BOOK OF ABSTRACTS

Analysis/Stochastic Analysis Intercontinental

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Mathematics Department

Huxley Building, **Room HXLY 130**

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NTERPLAY BETWEEN FOURIER AND GABOR BASES THROUGH TILING CONDITIONS

Given, $\Omega \subset \mathbb{R}^d$, Fuglede's conjecture states that $L_2(\Omega)$ admits a Fourier orthonormal basis if and only if Ω tiles $\mathbb{R}^d$ when it is translated with respect to a set $\Gamma \subset \mathbb{R}^d$. This means that $\mathbf{C}_{\gamma\in\Gamma}(\gamma + \Omega)$ forms an almost everywhere partition of $\mathbb{R}^d$. The conjecture holds true whenever the set Γ is a lattice. However, in general it has been disproved in both directions for dimensions $d \geq 3$. In the case $d = 1$ and 2 only partial results are known. For $d = 1$, Łaba proved that, if Ω is a union of two intervals, then the conjecture is true.

We will center our talk in the connections between Łaba's result and the Gabor systems with window the characteristic function of Ω, where Ω is the union of two intervals. Recall that given a function g, called window, the Gabor system consists on the set of modulations and translations of g. Then, we will show that the Gabor system with window the characteristic function of Ω is a basis if and only if Ω tiles R with respect to a set of translations. In this case, we will describe the structure of the (time-frequency) sets associated to this bases.

This talk is based in a joint work with Jorge Antezana and Mihalis Kolountzakis.

Jorge Antezana

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QUASICRYSTALS IN HARMONIC ANALYSIS

Quasicrystals are non-periodic structures discovered by Shechtman in 1984, while he was studying materials whose X-ray diffractions spectra present such non-periodic behaviors (see [7]).

Nowadays, one of the best mathematical ways to model quasicrystals are the so called model sets introduced by Meyer in [6] many years before the discovery of Shechtman. The aim of Meyer was to study approximation of algebraic characters by continuous ones in locally compact abelian groups (see also [3]).

Recently, important applications of quasicrystals to Fourier Analysis have been found (see [5], [2], [4]). In this talk we will discuss some of these applications, making focus in those related with problems of sampling and interpolation in Paley Wiener spaces.

References

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Inés Armendariz

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Metastability in condensing zero-range processes

We consider zero-range processes with jump rates that decrease with the occupation number, which are known to exhibit a condensation phenomenon where a fraction of all particles concentrates on a single lattice site. We derive a scaling limit for the asymptotic stationary dynamics of the condensate location in the thermodynamic limit on a one-dimensional torus. Our proof follows previously developed methods using potential theory and a martingale approach. Joint work with A. de Masi, S. Grosskinsky, M. Loulakis and E. Presutti.

Marco Bertola (SISSA, Italy)

The matrix models of Kontsevich, Kontsevich—Penner and generalizations: an approach based on inverse monodromy

The Kontsevich matrix integral (aka "Matrix Airy function"), in the appropriate formal limit, generates the intersection numbers on $\mathcal M_{g,n}$. In the same formal limit it is also a particular tau function of the KdV hierarchy; truncation of the times yields thus tau functions of the first Painlev\'e\ hierarchy.
This, however is a purely formal manipulation that pays no attention to issues of convergence.

The talk will try to address two issues:
Issue 1: how to make an analytic sense of the convergence of the Kontsevich integral to a tau function for a member of the Painlev\'e\ I hierarchy?
Which particular solution(s) does it converge to?
Where (for which range of the parameters)?

There exist now generalizations due originally to Penner (and used recently by Alexandrov, Buryak, Solomon, Tessler, Pandharipande, Brezin and Hikami) that generate similar intersection numbers on the moduli space of pointed Riemann surfaces with boundary ("open" intersection numbers). Some aspects are still conjectural, due mainly to the difficult definition of the moduli space itself. The approach to the integral that we propose leads to an isomonodromic

description that allows a closed—form expression (although involving several level of nested sums) of all these numbers.

This is joint work with Giulio Ruzza (SISSA) and based also on prior work with Boris Dubrovin and Di Yang.

Jose Antonio Carrillo de la Plata

Mathematics Department

Imperial College London

Nonlinear aggregation-diffusions in the diffusion-dominated and fair-competitions regimes

We analyse under which conditions equilibration between two competing effects, repulsion modelled by nonlinear diffusion and attraction modelled by nonlocal interaction, occurs. I will discuss several regimes that appear in aggregation diffusion problems with homogeneous kernels. I will first concentrate in the fair competition case distinguishing among porous medium like cases and fast diffusion like ones. I will discuss the main qualitative properties in terms of stationary states and minimizers of the free energies. In particular, all the porous medium cases are critical while the fast diffusion are not. In the second part, I will discuss the diffusion dominated case in which this balance leads to continuous compactly supported radially decreasing equilibrium configurations for all masses. All stationary states with suitable regularity are shown to be radially symmetric by means of continuous Steiner symmetrisation techniques. Calculus of variations tools allow us to show the existence of global minimizers among these equilibria. Finally, in the particular case of Newtonian interaction in two dimensions they lead to uniqueness of equilibria for any given mass up to translation and to the convergence of solutions of the associated nonlinear aggregation-diffusion equations towards this unique equilibrium profile up to translations as time tends to infinity. This talk is based on works in collaboration with S. Hittmeir, B. Volzone and Y. Yao and with V. Calvez and F. Hoffmann.

Dan Crisan

Department of Mathematics

Imperial College London

On a two-dimensional pseudo-gravity model

I will present a macroscopic model describing the evolution of a cloud of particles confined in a magneto-optical trap. The behavior of the particles is mainly driven by self-consistent attractive forces. In contrast to the standard model of gravitational forces, the force field does not result from a potential; moreover, the nonlinear coupling is more singular than the coupling based on the Poisson equation. We establish the existence of solutions, under a suitable smallness condition on the total mass, or, equivalently, for a sufficiently large diffusion coefficient. When a symmetry assumption is fulfilled, the solutions satisfy strengthened estimates (exponential moments). I will also discuss the convergence of the N-particles description towards the PDE system in the mean field regime.

This is joint work with Julien Barré (Orleans) and Thierry Goudon (Nice).

Pierre Degond

Department of Mathematics

Imperial College London

Coarse-graining of collective dynamics models

In this talk, we will report on some new individual-based models of collective dynamics and their coarse-graining into continuum models. The applications span from collective cell dynamics (such as social bacteria or sperm) to flocking of birds or fish. Models of social behavior are best set up at the individual scale where behavioral rules can be easily introduced and tested. However, the complexity of individual-based models increases rapidly with the number of individuals and their calibration or control can hardly be implemented at this level. To overcome this limitation, one often uses continuum model that describe the system through average quantities such as densities or mean orientation. But the downside of most models in the literature is that the link between the rules at the individual behavior and the coefficients in the macroscopic model are not known exactly and are at best extrapolated from heuristic consideration. Here, we propose a systematic and mathematical rigorous way to derive continuum models from collective dynamics models. It relies on the introduction of a new concept, the "generalized collision invariants", which permit to overcome the lack of physical invariance in most systems undergoing collective dynamics. In this talk, we will review some recent developments of these concepts and how they can be used to model systems of practical scientific importance.

Daniel E. Galicer

IMAS, CONICET - UNIVERSIDAD DE BUENOS AIRES

CLUSTER VALUES FOR ALGEBRAS OF ANALYTIC FUNCTIONS

The Cluster Value Theorem is known for being a weak version of the classical Corona Theorem. Given a Banach space X, we study the Cluster Value Problem for the ball algebra $A_u(B_X)$, the Banach algebra of all uniformly continuous holomorphic functions on the unit ball B_X; and also for the Fréchet algebra $H_b(X)$ of holomorphic functions of bounded type on X. We show that Cluster Value Theorems hold for all of these algebras whenever the dual of X has the bounded approximation property. These results are an important advance in this problem, since the validity of these theorems was known only for trivial cases (where the spectrum is formed only by evaluation functionals) and for the infinite dimensional Hilbert space. If time permits, we will present some structural results for the spectrum of these algebras.

Joint work with Daniel Carando, Santiago Muro and Pablo Sevilla-Peris.

Pablo Groisman

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Branching-selection particle systems: travelling waves, quasi-stationary

distributions and selection principles.

ABSTRACT: We will discuss three different models of branching with selection. In all of them N particles evolve independently according to a Markov process unless a death occurs. This could happen at Poissonian times or at stopping times defined by the movement of the particles. At these times one particle is eliminated from the system and it is replaced by a new one according to some rule. We interpret this transition as a branching-selection mechanism. We will discuss common features of these systems such as their hydrodynamic limits and a selection principle: the convergence under the invariant measure, as the number of particles goes to infinity, to a minimal quasi-stationary distribution or a minimal travelling-wave depending on the model

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Gustav Holzegel

Department of Mathematics

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The Black Hole Stability Problem

I will discuss the black hole stability problem in general relativity and some of its geometric and analytical challenges from the point of view of non-linear partial differential equations. I will also describe progress that has been made in the past ten years including a recent theorem (obtained in collaboration with M. Dafermos and I. Rodnianski) proving the linear stability of the Schwarzschild solution.

Christain Olivera

IMECC-UNICAMP

Regularization by noise in (2x 2) hyperbolic systems of conservation law.

In this talk we study a non-strictly hyperbolic system of conservation law by stochastic perturbation. We show the existence and uniqueness of the solution. We do not assume that BV- regularity for the initial conditions. The proofs are based in concept of entropy solution and in the characteristics method (in the influence of noise). This is the first result on the regularization by noise in hyperbolic systems of conservation law.

Michela Ottobre

Mathematics Department

Heriot-Watt University

A kinetic model of self-propelled particles in one dimension

One of the new challenges of statistical mechanics arises from the study of interacting particle systems of self-propelled particles. Such models are at the root of many biological phenomena, such as bacterial migration, flocking of birds etc. In this talk we will consider a non-linear PDE for a Viksek-type model. The PDE at hand is i) not in gradient form and ii) it is non-uniformly elliptic (but hypoelliptic instead). Moreover, as typical in this framework, the dynamics exhibits multiple equilibria. This is a joint work with P.Butta (La Sapienza, Rome) and B. Zegarlinski (Imperial College).

Greg Pavliotis

Department of Mathematics

Imperial College London

Noise-induced transitions and mean field limits for multiscale diffusions.

In this talk I will present some recent results on the long time behaviour of the overdamped Langevin dynamics for Brownian particles moving in a multiscale, rugged energy landscape. The dynamics of such processes can be quite complicated, in particular in the low temperature regime, since metastable states, corresponding to local minima of the potential, can(co-)exist at all scales. We will show how we can obtain a coarse-grained description for the dynamics at large scales, given by a stochastic differential equation with multiplicative noise, despite the fact that the noise in the original dynamics is additive. We then show that the combined effect of noise and multiscale structure leads to hysteresis effects in the bifurcation diagram for the equilibrium coarse-grained dynamics. In the second part of the talk I will present recent results on the mean field limit of systems of interacting diffusions in a multiscale confining potential. The mean field limit is described by a nonlinear, nonlocal Fokker-Planck equation of McKean-Vlasov type that exhibits phase transitions. The effect of the multiscale structure of the potential on the phase diagram will be discussed in detail.

Ewelina Zatorska

Mathematics Department

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On Particle interactions mediated by dynamical network.

Starting from a microscopic model of particles interacting through a dynamical network of links, via the mean eld limit, I will formally derive coupled kinetic equations for the particle and link densities. I will then consider a fast

linking/unlinking limit and analyse qualitatively the resulting aggregation-diffusion equation. I will give a precise condition for the phase transition, and, using the central manifold reduction, I will characterize the type of bifurcation at the instability onset. I will also provide the numerical verification of these results.
This is a talk based on two papers in collaboration with: J. Barre, J.A. Carrillo, P. Degond, and D. Peurichard.

Contributed Talks

List of Speakers

Rauan Akylzhanov

Imperial College London

Chiara Taranto

Imperial College London

Paul Dobson

University of Warwick

Nurgissa Yessirkegenov

Imperial College London

Yong Sul Won

Imperial College London

Titles & Abstracts

Rauan Akylzhanov

Imperial College London

L_p-L_q Fourier multipliers on locally compact groups

Imperial College London Department of Mathematics Imperial College London 180 Queen's Gate, London SW7 2AZ United Kingdom

Joint work with: Michael Ruzhansky

We discuss the L_p-L_q boundedness of both spectral and Fourier multipliers on general locally compact separable unimodular groups G for the range $1 < p \leq 2 \leq q < \infty$. As a consequence of the established Fourier multiplier theorem we also derive a spectral multiplier theorem on general locally compact separable unimodular groups. We then apply it to obtain embedding theorems as well as time-asymptotics for the L_p-L_q norms of the heat kernels for general positive unbounded invariant operators on G. We illustrate the obtained results for sub-Laplacians on compact Lie groups and on the Heisenberg group, as well as for higher order operators. We show that our results imply the known results for L_p-L_q multipliers such as Ho¨rmander's Fourier multiplier theorem on Rn or known results for Fourier multipliers on compact Lie groups. The new approach developed in this paper relies on advancing the analysis in the group von Neumann algebra and its application to the derivation of the desired multiplier theorems.

Chiara Taranto

Imperial College London

WELL-POSEDENESS OF THE ROCKLAND WAVE EQUATION ON GRADED GROUPS AND SUB-LAPLACIAN GEVREY SPACES

Joint work with: Michael Ruzhansky

In a recent work [2], C. Garetto and M. Ruzhansky investigate the Cauchy problem for the time-dependent wave equation for sums of squares of vector fields on compact Lie groups. In particular, they establish the well-posedness in spaces that compare to the Gevrey spaces. We generalise their result to graded groups and to more general operators, [3]. Furthermore, modelled on the spaces of Gevrey-type appearing in [2], we define the sub-Laplacian Gevrey spaces on manifolds and partially characterise these spaces. A full characterisation for the sub-Laplacian Gevrey spaces is achieved for certain groups whose symbolic calculus is well-known, such as the Heisenberg group. In this talk, I will first introduce some preliminary facts about the Fourier analysis and the quantisation on Lie groups, focusing on the case of the Heisenberg group. Subsequently, I will present the results mentioned above.

References

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Paul Dobson

University of Warwick

Degenerate Diffusion Processes with (possibly) multiple equilibria

In this talk we present a class of diffusion processes satisfying the so-called UFG condition, introduced by Kusuoka and Stroock in the eighties. The UFG condition is weaker than Hömander's condition (i.e. the class of UFG processes contains the class of uniformly hypoelliptic processes). These processes can exhibit more than one invariant measure. It is well known that for dynamics exhibiting multiple equilibria, the problem of determining the basin of attraction of each equilibrium is in general out of reach. However, the geometrical structure of such a condition allows one to completely describe the set of initial data converging to a given equilibrium. In this talk we discuss how to apply control theoretic techniques to determine where an invariant measure can be supported and for which initial datum we have convergence to a given invariant measure.

Nurgissa Yessirkegenov

Imperial College London

"Best constants in Sobolev and Gagliardo-Nirenberg inequalities on graded groups and ground states for higher order nonlinear subelliptic equations."

I will talk about the dependence of the best constants in Sobolev and Gagliardo-Nirenberg inequalities on the precise form of the Sobolev space norm. The analysis is carried out on general graded Lie groups, thus including the cases of Rn, Heisenberg, and more general stratified Lie groups. The best constants are expressed in the variational form as well as in terms of the ground state solutions of the corresponding nonlinear subelliptic equations. The orders of

these equations can be high depending on the Sobolev space order in the Sobolev or Gagliardo-Nirenberg inequalities, or may be fractional. Applications are obtained also to equations with lower order terms given by different hypoelliptic operators. This talk is based on joint works with Michael Ruzhansky and Niyaz Tokmagambetov.

Yong Sul Won

Imperial College London

Population dynamics in heterogeneous environments.

We study a nonlinear reaction-diffusion system undergoing a logistic type competition. The diffusivity of species depends on the ratio of its density and the availability of food source. This nonlinear feature provides us with not only a more realistic model of population dispersal but also interesting mathematical challenges, too. We will present some existence theorems with an aid of log-Sobolev inequality.

This is work in progress.
