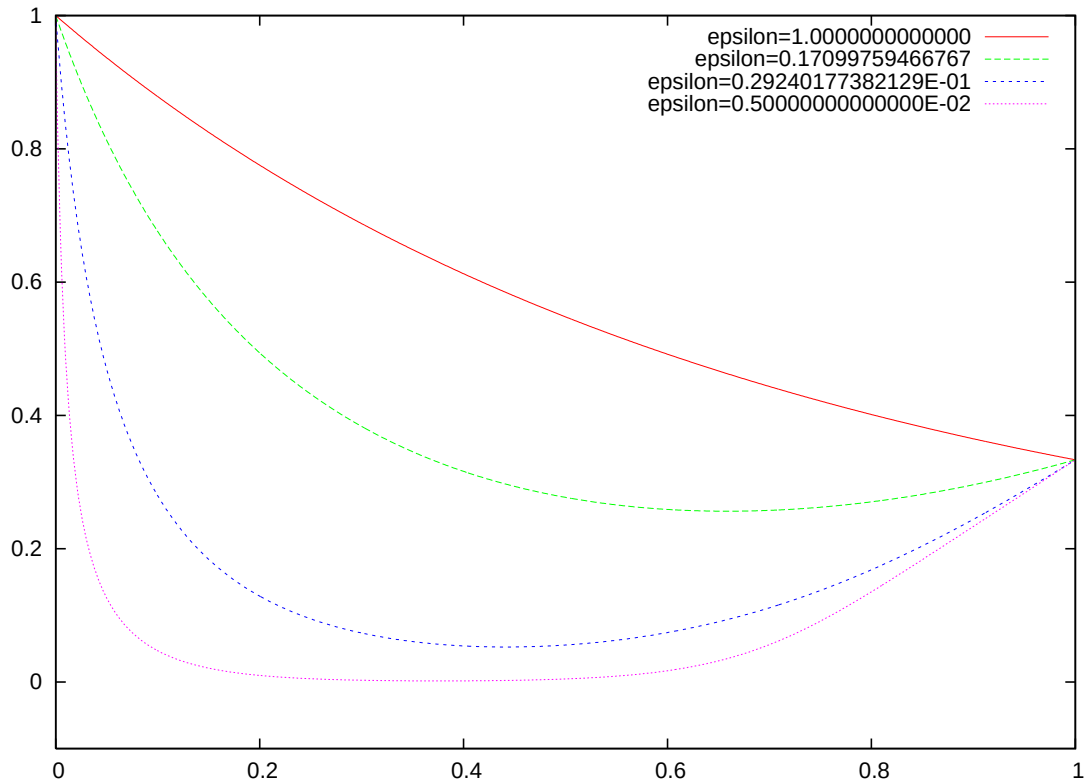
$y(x)$ versus x.

Test Problem 26: $\epsilon y'' + yy' - y = 0, \quad y(0) = 1, \;\; y(1) = -\frac{1}{3}.$



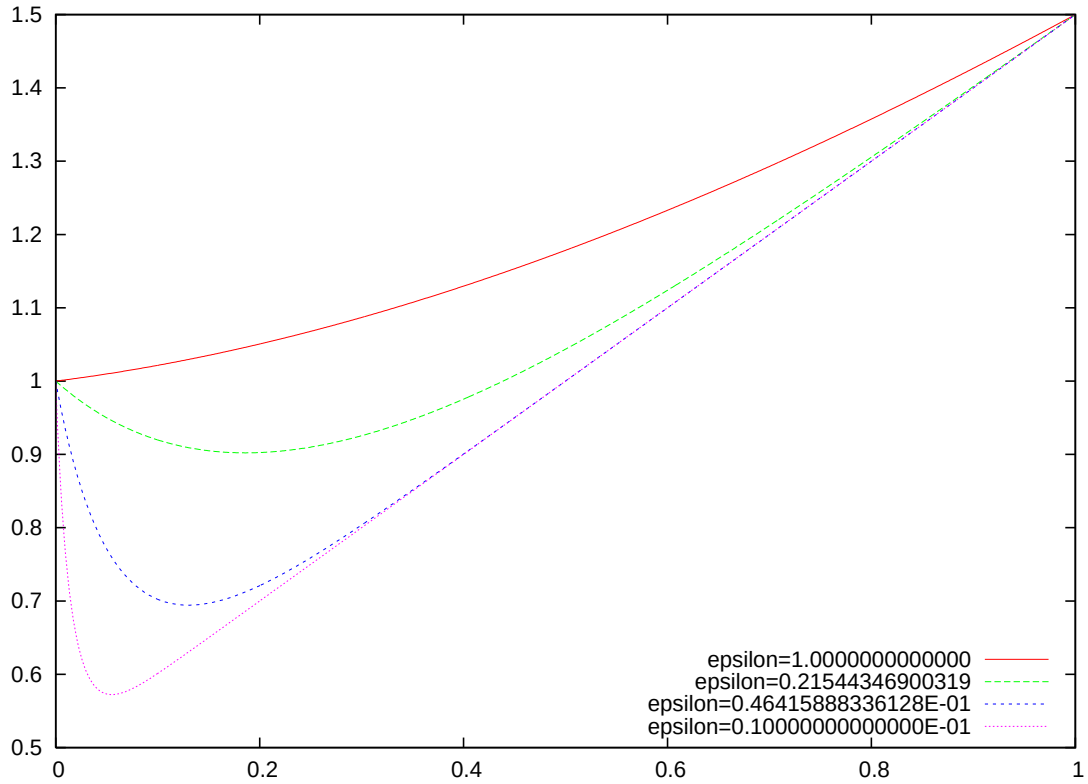
$y(x)$ versus x.

Test Problem 27: $\epsilon y'' + yy' - y = 0, \quad y(0) = 1, \;\; y(1) = \frac{1}{3}$.



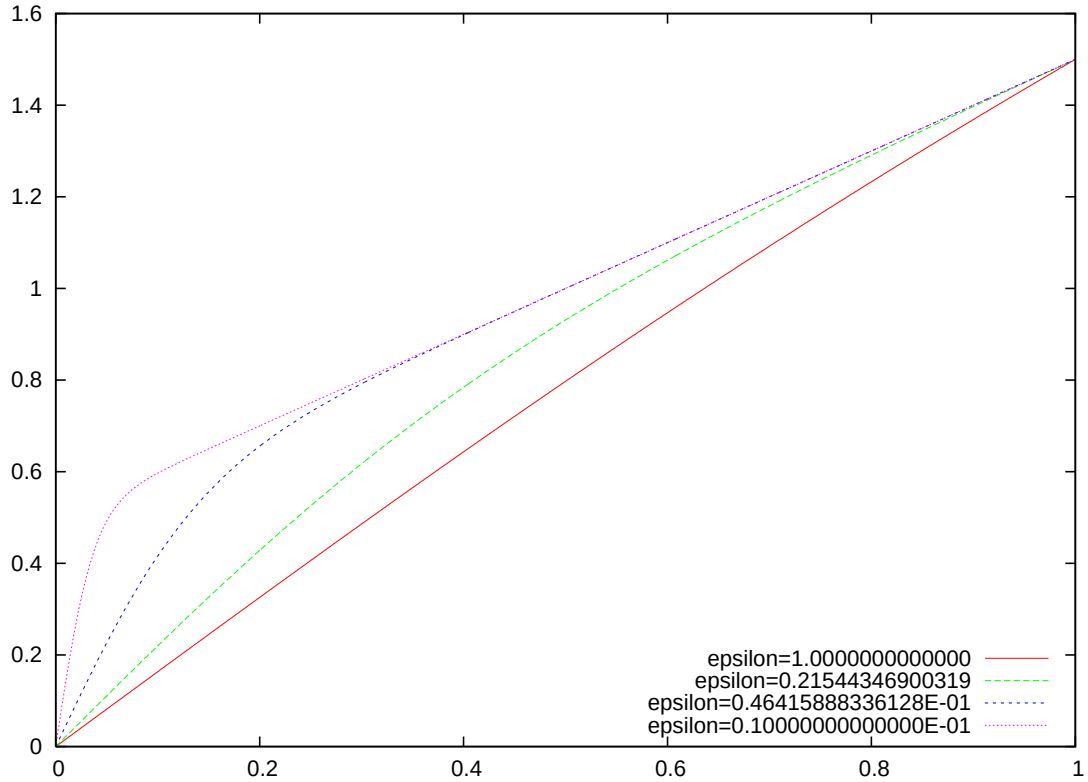
$y(x)$ versus x.

Test Problem 28: $\epsilon y'' + yy' - y = 0, \quad y(0) = 1, \;\; y(1) = \frac{3}{2}$.



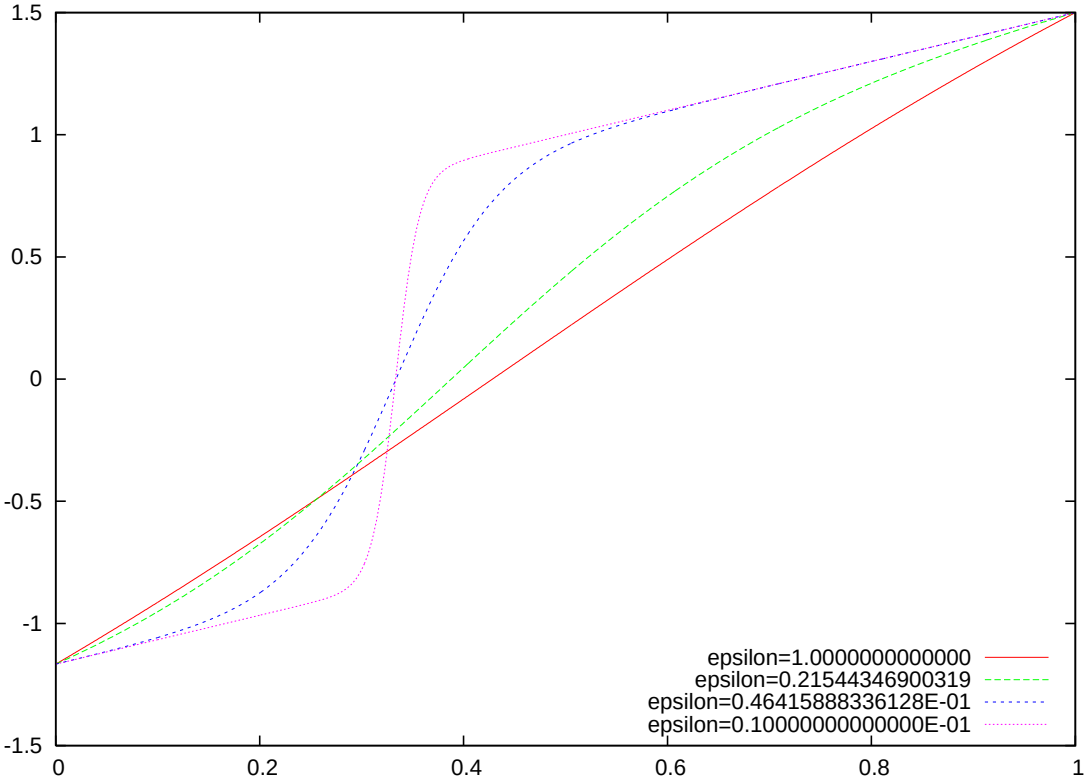
$y(x)$ versus x.

Test Problem 29: $\epsilon y'' + yy' - y = 0, \quad y(0) = 0, \;\; y(1) = \frac{3}{2}$.



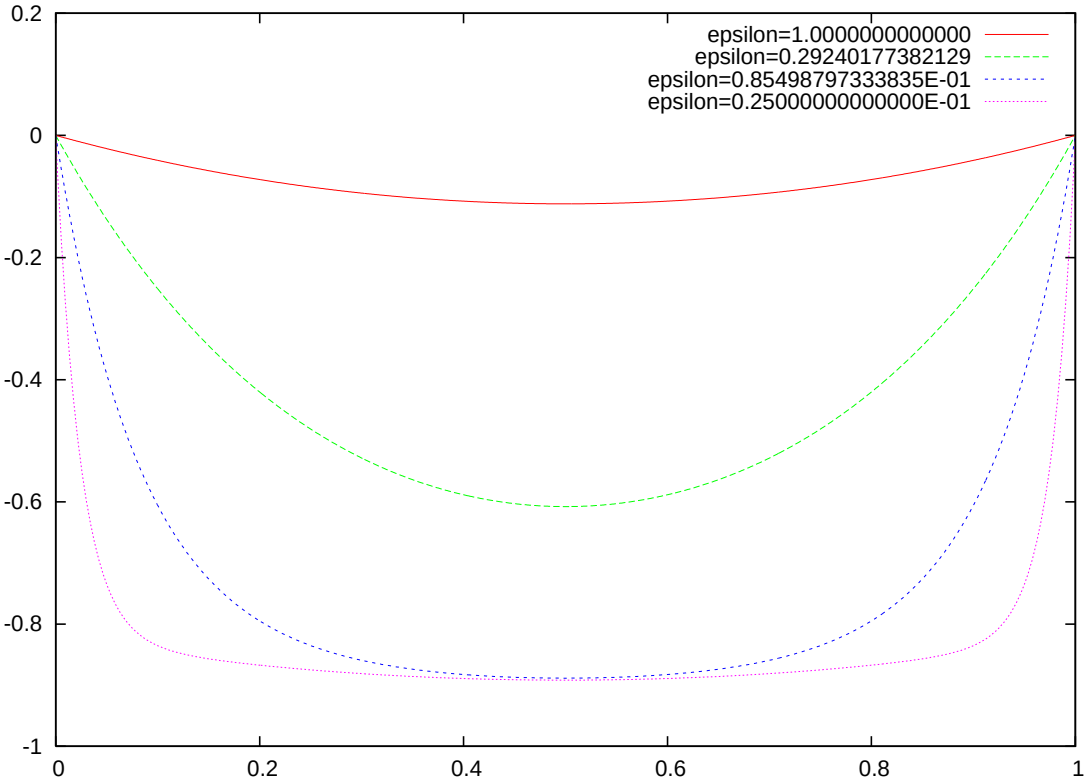
$y(x)$ versus x.

Test Problem 30: $\epsilon y'' + yy' - y = 0, \quad y(0) = -\frac{7}{6}, \;\; y(1) = \frac{3}{2}$.



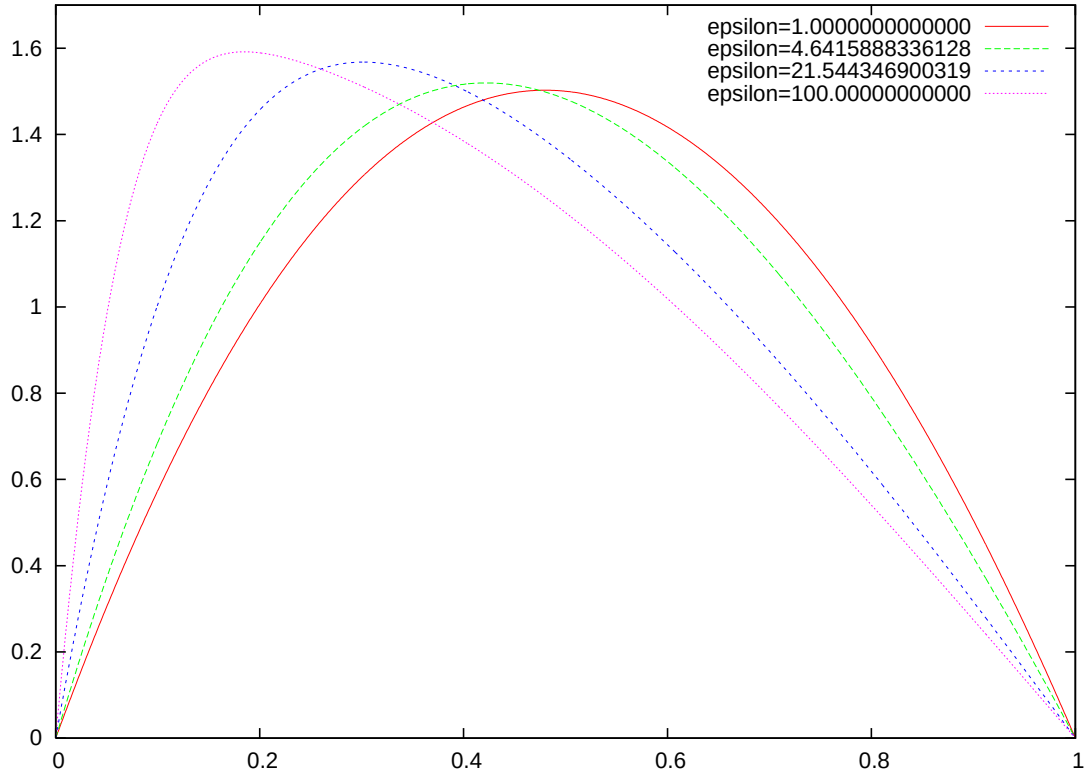
$y(x)$ versus x.

Test Problem 31: $y' = \sin\theta,\ \ \theta' = M,\ \ \epsilon M' = -Q,\ \ \epsilon Q' = (y-1)\cos\theta - MT,$
$T = \sec\theta + \epsilon Q\tan\theta,\quad y(0) = y(1) = 0,\ \ M(0) = M(1) = 0.$



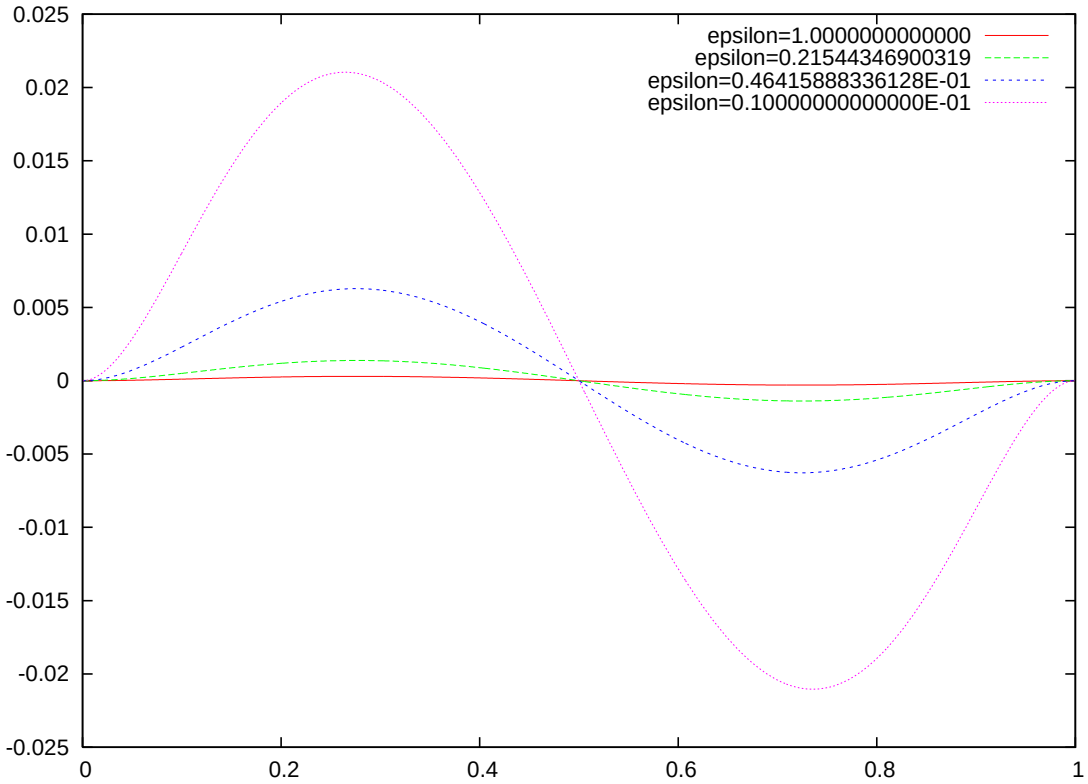
$M(x)$ versus x.

Test Problem 32: $y'''' = \epsilon(y'y'' - yy'''),\quad y(0) = y'(0) = 0,\ \ y(1) = 1,\ \ y'(1) = 0.$



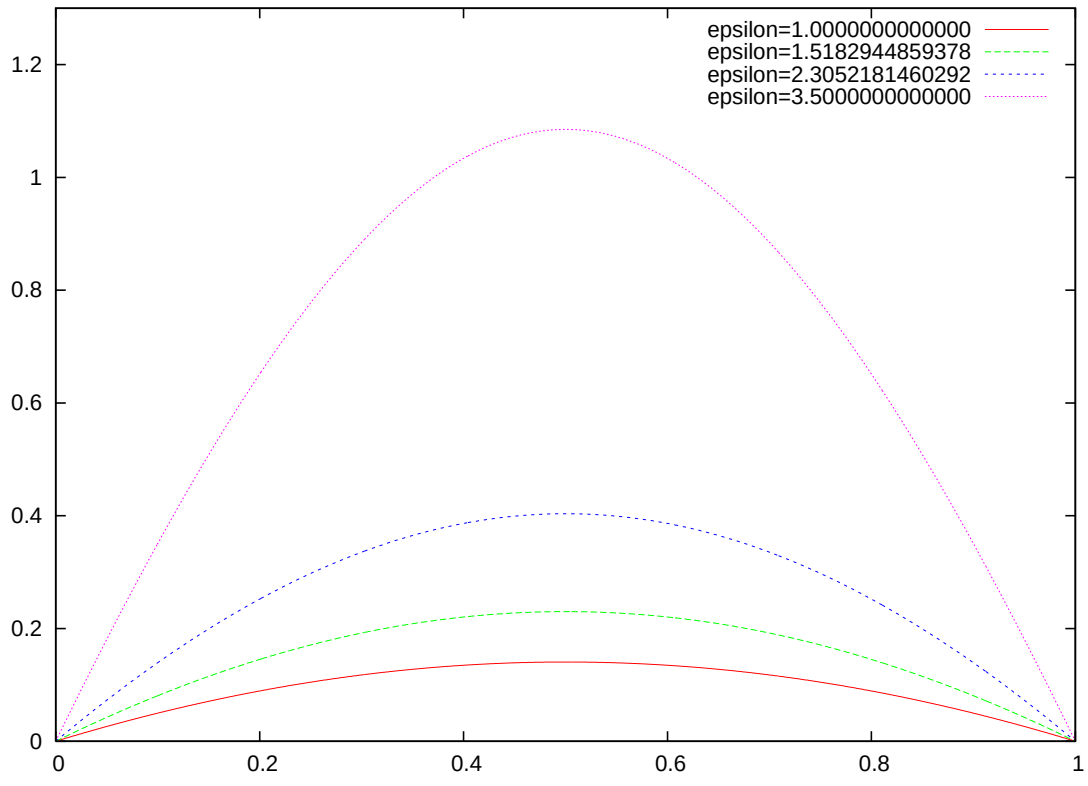
$y'(x)$ versus x.

Test Problem 33: $\epsilon z'''' = -zz''' - yy', \;\; \epsilon y'' = yz' - zy', \;\; y(0) = -1, \;\; y(1) = 1, \;\; z(0) = z'(0) = z(1) = z'(1) = 0.$



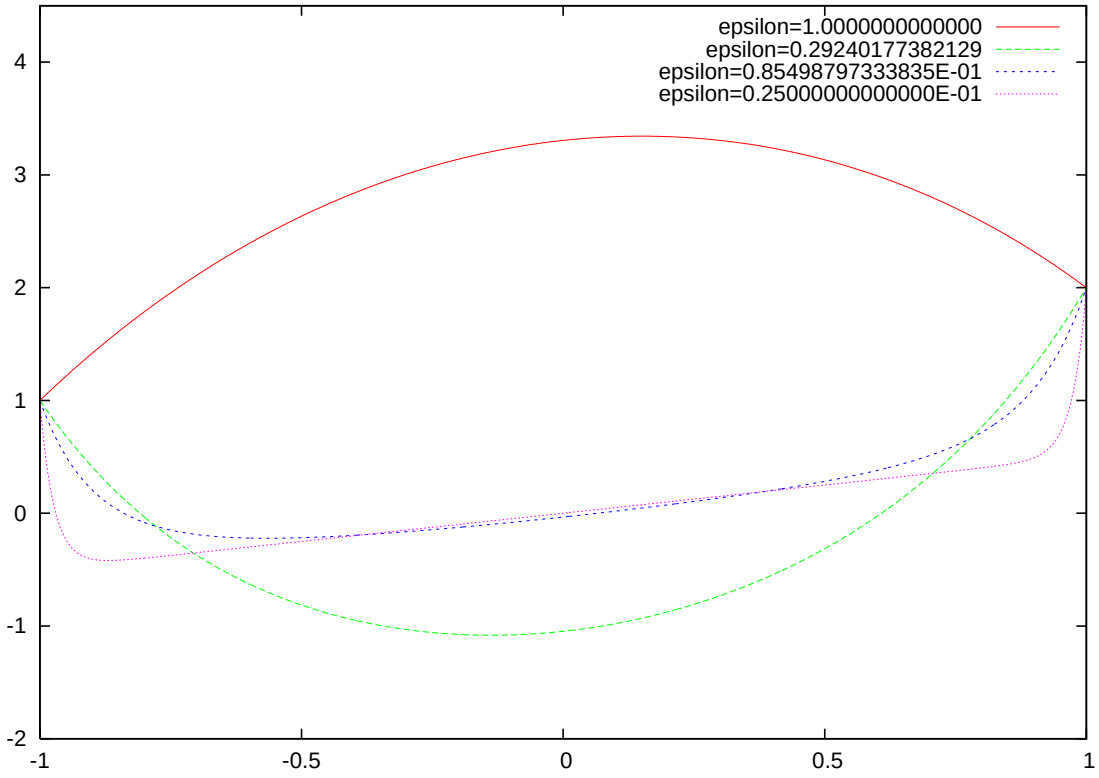
$z(x)$ versus x.

Test Problem 34: $y'' = -\epsilon \exp(y), \quad y(0) = y(1) = 0$



$y(x)$ versus x.

Test Problem 35: $\epsilon y'' = xy' - y, \quad y(-1) = 1, \;\; y(1) = 2.$



$y(x)$ versus x.