

Basket Options Valuation for a Local Volatility Jump-Diffusion Model with the Asymptotic Expansion Method

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Abstract In this paper we discuss the basket options valuation for a jump-diffusion model. The underlying asset prices follow some correlated local volatility diffusion processes with systematic jumps. We derive a forward partial integral differential equation (PIDE) for general stochastic processes and use the asymptotic expansion method to approximate the conditional expectation of the stochastic variance associated with the basket value process. The numerical tests show that the suggested method is fast and accurate in comparison with the Monte Carlo and other methods in most cases.

Keywords Basket options pricing, local volatility jump-diffusion model, forward PIDE, asymptotic expansion.

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1 Introduction

A basket option is an exotic option whose payoff depends on the value of a portfolio of assets. Basket options are in general difficult to price and hedge due to the lack of analytic characterization of the distribution of the sum of correlated random variables. Monte Carlo simulation is often used to price basket options, which is simple, accurate, but time-consuming. There has been some extensive research recently for fast and accurate pricing methods.

Most work in the literature assumes that underlying asset prices follow geometric Brownian motions. The basket value is then the sum of correlated lognormal variables. The main idea of the analytic approximation method is to find a simple random variable to approximate the basket value and then to use it to get a closed form pricing formula. The approximate random variable is required to match some moments of the basket value. Levy (1992) uses a lognormal variable to approximate the basket value with the matched first and second moments. The results are remarkably good but there is no error estimation. Curran (1994) introduces the idea of conditioning variable and conditional moment matching. The option price is decomposed into two parts: one can be calculated exactly and the other approximately by conditional moment matching method. Rogers and Shi (1995) derive the lower and upper bounds. Vanmaele et al. (2004) suggest a moment matching comonotonic approximation for basket options. Lord (2006) gives a survey on pricing methods for basket options. Efforts have been made to extend to more general asset price models. Albrecher and Predota (2004) discuss the normal inverse gamma Lévy process, and Flamouris and Giamouridis (2007) the Bernoulli jump diffusion model.

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Xu and Zheng (2009) suggest a jump-diffusion model for underlying asset price processes. The innovative feature of that model is that, apart from correlated Brownian motions, there are two types of Poisson jumps: a systematic jump that affects all asset prices and idiosyncratic jumps that only affect specific asset prices. Such a model can characterize well the market-wide phenomenon and individual events. Xu and Zheng (2009) use the partial exact approximation (PEA) method to find a closed form approximate solution which is guaranteed to lie between the lower and upper bounds. The numerical tests show that the PEA method has superior performance in comparison with other methods such as the lower bound, reciprocal gamma, and lognormal approximations. The limitation of the PEA method is that it depends crucially on the conditioning variable which is derived from the estimation of the basket value and the closed form solutions of individual asset prices. This may not be possible for general processes. For example, if individual asset prices follow some local volatility models, see Dupire (1994), then there are in general no closed form solutions and the PEA method cannot be applied.

To price basket options for general asset price processes one may study directly the basket value and its associated stochastic processes which may contain stochastic volatilities and/or stochastic jump intensities and sizes. Dupire (1994) shows that any diffusion model with stochastic volatility can be replaced by a local volatility model without changing the European option price and the marginal distribution of the underlying asset price thanks to the uniqueness of the solution to the corresponding pricing equation, a parabolic PDE. In fact, Gyöngy (1986) discovers the equivalence of a non-Markovian model with a Markovian model and proves that marginal distributions of any Itô processes can be matched by those of Markovian local volatility processes, that is, the value of the square of the local volatility is equal to the expectation of the square of the stochastic volatility conditional on the final stock price being equal to the strike price. Without the uniqueness of the resulting SDE one cannot claim the equivalence of stochastic volatility models and local volatility models but one can still derive the same pricing equation.

The pricing equation for general asset price processes may contain coefficients expressed in terms of some conditional expectations. It is in general difficult to compute these conditional expectations as there is no closed form solution to the related SDE. One may try to find some good approximations. Antonov et al. (2009) use the Markovian projection onto a displaced diffusion and Avellaneda et al. (2002) apply the steepest decent search with Varadhan's formula, both methods require to solve some minimization problems. Xu and Zheng (2009) derive a closed form approximation to the conditional expectation with a weighted sum of the lower bound and the conditional second moment adjustments.

In this paper we discuss the European basket options pricing for a local volatility jump-diffusion model. The main idea is to reduce a multi-dimensional local volatility jump-diffusion model problem to a one-dimensional stochastic volatility jump-diffusion model, then to derive a forward PIDE for the basket options price with an unknown conditional expectation, or local volatility function, and finally to apply the asymptotic expansion method to approximate the local volatility function. The main contributions of the paper to the existing literature of the basket options pricing are the following: we propose a correlated local volatility jump-diffusion model for underlying asset price processes and derive a forward PIDE for general asset price processes with stochastic volatilities and stochastic jump compensators, which may be used for other applications in pricing and calibration, and we find the approximation of the conditional expectation with the asymptotic expansion method. Numerical tests show that the method discussed in the paper, the asymptotic expansion method, performs very well for most cases in comparison with the Monte Carlo method and the PEA method discussed in Xu and Zheng (2009).

The paper is organized as follows. Section 2 formulates the basket options pricing problem, reviews some pricing results on jump-diffusion asset price models and forward PIDEs. Section 3 discusses the computation of the conditional expectation and applies the asymptotic expansion method to approximate the local volatility function. Section 4 elaborates the numerical implementation and

compares the numerical performance of different methods in pricing basket options. Section 5 is the conclusion. The appendix contains the outline of the derivation of a forward PIDE for a general stochastic process.

2 Forward PIDE for Basket options pricing

Assume a portfolio is composed of n assets and the risk-neutral asset prices S_i satisfy the following SDEs:

$$\frac{dS_i(t)}{S_i(t-)} = r(t)dt + \sigma_i(t, S_i(t-))dW_i(t) + \int_{\mathbb{R}} (e^x - 1)[\mu(dx, dt) - \nu(dx, dt)] \quad (1)$$

where W_i are standard Brownian motions with correlation matrix (ρ_{ij}) , μ is a random measure, ν is its compensator, σ_i are bounded local volatility functions, and r is a deterministic risk-free interest rate function. The basket value $S(t)$ at time t is defined by

$$S(t) = \sum_{i=1}^n w_i S_i(t),$$

where w_i are positive constant weights and S_i satisfy SDE (1). Define

$$W(t) = \int_0^t \frac{1}{V(u)} \sum_{i=1}^n w_i \sigma_i(u, S_i(u)) \frac{S_i(u)}{S(u)} dW_i(u)$$

where

$$V(t)^2 := \frac{1}{S(t)^2} \sum_{i,j=1}^n w_i w_j \sigma_i(t, S_i(t)) \sigma_j(t, S_j(t)) S_i(t) S_j(t) \rho_{ij}.$$

Then W is a standard Brownian motion and the basket value S follows the SDE

$$\frac{dS(t)}{S(t-)} = r(t)dt + V(t)dW(t) + \int_{\mathbb{R}} (e^x - 1)[\mu(dx, dt) - \nu(dx, dt)] \quad (2)$$

with the initial price $S(0) = \sum_{i=1}^n w_i S_i(0)$. Note that $V(t)$ is a stochastic volatility which depends on individual asset prices, not just the basket price, and (2) is a stochastic volatility jump-diffusion asset price model for the basket option problem. We next review some related pricing results for the jump-diffusion asset price process (2).

Andersen and Andreasen (2000) assume that $V(t) = \sigma(t, S(t))$, i.e., the local volatility model, and the compensator has a time dependent form $\nu(dx, dt) = \lambda(t)\zeta(x, t)dxdt$ where $\lambda(t)$ is a nonnegative deterministic intensity function and $\zeta(x, t)$ a time dependent density function of jump sizes. Define $m(t) = \int_{\mathbb{R}} (e^x - 1)\zeta(x, t)dx$. Then the European call option price $C(T, K)$ at time 0 as a function of maturity $T > 0$ and exercise price $K \geq 0$ satisfies a forward PIDE:

$$\begin{aligned} C_T(T, K) &= (-r(T) + \lambda(T)m(T))KC_K(T, K) + \frac{1}{2}\sigma(T, K)^2 K^2 C_{KK}(T, K) \\ &\quad + \lambda(T) \left(\int_{-\infty}^{\infty} C(T, Ke^{-y})e^y \zeta(y; T)dy - (1 + m(T))C(T, K) \right) \end{aligned} \quad (3)$$

with the initial condition $C(0, K) = (S(0) - K)^+$.

Andersen and Andreasen (2000) also discuss the stochastic volatility jump-diffusion model (2) with the same compensator $\nu(dx, dt) = \lambda(t)\zeta(x, t)dxdt$ and point out that the European call option price satisfies the same PIDE (3) with the local volatility function σ replaced by

$$\sigma(T, K)^2 = \mathbf{E}[V(T)^2 | S(T) = K]. \quad (4)$$

Carr et al. (2004) generalize the asset price model of Andersen and Andreasen (2000) to a general local volatility and local Lévy type model with the compensator having a form $\nu(dx, dt) = a(S(t), t)k(x)dxdt$, where $a(S, t)$ is a deterministic local speed function that measures the speed at which the Lévy process is running at time t and stock price S , and $k(x)dx$ specifies the arrival rate of jumps of size x per unit of time. Carr et al. (2004) derive a forward PIDE for the European call option price $C(T, K)$ at time 0 as

$$\begin{aligned} C_T(T, K) = & -r(T)KC_K(T, K) + \frac{1}{2}\sigma(T, K)^2K^2C_{KK}(T, K) \\ & + \int_0^\infty a(T, z)C_{zz}(T, z)z\psi_e\left(\ln \frac{K}{z}\right)dz \end{aligned} \quad (5)$$

with the initial condition $C(0, K) = (S(0) - K)^+$, where ψ_e is the double-exponential tail of the Lévy measure given by

$$\psi_e(y) = \begin{cases} \int_y^\infty (e^x - e^y)k(x)dx & \text{for } y > 0 \\ \int_{-\infty}^y (e^y - e^x)k(x)dx & \text{for } y < 0. \end{cases}$$

Kindermann et al. (2008) show the existence and uniqueness of the solution to the PIDE (5) under some continuity and uniform positive definiteness conditions. Carr and Wu (2009) generalize the local volatility asset price process further to a stochastic volatility asset price process with a stochastic jump compensator $\nu(dx, dt) = \bar{a}(t)k(x)dxdt$ and $\bar{a}(t)$ being the stochastic instantaneous variance. Carr and Wu (2009) use the model and the fast Fourier transform to value stock options and credit default swaps in a joint framework. We can show that the European call option price $C(T, K)$ satisfies the PIDE (5) for a general stochastic process \bar{a} with the local volatility function σ given by (4) and the local speed function a given by

$$a(T, z) = \mathbf{E}[\bar{a}(T)|S(T) = z] \quad (6)$$

where $\mathbf{E}[\bar{a}(T)|S(T)]$ is the conditional expectation of $\bar{a}(T)$ given $S(T)$. The derivation of the PIDE (5) with the local volatility function (4) and the local speed function (6) is given in the appendix.

3 Approximation of Local Volatility Function

In this section we focus on the approximation of the portfolio value for a special random measure μ with a compensator in the form of $\nu(dx, dt) = \lambda\phi_{\eta, \gamma^2}(x)dxdt$ where $\lambda > 0$ is a constant and ϕ_{η, γ^2} is the density function of a normal variable with mean η and variance γ^2 . Random measure μ is then a Poisson measure and $\int_{\mathbb{R}}(e^x - 1)\mu(dx, dt)$ is a differential form of a compound Poisson process $Z(t) := \sum_{l=1}^{N(t)}(e^{Y_l} - 1)$ with N being a Poisson process with intensity λ and $\{Y_l\}$ iid normal variables with mean η and variance γ^2 , and $e^{Y_l} - 1$ is the proportional change of the asset prices at the l th jump of the Poisson process N . Without loss of generality we also assume that $r(t) = 0$ for all t (otherwise we can work on discounted asset price processes). Denote $m = \mathbf{E}[e^{Y_l} - 1] = e^{\eta + \frac{1}{2}\gamma^2} - 1$. We can write SDE (1) in an equivalent form

$$\frac{dS_i(t)}{S_i(t-)} = -\lambda m dt + \sigma_i(t, S_i(t-))dW_i(t) + dZ(t) \quad (7)$$

with W , N , and $\{Y_l\}$ being independent of each other. If $\sigma_i(t, S)$ equals a constant σ_i for all (t, S) then the asset price process (7) is the well-known Merton model with discontinuous asset returns (Merton (1976)). The basket price S satisfies SDE (2) which can be equivalently written as

$$\frac{dS(t)}{S(t-)} = -\lambda m dt + V(t)dW(t) + dZ(t) \quad (8)$$

with the initial condition $S(0) = \sum_{i=1}^n w_i S_i(0)$. (8) is a special case of Andersen and Andreasen's model with $\lambda(t) = \lambda$ for all t and $\zeta(x, t) = \phi_{\eta, \gamma^2}(x)$ for all (x, t) . The European basket call option price $C(T, K)$ at time 0 satisfies the PIDE (3), or equivalently,

$$\begin{aligned} C_T(T, K) &= \lambda m K C_K(T, K) + \frac{1}{2} \sigma(T, K)^2 K^2 C_{KK}(T, K) \\ &\quad + \lambda \int_{-\infty}^{\infty} C(T, K e^{-y}) e^y \phi_{\eta, \gamma^2}(y) dy - \lambda(1 + m) C(T, K) \end{aligned} \quad (9)$$

with the initial condition $C(0, K) = (S(0) - K)^+$ and the local volatility function

$$\begin{aligned} \sigma(T, K)^2 &= \mathbf{E}[V(T)^2 | S(T) = K] \\ &= \frac{1}{K^2} \sum_{i,j=1}^n w_i w_j \rho_{ij} \mathbf{E}[\hat{\sigma}_i(T, S_i(T)) \hat{\sigma}_j(T, S_j(T)) | S(T) = K] \end{aligned} \quad (10)$$

and $\hat{\sigma}_i(T, S_i(T)) = \sigma_i(T, S_i(T)) S_i(T)$. The main difficulty is how to compute the conditional expectation (10) as asset prices S_i in general have no closed-form expressions. If $\sigma_i(t, S) = \sigma_i$, a constant, for all (t, S) , then there is a closed form solution to SDE (7) and there are some efficient approximation techniques for the basket value process S , see Xu and Zheng (2009) and references therein for details. Piterbarg (2007) uses the Taylor formula to approximate $\hat{\sigma}_i(T, S_i(T))$ to the first order with respect to $S_i(T)$ at point $F_i \equiv S_i(0)e^{rT}$ to get

$$\hat{\sigma}_i(T, S_i(T)) \approx p_i + q_i(S_i(T) - F_i)$$

where $p_i = \hat{\sigma}_i(T, F_i)$ and $q_i = \frac{\partial}{\partial F_i} \hat{\sigma}_i(T, F_i)$. We use the same first order approximation, also note that $r = 0$ here, to get

$$\hat{\sigma}_i(T, S_i(T)) \hat{\sigma}_j(T, S_j(T)) \approx p_i p_j + p_j q_i(S_i(T) - S_i(0)) + p_i q_j(S_j(T) - S_j(0)).$$

If we define $\hat{\sigma}(T, K)^2 = \sigma(T, K)^2 K^2$, then

$$\hat{\sigma}(T, K)^2 \approx \sum_{i,j=1}^n w_i w_j \rho_{ij} p_i p_j (1 + \varphi_i(T, K) + \varphi_j(T, K)) \quad (11)$$

where $\varphi_i(T, K) = \frac{q_i}{p_i} \mathbf{E}[S_i(T) - S_i(0) | S(T) = K]$.

To obtain an analytical approximation to $\mathbf{E}[S_i(T) - S_i(0) | S(T) = K]$, we use the asymptotic expansion approach related to small diffusion and small jump intensity and size, see Benhamou et al. (2009). The perturbation and its purpose are different in this paper. In Benhamou et al. (2009) the authors expand a parameterized process to the second order and apply it directly to price European options. In this paper we use a different parameterized process and expand it to the first order to get the analytic tractability and use it to approximate the conditional expectation of stochastic variance. In other words, we use the asymptotic expansion to find the *unknown* local volatility function and then use it in the forward PIDE, while Benhamou et al. (2009) use a different asymptotic expansion to a process with a *known* local volatility function and then find the options value directly. Assume $\epsilon \in [0, 1]$ and define

$$dS_i^\epsilon(t) = -\lambda m^\epsilon S_i^\epsilon(t-) dt + \epsilon \hat{\sigma}_i(t, S_i^\epsilon(t)) dW_i(t) + S_i^\epsilon(t-) dZ^\epsilon(t)$$

with the initial condition $S_i^\epsilon(0) = S_i(0)$, where $m^\epsilon = \mathbf{E}[e^{\epsilon Y_1} - 1] = e^{\epsilon \eta + \frac{1}{2} \epsilon^2 \gamma^2} - 1$ and $Z^\epsilon(t) = \sum_{l=1}^{N(t)} (e^{\epsilon Y_l} - 1)$. Note that $S_i^1(T) = S_i(T)$. If we define $S_{i,k}(t) = \left. \frac{\partial^k S_i^\epsilon(t)}{\partial \epsilon^k} \right|_{\epsilon=0}$, then the first order asymptotic expansion around $\epsilon = 0$ for $S_i^\epsilon(T)$ is

$$S_i^\epsilon(T) \approx S_{i,0}(T) + S_{i,1}(T) \epsilon. \quad (12)$$

Kunitomo and Takahashi (2001) use a similar asymptotic approximation (12) in pricing interest rate derivatives. We can find $S_{i,0}(T)$ and $S_{i,1}(T)$ as follows: $S_{i,0}$ satisfies the equation $dS_{i,0}(t) = 0$ with the initial condition $S_{i,0}(0) = S_i(0)$, therefore, $S_{i,0}(t) \equiv S_i(0)$ for all t . $S_{i,1}$ satisfies the equation

$$dS_{i,1}(t) = -\lambda\eta S_i(0)dt + \hat{\sigma}_i(t, S_i(0))dW_i(t) + S_i(0)dZ_1(t)$$

with the initial condition $S_{i,1}(0) = 0$, where $Z_1(t) = \frac{\partial}{\partial \epsilon} Z^\epsilon(t)|_{\epsilon=0} = \sum_{l=1}^{N(t)} Y_l$. Here we have used the result $S_{i,0}(t) = S_i(0)$. Therefore,

$$S_{i,1}(T) = -\lambda\eta S_i(0)T + \int_0^T \hat{\sigma}_i(t, S_i(0))dW_i(t) + S_i(0) \sum_{l=1}^{N(T)} Y_l.$$

The asset value $S_i(T)$ at time T may be approximated by

$$S_i(T) = S_i^1(T) \approx S_{i,0}(T) + S_{i,1}(T) = S_i(0) + S_{i,1}(T)$$

and the basket value by

$$S(T) \approx S(0) + \sum_{i=1}^n w_i S_{i,1}(T) := S_c(T). \quad (13)$$

Note that we have chosen $\epsilon = 1$ in (12) to get the approximation above, the similar approach is also used in Ju (2002) for Asian and basket options and Kawai (2003) for swaptions. Conditional on $N(T) = k$, the variable $S_{i,1}(T)$, written as $S_{i,1}(T, k)$, is a normal variable with mean $(-\lambda T + k)\eta S_i(0)$ and variance $\int_0^T \hat{\sigma}_i^2(t, S_i(0))dt + k\gamma^2 S_i(0)^2$, and the variable $S_c(T)$, written as $S_c(T, k)$, is also a normal variable with mean

$$\mu_c(k) = (1 - \lambda T\eta + k\eta)S(0) \quad (14)$$

and variance

$$\sigma_c(k)^2 = \sum_{i,j=1}^n w_i w_j \left[\left(\int_0^T \hat{\sigma}_i(t, S_i(0)) \hat{\sigma}_j(t, S_j(0)) dt \right) \rho_{ij} + k\gamma^2 S_i(0) S_j(0) \right]. \quad (15)$$

Therefore,

$$\begin{aligned} \mathbf{E}[S_i(T) - S_i(0) | S(T) = K] &\approx \mathbf{E}[S_{i,1}(T) | S_c(T) = K] \\ &= \sum_{k=0}^{\infty} P(N(T) = k) \mathbf{E}[S_{i,1}(T, k) | S_c(T, k) = K]. \end{aligned}$$

Since $S_{i,1}(T, k)$ and $S_c(T, k)$ are normal variables, we can find $\mathbf{E}[S_{i,1}(T, k) | S_c(T, k) = K]$ exactly as

$$\mathbf{E}[S_{i,1}(T, k) | S_c(T, k) = K] = \mathbf{E}[S_{i,1}(T, k)] + \frac{C_i(k)}{\sigma_c(k)^2} (K - \mu_c(k))$$

where $C_i(k)$ is the covariance of $S_{i,1}(T, k)$ and $S_c(T, k)$, given by

$$C_i(k) = \sum_{j=1}^n w_j \left[\rho_{ij} \left(\int_0^T \hat{\sigma}_i(t, S_i(0)) \hat{\sigma}_j(t, S_j(0)) dt \right) + k\gamma^2 S_i(0) S_j(0) \right].$$

From $\mathbf{E}[S_{i,1}(T, k)] = (-\lambda T + k)\eta S_i(0)$ and $\mathbf{E}[N(T)] = \lambda T$ we can see that

$$\sum_{k=0}^{\infty} P(N(T) = k) \mathbf{E}[S_{i,1}(T, k)] = \mathbf{E}[(-\lambda T + N(T))\eta S_i(0)] = 0.$$

Therefore

$$\mathbf{E}[S_i(T) - S_i(0)|S(T) = K] \approx \sum_{k=0}^{\infty} P(N(T) = k) \frac{C_i(k)}{\sigma_c(k)^2} (K - \mu_c(k))$$

and $\varphi_i(T, K)$ in (11) can be written as

$$\varphi_i(T, K) = \frac{q_i}{p_i} \sum_{k=0}^{\infty} P(N(T) = k) \frac{C_i(k)}{\sigma_c(k)^2} (K - (1 - \lambda T \eta + k \eta) S(0))$$

and $\hat{\sigma}(T, K)^2$ in (11) as

$$\hat{\sigma}(T, K)^2 = a(T) + b(T)K - c(T)S(0)$$

where

$$\begin{aligned} a(T) &= \sum_{i,j=1}^n w_i w_j \rho_{ij} p_i p_j \\ b(T) &= \sum_{i,j=1}^n \sum_{k=0}^{\infty} \frac{P(N(T) = k)}{\sigma_c(k)^2} w_i w_j \rho_{ij} p_i p_j \left(\frac{q_i}{p_i} C_i(k) + \frac{q_j}{p_j} C_j(k) \right) \\ c(T) &= \sum_{i,j=1}^n \sum_{k=0}^{\infty} \frac{P(N(T) = k)}{\sigma_c(k)^2} w_i w_j \rho_{ij} p_i p_j \left(\frac{q_i}{p_i} C_i(k) + \frac{q_j}{p_j} C_j(k) \right) (1 - \lambda T \eta + k \eta). \end{aligned}$$

The local volatility function $\sigma(T, K)$ in (10) can therefore be approximated by

$$\sigma(T, K) = \frac{\sqrt{a(T) + b(T)K - c(T)S(0)}}{K}. \quad (16)$$

4 Numerical Results

In this section we do some numerical tests for the European basket call options pricing with the underlying asset price processes (1). We use three different methods to facilitate the comparison: the full Monte Carlo (MC), the asymptotic expansion (AE), and the control variate (CV) method.

The MC method provides the benchmark results. We use the control variate technique to reduce the variance. In (13) the basket value $S(T)$ is approximated by the first order asymptotic expansion $S_c(T)$ which is used here as a control variate in MC simulation. The basket option price with the control variate $S_c(T)$ is given by

$$\mathbf{E}[(S_c(T) - K)^+] = \sum_{k=0}^{\infty} P(N(T) = k) \mathbf{E}[(S_c(T, k) - K)^+]. \quad (17)$$

Since $S_c(T, k)$ is a normal variable with mean $\mu_c(k)$ and variance $\sigma_c(k)^2$, see (14) and (15), a trivial calculation shows that

$$\mathbf{E}[(S_c(T, k) - K)^+] = \sigma_c(k) \phi \left(\frac{K - \mu_c(k)}{\sigma_c(k)} \right) + (\mu_c(k) - K) \Phi \left(\frac{-K + \mu_c(k)}{\sigma_c(k)} \right) \quad (18)$$

where ϕ is the density function of a standard normal variable and Φ its cumulative distribution function.

The AE method is to solve the PIDE (9) with the approximate local volatility function (16). We find the numerical solution with the log transform of variables and the explicit-implicit finite difference method of Cont and Voltchkova (2005).

The CV method approximates the basket value $S(T)$ with a tractable variable $S_c(T)$ and finds a closed form pricing formula (17) and (18). This approach is essentially in the same spirit as that of

Benhamou et al. (2009) with the difference that we only expand to the first order while Benhamou et al. (2009) to the second order.

The following data are used in all numerical tests: the number of assets in the basket $n = 4$, the portfolio weights of each asset $w_i = 0.25$ for $i = 1, \dots, n$, the correlation coefficients of Brownian motions $\rho_{ij} = 0.3$ for $i, j = 1, \dots, n$, the initial asset prices $S_i(0) = 100$ for $i = 1, \dots, n$, the risk free interest rate $r = 0$, the dividend rate $q = 0$, the exercise price $K = 100$.

Table 1

Table 1 displays the numerical results of the European basket call option prices with the MC, AE, CV methods. The first column is the maturity ($T = 1, 3$), the second and the third the coefficients of local volatility functions ($\sigma_i(t, S) = \alpha S^{\beta-1}$ with $\alpha = 0.1, 0.2, 0.5$ and $\beta = 1, 0.8, 0.5$), the fourth the MC results with standard deviations in the brackets, the fifth the AE results with relative percentage errors in comparison with the MC results, the sixth the CV results with errors, columns four to six correspond to the case of jump intensity $\lambda = 0.3$, and columns seven to nine are similarly defined for $\lambda = 1$. The last row displays the average standard deviations of the MC method and the average errors of the AE and CV methods. For normal variable $Y_l \sim N(\eta, \gamma^2)$ we set $\eta = -0.08$ and $\gamma = 0.35$. Whenever there is a jump event the jump size is relatively small (about 2% of the value lost). The choice of η, γ and intensity $\lambda = 0.3$ follow those of Benhamou et al. (2009) where the authors claim that these parameters are not small, especially for the jump intensity λ and the jump volatility γ .

It is clear that the overall performance of the AE method is excellent. All relative errors are less than 0.5% except for the four cases when the local volatility function is $\sigma_i(t, S) = 0.5$. This is the case corresponding to the high volatility in the Black-Scholes setting and is irrelevant to the maturity T and jump intensity λ . This is the phenomenon also reported by other researchers. The CV method is not satisfactory with average relative error about 7%. Matlab is used for the computations. When $T = 1$ we run 30,000 simulations for each case and repeat 10 times to get the average value, which is used as the Monte Carlo result. We choose the time step size $1/512$ and state step size $1/1024$ for the explicit-implicit finite difference method, it takes 40 seconds for the AE method and more than 30 minutes for the MC method. When $T = 3$ we run 100,000 simulations for each case and repeat 10 times to get the average Monte Carlo result and choose the same step sizes as those for $T = 1$, it takes 2 minutes for the AE method and more than 6 hours for the MC method. The AE method is much faster than the MC method while the accuracy is reasonable for most cases.

Table 2

Table 2 is similar to Table 1 with the only difference that the mean of Y_l is $\eta = -0.3$. Whenever there is a jump event the jump size is relatively large (about 21% of the value lost). The performance of the AE method is very similar to that in Table 1 with the average relative error 0.5%, but the performance of the CV method becomes much worse with the average relative error 18%. Since the CV method is similar in spirit to the method of Benhamou et al. (2009) there is a possibility that large errors may appear when $\eta = -0.3$ in Benhamou et al. (2009).

Table 3

Table 3 displays the results with three different methods: MC, AE, and the partial exact approximation (PEA) method suggested in Xu and Zheng (2009) when local volatility functions are given by $\sigma_i(t, S) = 0.2$ and random variables $Y_l \in N(\eta, 0)$, i.e., Y_l equal to a constant η . The reason to take constant jump sizes is due to the limitation of the PEA method which cannot deal with general

jump sizes. The purpose of the test is to see and compare the performance of the AE and PEA methods. The basic data are the same as those in Table 1 and 2. The first column is the jump intensity ($\lambda = 0.3, 1$), the second the jump size ($\eta = -0.25, -0.125, -0.0625$), the third maturity ($T = 1, 3$), the fourth the MC results, the fifth the PEA results with relative errors compared with the MC results, the last the AE results with relative errors. It is clear that both the PEA method and the AE method perform well with the relative error less than 1% for all cases, and the former is more accurate than the latter (the average relative error 0.1% vs 0.4%).

Table 4

Table 4 is similar to Table 3 with different local volatility functions $\sigma_i(t, S) = 0.5$. It is clear that the performance of the PEA method is much better than that of the AE method: the former has relative errors less than 1 percent for all cases while the latter has relative errors about 2 percent when $T = 1$ and jumps to about 6 percent when $T = 3$, irrespective to the jump intensities and sizes. We can reasonably say that the PEA method is a better approximation method for the European basket call options pricing when the local volatility functions are of the Black-Scholes type. However, the AE method is much more flexible and can handle general local volatility functions (and stochastic volatilities) and general jump variables, two cases which cannot be solved with the PEA method for the time being.

5 Conclusions

In this paper we have discussed the European basket options pricing for local volatility jump-diffusion models and derived a forward PIDE for general asset price processes. The asymptotic expansion (AE) method is used to approximate the local volatility function which is the square root of the conditional expectation of the stochastic variance. We have done numerical tests for different parameters to compare the performance of the AE method with those of other pricing methods. The numerical tests show that the AE method has small relative errors (less than 0.5%) compared with the Monte Carlo (MC) results for most parameters except when volatility is high (50%) in a Black-Scholes model and is much faster than the MC method. The AE method is much more accurate than the control variate (CV) method suggested in Benhamou et al. (2009). It is comparable in performance with the partial exact approximation (PEA) method suggested in Xu and Zheng (2009) in a Black-Scholes model when the volatility is not very high (about 20%) but is much more flexible than the PEA method as it can deal with general local volatility models and jump size distributions. We believe that the AE method provides a good approximation method for pricing basket European options with underlying asset prices satisfying some local volatility jump diffusion processes. The idea and methodology of the paper opens the way for other processes and refinements, for example, we may get better approximation if we asymptotically expand asset prices to the second order or we may introduce individual jump processes or different jump sizes for common jumps. There are also many open questions on estimating errors in approximating the local volatility function and in solving the PIDE. Further research is needed on these issues.

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Appendix

Outline of the proof of the PIDE (5) with the local functions (4) and (6). According to Protter (2003),

Theorem IV.68,

$$\begin{aligned} (S(T) - K)^+ &= (S(0) - K)^+ + \int_0^T 1_{[S(t-) > K]} dS(t) + \frac{1}{2} L_T^K \\ &\quad + \int_0^T \int_{-\infty}^{\infty} [1_{[S(t-) \leq K]} (e^x S(t-) - K)^+ + 1_{[S(t-) > K]} (K - e^x S(t-))^+] \mu(dx, dt) \end{aligned}$$

where L^K is a local time at K of process S . Taking the expectation on both sides, using Fubini's theorem and the martingale property, we have

$$\begin{aligned} &\mathbf{E}[(S(T) - K)^+] \\ &= (S(0) - K)^+ + \int_0^T (r(t) - q(t)) \mathbf{E}[1_{[S(t) > K]} S(t)] dt + \frac{1}{2} \mathbf{E}[L_T^K] \\ &\quad + \int_0^T \mathbf{E} \left[\int_{-\infty}^{\infty} [1_{[S(t) \leq K]} (e^x S(t) - K)^+ + 1_{[S(t) > K]} (K - e^x S(t))^+] \bar{a}(t) k(x) dx \right] dt. \end{aligned} \quad (19)$$

We have replaced $S(t-)$ by $S(t)$ due to the time integral taken with respect to the Lebesgue measure. Differentiating (19) with respect to T yields

$$\begin{aligned} \frac{\partial \mathbf{E}[(S(T) - K)^+]}{\partial T} &= (r(T) - q(T)) \mathbf{E}[1_{[S(T) > K]} S(T)] + \frac{1}{2} \frac{\partial \mathbf{E}[L_T^K]}{\partial T} \\ &\quad + \mathbf{E} \left[\int_{-\infty}^{\infty} L(T, K, x, S(T)) \bar{a}(T) k(x) dx \right] \end{aligned} \quad (20)$$

where

$$L(T, K, x, S(T)) = [1_{[S(T) \leq K]} (e^x S(T) - K)^+ + 1_{[S(T) > K]} (K - e^x S(T))^+].$$

Since the European call option price at time 0 with maturity T and exercise price K is given by

$$C(T, K) = e^{-\int_0^T r(t) dt} \mathbf{E}[(S(T) - K)^+]. \quad (21)$$

we have (Klebaner (2002))

$$\mathbf{E}[1_{[S(T) > K]}] = 1 - F_{S(T)}(K) = -\frac{\partial C(T, K)}{\partial K} e^{\int_0^T r(t) dt} \quad (22)$$

where $F_{S(T)}$ is the cumulative distribution function of $S(T)$, and

$$\frac{dF_{S(T)}(K)}{dK} = e^{\int_0^T r(t) dt} C_{KK}(T, K). \quad (23)$$

Note that the above equation and derivatives are defined in the sense of distribution. If $S(T)$ admits a continuous probability density function then $C(T, K)$ is twice continuously differentiable and (23) holds in the classical sense. Since

$$\mathbf{E}[(S(T) - K)^+] = \mathbf{E}[1_{[S(T) > K]} S(T)] - K \mathbf{E}[1_{[S(T) > K]}]$$

we can combine (21) with (22) to yield

$$\mathbf{E}[1_{[S(T) > K]} S(T)] = e^{\int_0^T r(t) dt} C(T, K) - K e^{\int_0^T r(t) dt} \frac{\partial C(T, K)}{\partial K}.$$

We also clearly have

$$\frac{\partial \mathbf{E}[(S(T) - K)^+]}{\partial T} = \frac{\partial}{\partial T} C(T, K) e^{\int_0^T r(t) dt} + C(T, K) e^{\int_0^T r(t) dt} r(T)$$

Following the same proof as in Klebaner (2002), Theorem 4, we can show that

$$\frac{\partial \mathbf{E}[L_T^K]}{\partial T} = \mathbf{E}[V(T)^2 K^2 | S(T) = K] e^{\int_0^T r(t) dt} C_{KK}(T, K). \quad (24)$$

The equation (24) and derivatives are defined in the sense of distribution. Klebaner (2002) proves (24) for a continuous semimartingale asset price process, it also works for the case here thanks to the property of the local time, that is,

$$\int_{-\infty}^{\infty} g(K) L_T^K dK = \int_0^T g(S(t-)) d\langle S^c \rangle_t$$

for all positive bounded functions g , where $\langle S^c \rangle_v$ is the quadratic variation of the continuous part of the process S . Everything then proceeds exactly the same. We now estimate the last term in (20). Using Fubini's theorem and the tower property, also noting (6) and (23), yield

$$\begin{aligned} & \mathbf{E} \int_{-\infty}^{\infty} [L(T, K, x, S(T)) \bar{a}(T)] k(x) dx \\ &= \int_{-\infty}^{\infty} \mathbf{E} [\mathbf{E}[L(T, K, x, S(T)) \bar{a}(T) | S(T)]] k(x) dx \\ &= \int_{-\infty}^{\infty} \mathbf{E} [L(T, K, x, S(T)) a(T, S(T))] k(x) dx \\ &= \int_{-\infty}^{\infty} \int_0^{\infty} L(T, K, x, z) a(T, z) dF_{S(T)}(z) k(x) dx \\ &= \int_{-\infty}^{\infty} \int_0^{\infty} L(T, K, x, z) a(T, z) e^{\int_0^T r(t) dt} C_{zz}(T, z) dz k(x) dx \\ &= e^{\int_0^T r(t) dt} \int_0^{\infty} a(T, z) C_{zz}(T, z) \left(\int_{-\infty}^{\infty} L(T, K, x, z) k(x) dx \right) dz \\ &= e^{\int_0^T r(t) dt} \int_0^{\infty} a(T, z) C_{zz}(T, z) z \psi_e \left(\ln \frac{K}{z} \right) dz \end{aligned}$$

where ψ_e is the double-exponential tail of the Lévy measure k . The last equality follows exactly Carr et al. (2004). Substituting everything into (20) and simplifying the expression we then get the PIDE (5) with local functions (4) and (6).

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λ			0.3			1		
T	α	β	MC (stdev)	AE (err%)	CV (err%)	MC (stdev)	AE (err%)	CV (err%)
1	0.1	1	5.91 (0.03)	5.91 (0.0)	6.14 (3.9)	11.86 (0.05)	11.83 (0.3)	12.7 (7.1)
			8.14 (0.02)	8.13 (0.1)	8.31 (2.1)	13.25 (0.06)	13.24 (0.1)	13.85 (4.5)
			15.50 (0.04)	15.18 (2.1)	15.52 (0.1)	18.89 (0.05)	18.60 (1.5)	19.25 (1.9)
	0.1	0.8	4.64 (0.02)	4.64 (0.0)	5.03 (8.4)	11.13 (0.03)	11.16 (0.3)	12.43 (11.7)
			5.47 (0.02)	5.47 (0.0)	5.74 (4.9)	11.60 (0.06)	11.58 (0.2)	12.56 (8.3)
			8.11 (0.03)	8.11 (0.0)	8.29 (2.2)	13.25 (0.04)	13.23 (0.2)	13.84 (4.5)
	0.1	0.5	4.06 (0.02)	4.08 (0.5)	4.81 (18.5)	10.96 (0.03)	10.98 (0.2)	12.40 (13.1)
			4.24 (0.01)	4.25 (0.2)	4.83 (13.9)	11.00 (0.02)	11.01 (0.1)	12.41 (12.8)
			4.85 (0.01)	4.85 (0.0)	5.19 (7.0)	11.24 (0.04)	11.26 (0.2)	12.44 (10.7)
3	0.1	1	12.18 (0.02)	12.16 (0.2)	12.75 (4.7)	22.94 (0.11)	22.99 (0.2)	24.36 (6.2)
			15.25 (0.03)	15.14 (0.7)	15.65 (2.6)	24.49 (0.13)	24.45 (0.2)	25.78 (5.3)
			27.23 (0.09)	25.64 (5.8)	27.21 (0.1)	33.03 (0.14)	31.55 (4.5)	34.02 (3.0)
	0.1	0.8	10.69 (0.02)	10.68 (0.1)	11.65 (9.0)	22.45 (0.10)	22.51 (0.3)	23.97 (6.8)
			11.64 (0.02)	11.62 (0.2)	12.29 (5.6)	22.81 (0.10)	22.79 (0.1)	24.19 (6.1)
			15.19 (0.03)	15.14 (0.3)	15.62 (2.8)	24.51 (0.07)	24.48 (0.1)	25.76 (5.1)
	0.1	0.5	10.16 (0.02)	10.16 (0.0)	11.53 (13.5)	22.35 (0.04)	22.43 (0.4)	23.90 (6.9)
			10.29 (0.02)	10.29 (0.0)	11.55 (12.2)	22.43 (0.07)	22.44 (0.0)	23.91 (6.6)
			10.92 (0.02)	10.91 (0.1)	11.77 (7.8)	22.51 (0.10)	22.57 (0.3)	24.01 (6.7)
Average			(0.03)	(0.6)	(6.6)	(0.07)	(0.5)	(7.1)

Table 1: The comparison of European basket call option prices with the Monte Carlo (MC), the asymptotic expansion (AE), and the control variate (CV) methods. The asset price processes are modelled by SDE (7). The table displays results with different maturities T , local volatility functions $\sigma_i(t, S) = \alpha S^{\beta-1}$, and jump intensities λ . The numbers inside brackets in the MC columns are the standard deviations and those in the AE and CV columns are the relative percentage errors in comparison with the MC results. The data used are: number of assets $n = 4$, weights $w_i = 0.25$, correlation of Brownian motions $\rho_{ij} = 0.3$, initial asset prices $S_i(0) = 100$, interest rate $r = 0$, exercise price $K = 100$, normal variable $Y_l \sim N(\eta, \gamma^2)$ with $\eta = -0.08$ and $\gamma = 0.35$.

λ			0.3			1		
T	α	β	MC (stdev)	AE (err%)	CV (err%)	MC (stdev)	AE (err%)	CV (err%)
1	0.1	1	6.99 (0.02)	7.00 (0.1)	8.4 (20.2)	15.23 (0.03)	15.28 (0.3)	18.20 (20.0)
			8.84 (0.01)	8.84 (0.0)	9.91 (12.1)	15.76 (0.05)	15.79 (0.2)	18.62 (18.2)
			15.89 (0.02)	15.62(1.7)	16.51 (3.9)	20.24 (0.04)	20.02 (1.1)	22.34 (10.4)
	0.1	0.8	6.45 (0.01)	6.45 (0.0)	8.13 (26.1)	15.14 (0.03)	15.17 (0.2)	18.09 (19.5)
			6.72 (0.01)	6.73 (0.2)	8.24 (22.6)	15.20 (0.04)	15.23 (0.2)	18.15 (19.4)
			8.83 (0.02)	8.83 (0.0)	9.89 (12.0)	15.75 (0.02)	15.79 (0.3)	18.62 (18.2)
	0.1	0.5	6.43 (0.01)	6.44 (0.2)	8.12 (26.3)	15.11 (0.03)	15.15 (0.3)	18.07 (19.6)
			6.44 (0.01)	6.44 (0.0)	8.12 (26.1)	15.12 (0.03)	15.16 (0.3)	18.07 (19.5)
			6.49 (0.01)	6.49 (0.0)	8.14 (25.4)	15.15 (0.05)	15.19 (0.3)	18.10 (19.5)
3	0.1	1	14.70 (0.02)	14.71 (0.1)	17.46 (18.8)	27.00 (0.07)	27.03 (0.1)	31.92 (18.2)
			16.85 (0.03)	16.79 (0.4)	19.11 (13.4)	28.08 (0.06)	28.04 (0.1)	32.93 (17.3)
			27.99 (0.04)	26.51 (5.3)	29.19 (4.3)	35.31 (0.06)	33.92 (3.9)	39.44 (11.7)
	0.1	0.8	14.27 (0.02)	14.29 (0.1)	17.12 (20.0)	26.64 (0.07)	26.74 (0.4)	31.63 (18.7)
			14.48 (0.02)	14.49 (0.1)	17.30 (19.5)	26.82 (0.07)	26.91 (0.3)	31.79 (18.5)
			16.81 (0.02)	16.80 (0.1)	19.09 (13.6)	28.07 (0.06)	28.09 (0.1)	32.92 (17.3)
	0.1	0.5	14.22 (0.01)	14.23 (0.1)	17.07 (20.0)	26.58 (0.04)	26.68 (0.4)	31.58 (18.8)
			14.23 (0.01)	14.25 (0.1)	17.07 (20.0)	26.62 (0.05)	26.69 (0.3)	31.59 (18.7)
			14.31 (0.02)	14.32 (0.1)	17.15 (20.0)	26.71 (0.06)	26.78 (0.3)	31.66 (18.5)
Average			(0.02)	(0.5)	(18.0)	(0.05)	(0.5)	(17.9)

Table 2: The comparison of European basket call option prices with the Monte Carlo (MC), the asymptotic expansion (AE), and the control variate (CV) methods. The asset price processes are modelled by SDE (7). The table displays results with different maturities T , local volatility functions $\sigma_i(t, S) = \alpha S^{\beta-1}$, and jump intensities λ . The numbers inside brackets in the MC columns are the standard deviations and those in the AE and CV columns are the relative percentage errors in comparison with the MC results. The data used are: number of assets $n = 4$, weights $w_i = 0.25$, correlation of Brownian motions $\rho_{ij} = 0.3$, initial asset prices $S_i(0) = 100$, interest rate $r = 0$, exercise price $K = 100$, normal variable $Y_l \sim N(\eta, \gamma^2)$ with $\eta = -0.3$ and $\gamma = 0.35$.

λ	η	T	MC (stdev)	PEA (err%)	AE (err%)
0.3	−0.25	1	7.35 (0.01)	7.35 (0.0)	7.35 (0.0)
		3	12.93 (0.01)	12.92 (0.1)	12.85 (0.6)
	−0.125	1	6.08 (0.01)	6.08 (0.0)	6.07 (0.2)
		3	10.57 (0.01)	10.56 (0.1)	10.49 (0.8)
	−0.0625	1	5.66 (0.01)	5.66 (0.0)	5.65 (0.2)
		3	9.83 (0.01)	9.82 (0.1)	9.74 (0.9)
1	-0.2212	1	10.78 (0.01)	10.77 (0.1)	10.78 (0.0)
		3	18.64 (0.01)	18.63 (0.1)	18.57 (0.4)
	-0.1175	1	7.28 (0.01)	7.28 (0.0)	7.28 (0.0)
		3	12.65 (0.01)	12.64 (0.1)	12.58 (0.6)
	-0.0606	1	6.02 (0.01)	6.02 (0.0)	6.01 (0.2)
		3	10.45 (0.01)	10.43 (0.2)	10.37 (0.8)
Average			(0.01)	(0.1)	(0.4)

Table 3: The comparison of the Monte Carlo (MC), the partial exact approximation (PEA), and the asymptotic expansion (AE) methods. The asset price processes are modelled by SDE (7). The table displays the results with different jump intensities λ , jump sizes η , and maturities T . The numbers inside brackets in the MC columns are the standard deviations and those in the PEA and AE columns are the relative percentage errors in comparison with the MC results. The data used are: number of assets $n = 4$, weights $w_i = 0.25$, correlation of Brownian motions $\rho_{ij} = 0.3$, initial asset prices $S_i(0) = 100$, interest rate $r = 0$, and exercise price $K = 100$, local volatility function $\sigma_i(t, S) = 0.2$, and jump variable $Y_l \in N(\eta, 0)$, a constant.

λ	η	T	MC (stdev)	PEA (err%)	AE (err%)
0.3	−0.25	1	14.71 (0.01)	14.66 (0.3)	14.42 (2.0)
		3	25.69 (0.04)	25.44 (1.0)	24.14 (6.0)
	−0.125	1	14.08 (0.01)	14.03 (0.4)	13.79 (2.1)
		3	24.61 (0.03)	24.39 (0.9)	23.07 (6.3)
	−0.0625	1	13.90 (0.01)	13.85 (0.4)	13.61 (2.1)
		3	24.32 (0.04)	24.11 (0.9)	22.77 (6.4)
1	-0.2212	1	16.60 (0.01)	16.55 (0.3)	16.32 (1.7)
		3	28.80 (0.04)	28.55 (0.9)	27.28 (5.3)
	-0.1175	1	14.64 (0.01)	14.59 (0.3)	14.35 (2.0)
		3	25.52 (0.05)	25.28 (0.9)	24.00 (6.0)
	-0.0606	1	14.05 (0.01)	14.00 (0.4)	13.76 (2.1)
		3	24.55 (0.04)	24.33 (0.9)	23.02 (6.2)
Average			(0.03)	(0.6)	(4.0)

Table 4: The comparison of the Monte Carlo (MC), the partial exact approximation (PEA), and the asymptotic expansion (AE) methods. The asset price processes are modelled by SDE (7). The table displays the results with different jump intensities λ , jump sizes η , and maturities T . The numbers inside brackets in the MC columns are the standard deviations and those in the PEA and AE columns are the relative percentage errors in comparison with the MC results. The data used are: number of assets $n = 4$, weights $w_i = 0.25$, correlation of Brownian motions $\rho_{ij} = 0.3$, initial asset prices $S_i(0) = 100$, interest rate $r = 0$, and exercise price $K = 100$, local volatility function $\sigma_i(t, S) = 0.5$, and jump variable $Y_l \in N(\eta, 0)$, a constant.