stochastics, control and finance

WORKSHOP

In honour of

Professor Mark H.A. Davis

on the occasion of his 65th birthday

Imperial College London

12 – 14 April 2010

Organising Committee

Dirk Becherer (Humboldt University of Berlin)
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Sponsors

European Science Foundation
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programme

Monday, April 12

08:30-09:00 *Registration at Electrical Engineering Building*

Session 1. Chair: Mark Davis
09:00-09:10 Tomas Björk (welcome)
09:10-09:40 Richard Vinter
09:45-10:15 Andrew Heunis

10:20-10:50 Coffee Break

Session 2. Chair: Alex Mijatović
10:50-11.20 Martin Schweizer
11:25-11:55 Ralf Korn
12:00-12:30 Vicky Henderson

12:35-14:00 Break

Session 3. Chair: Giulia Di Nunno
14:00-14:30 Xunyu Zhou
14:35-15:05 Tomas Björk

15:10-15:40 Coffee Break

Session 4. Chair: Dirk Becherer
15:40-16:10 Paul Embrechts
16:15-16:45 Eduardo Epperlein
16:50-17:20 Alexander Lipton

Tuesday, April 13

Session 1. Chair: Arne Løkka
09:00-09:30 Ioannis Karatzas
09:35-10:05 Michel Vellekoop

10:10-10:40 Coffee Break

Session 2. Chair: Martijn Pistorius
10:40-11.10 Bernt Øksendal
11:15-11:45 Nizar Touzi
11:50-12:20 Saul Jacka

12:25-14:00 Break

Session 3. Chair: Dorje Brody
14:00-14:30 Michael Dempster
14:35-15:05 David Hobson

15:10-15:40 Coffee Break

Session 4. Chair: Sébastien Lleo
15:40-16:10 Eckhard Platen
16:15-16:45 Jim Gatheral
16:50-17:20 Vladimir Lucić

Wednesday, April 14

Session 1. Chair: Nick Bingham
09:00-09:30 Terry Lyons
09:35-10:05 Martin Clark

10:10-10:40 Coffee Break

Session 2. Chair: Michael Monoyios
10:40-11.10 Thaleia Zariphopoulou
11:15-11:45 Peter Bank
11:50-12:20 Lane Hughston

12:25-14:00 Break

Session 3. Chair: Albina Danilova
14:00-14:30 Walter Schachermayer
14:35-15:05 Jan Obłój

15:10-15:40 Coffee Break

Session 4. Chair: Mihail Zervos
15:40-16:10 Chris Rogers
16:15-16:45 Mark Davis
16:50-17:00 Thaleia Zariphopoulou (conclude)

abstracts

Market indifference prices

Peter Bank
Technical University of Berlin

We consider a financial market model where a single large investor submits orders to a finite number of market makers. These orders are filled at what we call market indifference prices and they lead to a new efficient allocation of risk among the market makers. We show how this allocation depends on the size of the order, discuss the cash compensation between market makers and the large investor, and explain how convex duality techniques allow for a quantitative analysis of the permanent market impact resulting from a transaction. (This is joint work with Dmitry Kramkov.)

Time inconsistent stochastic control

Tomas Björk
Stockholm School of Economics

We present a theory for stochastic control problems which, in various ways, are time inconsistent in the sense that they do not admit a Bellman optimality principle. We attach these problems by viewing them within a game theoretic framework, and we look for subgame perfect Nash equilibrium points.
For a general controlled Markov process and a fairly general objective functional we derive an extension of the standard Hamilton-Jacobi-Bellman equation, in the form of a system of non-linear equations, for the determination for the equilibrium strategy as well as the equilibrium value function. All known examples of time inconsistency in the literature are easily seen to be special cases of the present theory. We prove that for every time inconsistent problem, there exists an associated time consistent problem such that the optimal control and the optimal value function for the consistent problem coincides with the equilibrium control and value function respectively for the time inconsistent problem. We also study some concrete examples.

Approximation of filters for homogeneous diffusions by matched Beneš filters

Martin Clark
Imperial College London

TBA

Asset management via risk-sensitive stochastic control

Mark Davis
Imperial College London

This talk summarizes joint work with Sébastien Lleo on risk-sensitive portfolio optimization with jump-diffusion asset price processes whose growth rates depend on a factor process X_t. When X_t is a diffusion process the the problem reduces, using a change of measure idea introduced by Kuroda and Nagai, to a problem of controlled diffusion whose solution is characterized by the classical $C^{1,2}$ solution of the corresponding HJB equation. When X_t also has jumps the HJB equation has a non-local term and we proceed via viscosity solutions.

Ultimately, however, we are able to establish $C^{1,2}$ regularity of the value function in this case also.

References: arXiv:0905.4740, arXiv:1001.1379

Estimating exponential affine models with correlated measurement errors: applications to fixed income and commodities

Michael Dempster
University of Cambridge

Exponential affine models (EAMs) are factor models popular in financial asset pricing which requires a dynamic term structure, such as for interest rates and commodity futures. When implementing EAMS it is usual to first specify the model in state space form (SSF) and then to estimate its parameters using the Kalman filter. To specify the SSF a structure of the measurement error must be provided which is not specified in the EAM itself. Different specifications of the measurement errors will result in different SSFs, leading to different parameter estimates. Using market data for both fixed income and commodities we provide evidence that measurement errors are cross-sectionally and serially correlated, which is not consistent with the identically distributed (iid) assumptions commonly adopted in the literature. Using simulated data we show that measurement error assumptions affect parameter estimates, especially in the presence of serial correlation. We provide a new specification, the augmented state space form (ASSF), as a solution to removing these biases and show that the ASSF give much better estimates than the basic SSF.
(Joint work with Ke Tang, Renmin University of China)

Mathematics and the financial crisis

Paul Embrechts
ETH Zurich

Mathematicians (quants, financial engineers) have been branded as contributors to the financial crisis, some would even go so far as to say "main contributors". I will highlight the ongoing discussion from my personal point of view and experience. Issues touched upon will include "model uncertainty", "micro-correlations", "extremes", "warnings" and "guilt". The talk is based on the following paper: "The devil is in the tails: actuarial mathematics and the subprime mortgage crisis, 2010, ASTIN Bulletin, to apear)" (C. Donnelly and P.Embrechts)

iVAST: an 'integrated VaR And Stress Testing' approach to risk capital

Eduardo Epperlein
Citigroup

Financial models, and in particular Value-at-Risk models, have been blamed as a source of evil and the primary cause for the credit crisis. Stress Testing has been often prescribed as a panacea for all crisis problems. VaR and Stress Testing techniques have been around for a very long time, and both suffer from their own strengths and weaknesses. The iVAST methodology integrates both VaR and Stress Testing in a coherent framework to be used for estimating risk capital.

Optimal order execution

Jim Gatheral
Bank of America Merrill Lynch and Baruch College, CUNY

In this talk, we review the models of Algmren and Chriss, Obizhaeva and Wang, and Alfonsi, Fruth and Schied. We use variational calculus to derive optimal execution strategies in these models, and show that static strategies are dynamically optimal, in some cases by explicitly solving the HJB equation. Also, we will present some new generalizations of the Obizhaeva and Wang model given in a recent paper by Gatheral, Schied and Slynko, again deriving explicit closed-form optimal execution strategies. Time permitting, we will explore some implications for optimal strategies of time-varying and hidden liquidity.

Prospect theory, partial liquidation and the disposition effect

Vicky Henderson
University of Oxford

We solve an optimal stopping problem for an agent with prospect theory preferences who seeks to sell a portfolio of (divisible) claims on an underlying asset. Our methodology enables us to consider different formulations of prospect preferences in the literature, and diffusion price processes. We find that these differences in specification are important - for instance, with piecewise power functions (but not piecewise exponentials) the agent may voluntarily liquidate at a loss relative to break-even. This is consistent with the disposition effect documented in empirical and experimental studies. The ability to partially liquidate also has significant consequences. The prospect agent liquidates the entire position at once, in contrast to behavior under standard concave preferences.
Paper available at: http://www.oxford-man.ox.ac.uk/research/abstracts/2009OMI04.html

Quadratic minimization with portfolio and terminal wealth constraints

Andrew Heunis
University of Waterloo

We study a problem of stochastic optimal control in a standard complete market, with the goal of minimizing the expected value of a general quadratic loss function of the wealth at close of trade when there is a general convex constraint on the portfolio together with "portfolio insurance" in the form of a specified almost-sure lower bound on the wealth at close of trade. In the terminology of stochastic optimal control the problem involves both a "control constraint" (on the portfolio) as well as an almost-sure "state-constraint" (on the wealth). We use a conjugate-duality approach of Rockafellar and Moreau, the essence of which is to appropriately "perturb" the given problem, then calculate concave conjugates in terms of the perturbation to synthesize a Lagrangian function and a dual cost function, together with Kuhn-Tucker optimality relations which characterize the saddle-points of the Lagrangian. Existence of a Lagrange multiplier, which "enforces" both the convex constraint on the portfolio and the almost-sure constraint on the wealth, is established subject to a natural Slater-type condition on the wealth constraint; the Lagrange multiplier comprises an Ito process paired with a finitely-additive measure on the event sigma-algebra at the close of trade. The optimality relations are then used to synthesize an optimal portfolio in terms of the Lagrange multiplier.

Constructing diffusions consistent with optimal stopping values

David Hobson
University of Warwick

Consider an optimal stopping problem with a one-parameter objective function and suppose we are given the expected discounted values for the problem for a continuous range of parameter values. Under mild regularity conditions on the payoff function we show how to construct a time-homogeneous diffusion consistent with the given values. The forward problem of determining the expected values given a process is related to the inverse problem through a generalised duality relation with respect to the log-transformed payoff function.

Rational term-structure models with geometric Levy martingales

Lane Hughston
Imperial College London

In the "positive interest" models of Flesaker & Hughston (1996) the nominal discount bond system is represented by a one-parameter family of positive martingales. In the present paper we extend the analysis to include a variety of distributions for the martingale family, parameterised by a function $\phi(x)$ that determines the behaviour of the market risk premium. These distributions include jump and diffusion characteristics that generate various interesting properties for discount bond returns. For example, one can generate skewness and excess kurtosis in the discount bond returns by choosing the martingale family to be given by (a) exponential gamma processes, or (b) exponential variance-gamma processes. The models are "rational" in the sense that the discount bond price process is given by the ratio of a pair of sums of positive martingales. Our findings lead to semi-analytical and Fourier-inversion style solutions for the prices of European options on discount bonds, foreign exchange rates, and foreign discount bonds. The paper is motivated in part by the results of Filipović, Tappe & Teichmann (2009), who demonstrated that the term structure density approach of Brody & Hughston (2001) admits a natural extension to general positive term-structure models driven by a class of Lévy processes.
References: [1] B. Flesaker & L. P. Hughston (1996) Positive interest. *Risk*, **9**, 46-49. [2] D. Filipović, S. Tappe & J. Teichmann (2009) Term structure models driven by Wiener process and Poisson measures: existence and positivity (arXiv:0905.1413). [3] D. C. Brody & L. P. Hughston (2001) Interest rates and information geometry. *Proc. Roy. Soc. Lond.* **A457**, 1343-1364. Co-authors: D. C. Brody (Department of Mathematics, Imperial College London), and E. Mackie (Imperial College Business School, and Department of Mathematics, Imperial College London)

Trading with transaction costs: the fundamental theorem

Saul Jacka
University of Warwick

We will consider the general model for trading with proportional trading costs in discrete time. We shall show how to suitably extend the fundamental theorem of asset pricing to this setting. This includes giving a suitable generalisation of the concept of an Equivalent Martingale Measure.

Martingale approach to stochastic differential games of control and stopping

Ioannis Karatzas
INTECH Investment Management LLC and Columbia University

In 1973 Mark Davis published in the *SIAM Journal on Control & Optimization* two papers: "Dynamic Programming Conditions for Partially Observable Stochastic Systems" (with Pravin Varaiya), and "On the Existence of Optimal Strategies in Stochastic Control". These works laid the foundations for what became known as the martingale approach to stochastic control. We revisit these seminal papers in the context of stochastic differential games of control and stopping, and of optimal stopping for a decision maker who uses convex risk measures to evaluate future rewards. (Joint work with Mona Zamfirescu, Erhan Bayraktar and Song Yao.)

Portfolio optimization and transaction costs in action

Ralf Korn
University of Kaiserslautern

The paper by Davis and Norman (1990) has set the scene for the mathematical research on portfolio selection in the presence of transaction costs. In the talk different approaches to the treatment of transaction costs will be presented together with their conceptual advantages and the problems that occur when applying them to real-world settings. As a suggestion, we show how to modify a somewhat artificial approach to obtain good portfolio strategies under transaction costs.

Credit value adjustment in theory and practice

Alexander Lipton
Bank of America Merrill Lynch

In this talk we demonstrate how to evaluate credit value adjustment for various derivatives. We discuss both theoretical and practical aspects of the problem. As a particularly important example, we study credit default swaps and show how to modify their prices in the presence of counterparty risk.

On boundary conditions for computing densities in CIR-based models via PDE methods

Vladimir Lucic
Barclays Capital

The CIR process has a long history of applications in finance, spanning several decades and a variety of asset classes. Despite its apparent simplicity the CIR diffusion exhibits rich behaviour, and consequently related numerical schemes are far from straightforward. In the Monte-Carlo setting this problem has generated a considerable amount of research over the years. With regard to Kolmogorov (backward) PDEs, it is only recently that heuristic boundary conditions for the Heston model which have been used since early '90s have been put on firm theoretical grounds.
In this work we investigate boundary conditions for the Fokker-Planck (forward) PDEs arising from CIR-type diffusions. We start from a result due to Feller addressing the one-dimensional

case, and proceed towards more complex processes. The main practical applications of this work pertain to forward-inductive calibration schemes where one of the state variables follows the CIR process. One such application relates to the local stochastic volatility model, which will be examined in more detail.

The signature of a path

Terry Lyons
University of Oxford

Paths represent, in mathematical language, sequentially unfolding of information or changing state. We are often interested in data streams for their potential effect on other systems. For this purpose it can be instructive and useful to have effective approximate descriptions of the stream. Surprisingly there is a transform, which has a universal character, that captures all the relevant information in a structured way. In effect, tree reduced paths form a group, the enveloping algebra of this group is the Tensor Sequence space which is graded. The canonical representation of the reduced paths into this algebra is faithful, the tensor is referred to as the signature of the path. Truncation of the signature to tensors of degree at most n gives an effective (nilpotent) family of approximate descriptions. The abstraction leads, surprisingly, to effective computational tools. It also creates new challenges. The ideas presented here have a long pedigree. Algebraic calculations of asymptotic expansions go back to magnus and chen and are well known in control theory. Young made important contributions to integration theory for p variation paths. Lyons (et al.) put the algebra and analysis together to get a theory of differential equations driven by Rough Paths. Hambly and Lyons, building on Chen's work, established the signatre as a complete description for paths of finite length. Every rough path has a signature, but it remains open whether it provides a complete description up to TreeLike equivalence.

Utility theory front to back – inferring utility from agents' choices

Jan Obłój
University of Oxford

We propose a new approach to utility theory, and consumption/investment problems. Instead of specifying agent's utility function and deriving her actions, we assume we observe her actions (i.e. her consumption and investment strategies) and derive utility function for which the observed behaviour is optimal. This is done in a one-period model and in continuous time both in a deterministic and stochastic setting. In the setup of Black-Scholes market it turns out that the consumption and investment strategies have to satisfy a consistency condition (PDE) if they come from a classical utility maximisation problem. We further show that agent's important characteristics such as attitude towards risk (e.g. DARA) can be directly deduced from her consumption/investment choices.
Joint work with A. Cox and D. Hobson.

Backward stochastic differential equations with respect to general filtrations and applications to insider finance

Bernt Øksendal
University of Oslo

In this paper, we study backward stochastic differential equations with respect to general filtrations. The results are used to find the optimal consumption rate for an insider from a

cash flow modeled as a generalized geometric Itô-Lévy process. (Joint work with Tusheng Zhang of University of Manchester)

Empirical properties of a well diversified global stock index

Eckhard Platen
University of Technology Sydney

Most of the papers that study the distributional and fractal properties of financial instruments focus on stock prices or exchange rates. This leads typically to mixed results concerning the distributions of log-returns and some multi-fractal properties of exchange rates, stock prices, and regional indices. It will be suggested to use a very well diversified world stock index in various denominations as the main object of empirical analysis. Such index has been formed using daily and intraday data. It aggregates, in principle, the non-diversifiable risk of the stock market. Compared to other global stock market indices it has extremely low volatility and, thus, a high signal to noise ratio when denominated in a currency. Furthermore, by diversification such an index can be shown to approximate the growth optimal portfolio or numeraire portfolio, which is the central object of the, so called benchmark approach. The paper will demonstrate that the above mentioned diversified index is an ideal object for studying the statistical properties of given securities. For instance, when denominating the savings account of a currency in units of this diversified global world index, one observes the movements of the currency against the entire market. This provides a practically undisturbed observation of the currency dynamics against the whole of the market. In this manner, one can conveniently disentangle, e.g., the superposition of the characteristic properties of the two currencies generating a given exchange rate. The exchange rate is then obtained as the ratio of the two currency denominations of the benchmark. The proposed benchmark approach to the empirical analysis of financial data allows one to establish remarkable stylized facts. For instance, the log-returns of a well diversified global stock index, when denominated in a currency, are with high significance Student t distributed with about four degrees of freedom. The repeatedly documented multi-fractal appearance of financial time series turns out to be only very weak when analysed for a well diversified global index. The Hurst exponent of the observed mono-fractal behavior assumes typical values between 0.55 and 0.65. Accordingly, the quadratic variation vanishes asymptotically when reducing the observation time step size. These results can be contrasted with the mixed findings on empirical properties of FX rates or stock prices. A range of further empirical facts can be expected to be identifiable when using a well diversified index in the denomination of a given security as the object of study.

References: 1) Platen, E. & D. Heath (2006). A Benchmark Approach to Quantitative Finance. Springer. 2) Platen, E. & R. Rendek (2008). Empirical Evidence on Student-t Log-returns of Diversified World Stock Indices. J. Statist. Theory and Practice, Vol 2 (2), 233-251. 3) Breymann, W., D.Luethi & E. Platen (2009). Empirical Behavior of a World Stock Index from Intra-day to Monthly Time Scales. European Physical Journal B, 71, 511-522.

Diverse beliefs

Chris Rogers
University of Cambridge

This paper presents a general framework for studying diverse beliefs in dynamic economies. Within this general framework, the characterization of a central-planner general equilbrium

turns out to be very easy to derive, and leads to a range of interesting applications. We show how for an economy with log investors holding diverse beliefs, rational overconfidence is to be expected; volume-of-trade effects are effectively modelled; the Keynesian beauty contest can be modelled and analysed; and bubbles and crashes arise naturally. We remark that models where agents receive private information can formally be considered as models of diverse beliefs.

The fundamental theorem of asset pricing for continuous processes under small transaction costs

Walter Schachermayer
University of Vienna

A version of the fundamental theorem of asset pricing is proved for continuous asset prices with small proportional transaction costs. Equivalence is established between: (a) the absence of arbitrage with general strategies for arbitrarily small transaction costs $\varepsilon > 0$, (b) the absence of free lunches with bounded risk for arbitrarily small transaction costs $\varepsilon > 0$, and (c) the existence of ε-consistent price systems the analogue of martingale measures under transaction costs for arbitrarily small $\varepsilon > 0$. The proof proceeds through an explicit construction, as opposed to the usual separation arguments. The paper concludes comparing numéraire-free and numéraire-based notions of admissibility, and the corresponding martingale and local martingale properties for consistent price systems.

Portfolio choice and horizon-dependence

Martin Schweizer
ETH Zürich

We consider the standard problem of maximising utility from terminal wealth up to a finite time horizon T. We then study how the solution, i.e. the optimal final wealth at T and the optimal strategy on $[0, T]$, behave when viewed as processes indexed by T. This leads to mathematical problems and results of independent interest.
The results are based on joint work with Tahir Choulli (University of Alberta, Edmonton, Canada).

Model independent bound for option pricing: a stochastic control aproach

Nizar Touzi
Ecole Polytechnique

We develop a stochastic control approach for the derivation of model independent bounds for derivatives under various calibration constraints. Unlike the previous literature, our formulation seeks the optimal no arbitrage bounds given the knowledge of the distribution at some (or various) point in time. This problem is converted into a classical stochastic control problem by means of convex duality. We obtain a general characterization, and provide explicit optimal bounds in some examples.
(joint with Alfred Galichon and Pierre Henry-Labordere)

The early exercise boundary for American options on dividend-paying assets

Michel Vellekoop
University of Amsterdam

Standard equity option pricing models usually pay little attention to the inclusion of realistic dividend structures in the model for the underlying asset prices. In this talk we show how cash dividends can be included in option pricing schemes in a consistent way, and we study the poperties of American options when dividends are included. We derive a generalized version of a well-known integral equation for the early exercise boundary which allows the inclusion of dividend payments, and use this to illustrate the differences with the case where no dividends are present. We then prove regularity properties of the optimal exercise boundary when dividends are present.

Control strategies for confining trajectories of a stochastic system to a safe region of the state space

Richard Vinter
Imperial College London

We consider the problem of controlling a noisy diffusion process to prevent it leaving a safe region of the state space. Such problems arise in the control of dams and surge tanks to avoid overflow, control of telecommunications systems to avoid the failing of a commications link, control of queues to avoid excessive waiting times, and other areas. Maximizing the expected value of the exit time is unsatisfactory for many of these applications, because such an approach may give control strategies for which the probability of an early exit is unacceptably high. These concerns have given rise to a substantial literature, risk sensitive stochastic optimal control, in which the cost is modified to reduce the probability of an early exit, according to the level of risk 'averseness' implicit in the problem formulation. It is know well known that, for large families of risk sensitive optimal control problems parameterized by the noise intensity ϵ, the limiting control strategy as $\epsilon \to 0$, is the solution to a one-sided differential game. For some problems for which the state space dimension is less than two, it has been possible to solve this games problem by analytical means. But solution techniques for higher problems involving higher dimensional state spaces are largely lacking. We identify a broad class of exit problems, including some with high state dimension, for which the related games problem can be be computed. The high state dimension excludes the use of methods based on numerical solution of the Hamilton-Jacobi-Isaacs equation. Instead we propose a solution technique involving the on-line solution of a family optimal control problems. The resulting control strategy is, admittedly, computationally demanding, but this is little consequence in applications such as surge tank control, where process time constants are large, sampling rates are high, and the time available for online calculations is therefore substantial. (joint work with Martin Clark)

TBA

Thaleia Zariphopoulou
Oxford-Man Institute of Quantitative Finance

TBA

Hope, fear and aspiration

Xunyu Zhou
University of Oxford

We propose a new portfolio choice model in continuous time featuring three key human emotions in making choices: hope, fear and aspiration. By applying the recently developed quantile formulation, we solve this model completely. Three quantitative indices: those of fear, hope, and aspiration are proposed to study the impacts of the emotions respectively on the investment behaviors. This is a joint work with Xuedong He (Columbia).