

Report on Grant EP/C003012/1
Workshop on Pure Model Theory
UEA, Norwich, 4-8 July, 2005.

Organisers: Dr David Evans (UEA, Norwich, UK), Prof. Dugald Macpherson (U. of Leeds, UK), Prof. Anand Pillay (U. of Illinois, Urbana, USA, and U. of Leeds, UK), Prof. Frank Wagner (U. Lyon I, France).

1. The Context of the Workshop. The Workshop was a satellite meeting of the Isaac Newton Institute programme ‘Model Theory and Applications to Algebra and Analysis’ which ran from 17 January to 15 July 2005. It was one of the three main conferences associated with the programme. The other two, held at the INI, were the workshop ‘An Introduction to Recent Applications of Model Theory’ held in March, and ‘Model Theory, Algebraic and Analytic Geometry’ immediately following the workshop at UEA.

The main thrust of the NI programme and the associated conferences in Cambridge was on applications of model theory and its interactions with other parts of mathematics. In contrast, the Workshop at UEA concentrated mainly on developments in pure model theory, without, of course, excluding connections with other branches of mathematics.

2. Format of the Workshop. The Workshop took place on the campus of the UEA in Norwich and attracted 80 participants (exceeding the original estimate of 50 to 70). Of these, 73 were resident on campus, and the remaining 7 ‘commuted’ from the Newton Institute in Cambridge. 20 of the participants were PhD students and at least 50 of the participants had visited the Newton Institute during the semester. The following invited lectures were given:

Tutorials (each comprising three 1-hour lectures):

- Itay Ben-Yaacov (Madison), ‘Model theory in positive and continuous logics’
- Ludomir Newelski (Wrocław), ‘Multiplicity; Small profinite structures; type-generated equivalence relations’
- Bruno Poizat (Lyon), ‘AMALGAMES/Amalgams’

Principal speakers (1-hour talks):

- John Baldwin (Chicago), ‘Perspectives on expansions’
- Elisabeth Bouscaren (Paris), ‘Finitely axiomatizable strongly minimal sets’
- Byunghan Kim (MIT and Yonsei), ‘Generalized type-amalgamation and n-simplicity I’
- Chris Laskowski (Maryland), ‘Interpreting groups in strictly stable theories’
- Kobi Peterzil (Haifa), ‘Pillay’s group conjecture and its solution’
- Mike Prest (Manchester), ‘Functorial methods in model theory’
- Alex Usvyatsov (Jerusalem), ‘Dependent theories’
- Boris Zilber (Oxford), ‘Some counterexamples in pure model theory’
- Martin Ziegler (Freiburg), ‘Fields with black points’

Other Invited Speakers (half-hour talks):

- Hans Adler (Freiburg), ‘The lattice of algebraically closed sets’
- Alfred Dolich (McMaster), ‘On Model Theoretic Properties of Structures Expanding the Real Closed Field’
- Marko Djordjevic (Uppsala), ‘The finite submodel property and omega-categorical structures.’

- Assaf Hasson (Oxford), ‘On the Theory of Envelopes and Homogeneous Approximations’
- Martin Hils (Paris), ‘Generalised fusion’
- Alexei Kolesnikov (Michigan), ‘Generalized type-amalgamation and n -simplicity II’
- Alf Onshuus (Bogota), ‘th-forking’
- Nicholas Peatfield (Lisbon), ‘Generic functions and analytic Zariski structures’
- Massoud Pourmahdian (Tehran), ‘A simple Robinson theory with $Lstp$ not equal to stp ’
- Ziv Shami (Tel Aviv), ‘Unidimensionality and the forking topology’
- Alex Usvyatsov (Jerusalem), ‘Local stability in continuous logic’
- Evgueni Vassiliev (Florida), ‘Weak $nfcf$ and related conditions in simple theories’

The tutorials were as planned in the original proposal. Of the principal invited speakers mentioned in the proposal, Hrushovski and Shelah were unable to attend. Nevertheless, Peterzil reported on joint work with Hrushovski and Pillay which provided the final piece in the proof of a conjecture of Pillay on groups in o -minimal structures, and Usvyatsov gave a talk on work of Shelah related to ideas in this. Many of half-hour speakers were people who had completed their PhD’s within the last 5 years.

A problem session was held on the second day of the meeting; a poster session was offered, but not held due to a lack of contributions! The programme included space for discussions and a free afternoon (which was also used for discussions, thanks to the weather). Meals on campus were included as part of the conference package, and the communal dining arrangements provided a relaxed setting for further discussions and interactions between all participants.

3. The Scientific Content of the Workshop.

The following relates the talks given to the topics mentioned in the original proposal.

(i) Forking and simple theories. The talks by Kim and Kolesnikov discussed a hierarchy of simple theories based on strengthenings of the Independence Theorem: the n -amalgamation property and $K(n)$ -simplicity. Such properties have been applied in work of Kim, de Piro and Young on group configuration results for simple theories.

Simple theories were also discussed in the talks of Shami (on unidimensionality) and Vassiliev (on the simple analogue of the finite cover property).

Stable theories were the main focus of the talks by Laskowski and Bouscaren. Laskowski reported on joint work with Shelah on stable, non-superstable theories in a countable language, proving that these have DOP, or are deep, or interpret a strictly stable group. Bouscaren gave a survey about the question of existence of a finitely axiomatizable strongly minimal theory, and reported on recent progress on this question in joint work with Blossier.

(ii) Hrushovski constructions. The sequence of lectures by Poizat provided an overview of amalgamation constructions, in particular the Hrushovski constructions, and the possible logical frameworks in which to consider these. Connections with Ben-Yaacov’s series of talks were emphasised. Baldwin’s talk also discussed problems of axiomatizing the Hrushovski structures, and Ziegler reported on recent work with Baudisch and Martin-Pizzaro obtaining explicit axiomatizations of the fields of Morley rank 2 of Baldwin and Holland. These results have recently been extended, and Baudisch, Hils, Martin-Pizzaro and Wagner have just announced the construction of a ‘bad field’ of Morley rank 2.

Further talks on the Hrushovski constructions were given by Hils (on fusion of strongly minimal sets) and Peatfield (giving an approach to realising the field of complex numbers with a generic function and its derivatives as an analytic Zariski structure).

The final talk of the meeting was given by Zilber, who reported on ideas he had been developing during the Newton Institute semester on relating the Hrushovski constructions to non-commutative geometry. Specifically he outlined a connection between Poizat's bad field (of infinite Morley rank) and the quantum torus.

(iii) Non-elementary classes. Classically, model theory studies elementary classes using full first-order logic. In his series of talks, Ben-Yaacov discussed two alternatives to this setting which have been studied in recent years, allowing the application of model-theoretic methods to a larger family of classes of structures. These are positive model theory, developed by Ben-Yaacov (extending previous work of Shelah and Hrushovski), and continuous first-order logic, developed by Ben-Yaacov, Henson and Berenstein. The latter is particularly suited to the study of Banach spaces and measure algebras and had been the subject of a sequence of lectures at the Workshop held at the Newton Institute in March. The theme was continued in a talk by Usvyatsov on local (that is, formula-by-formula) stability in continuous logic.

Prest's talk described a model-theoretic / functor-category-theoretic 'dictionary' in the context of additive definable categories of structures, and indicated how it produces applications of model-theoretic methods in module theory.

(iv) Topological methods. The relationship between the space of types over a set and over its algebraic closure leads naturally to the notion of a profinite structure. The analysis of such spaces is central to Newelski's approach to Vaught's Conjecture for superstable theories. In his series of talks Newelski described how the study of profinite structures and tools developed in this approach (such as meagre forking, m -independence and M -rank) have evolved into an area of independent interest with many open problems and applications.

(v) Independence in unstable structures. One of the main results coming out of the Newton Institute programme was a proof of a conjecture of Pillay on the nature of definably compact groups in o -minimal structures, relating them to Lie groups. In his talk, Peterzil discussed the proof of the conjecture and in particular, the final part of the proof which is due to Hrushovski, Peterzil and Pillay. The proof uses many recent ideas from 'pure' model theory: the nature of forking in o -minimal structures (discussed in the talk of Dolich) and work of Shelah on theories without the independence property (a talk on this was given by Usvyatsov).

The notion of thorn forking, which unifies theories of independence in stable, o -minimal and supersimple theories, was discussed in the talks by Onshuus and Adler.

4. Use of Resources from EPSRC Grant.

(i) Participants' expenses. The EPSRC grant (together with a grant of 5000 pounds from the Newton Institute) paid accommodation, subsistence and travel expenses for principal invited speakers and organisers, and accommodation, subsistence and a contribution to travel expenses for other invited speakers. It provided half the accommodation and subsistence costs for a further 14 invited participants (Baudisch (Berlin); Blossier (Lyon); Casanovas (Barcelona); de Piro (Edinburgh); Ealy (Urbana); Hart (McMaster); Hodges (London); Kowalski (Oxford); Kudaibergenov (Almaty); Lippel (Notre Dame); Nuebling (Greifswald); Polkowska (Urbana); Tomasic (Lyon); Verbovskiy (Almaty)). The grant was also used to

pay half the accommodation and subsistence costs for 11 PhD students. Actual costs paid from the EPSRC grant are:

| | Pounds |
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| Travel for invited speakers: | 4698.04 |
| Conference packages for invited speakers and participants: | 7108.85 |
| Conference packages for PhD students: | 1187.70 |
| TOTAL: | 12994.59 |

(ii) *Staff.* The Workshop bought out 6.5 days of time of one of the clerical staff (employed at Clerical grade V) from the School of Mathematics at UEA to handle bookings, payment of expenses, and assembly and distribution of documentation.

The Workshop attracted 5000 pounds of funding from the Isaac Newton Institute (to pay participation costs of members of the INI programme) and 2000 pounds of funding from the London Mathematical Society (for UK PhD students and participants from the former Soviet Union).