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Schwarz-Christoffel Formula Solved

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A problem which has stumped mathematicians for nearly 150 years has been solved by an intrepid researcher at Imperial College in London.

Darren Crowdy made the breakthrough in the area of mathematics known as conformal mapping, a theoretical tool used by mathematicians, engineers, and scientists to translate information from a complicated shape to a simpler circular shape so as to make it is easier to analyze.

According to the Riemann Mapping Theorem, there exists a conformal map from the unit disk to any simply connected planar region (except the whole plane). However, finding such a map for a specific region is difficult. An important special case, for which a formula is known, is when the target region is polygonal. In that case we have the Schwarz-Christoffel formula, which was developed by the German mathematicians Hermann Amandus Schwarz and Elwin Bruno Christoffel in the mid-19th century. This theoretical tool has applications in modeling airflow patterns over intricate wing shapes in aeronautics, for instance, and is used in neuroscience to visualize the structure of the grey matter in the human brain.

Crowdy's insight involves what is called the Schottky group, which makes it possible to "add" boundaries of circles--and thus extend the Schwarz-Christoffel formula to holes. "If you give me any polygon with any number of polygonal holes, I can map it to a circle with the same number of circular holes," Crowdy said. "This formula," he stressed, "is an essential piece of the mathematical kit which is used the world over. Now, with my additions to it, it can be used in far more complex scenarios than before. In industry, for example, this mapping tool was previously inadequate if a piece of metal or other material was not uniform all over--for instance, if it contained parts of a different material, or had holes."

After Crowdy's breakthrough became known, mathematicians John Pfaltzgraff (University of North Carolina), Thomas DeLillo, and Alan Elcrat (both of Wichita State University) indicated that they uncovered the basic strategy--and a formula--years earlier. Michael Siegel (New Jersey Institute of Technology) said, "It's a breakthrough, and all these people contributed."

Crowdy's proof, "Schwarz-Christoffel Mappings to Unbounded Multiply Connected Polygonal Regions," appeared in *Mathematical Proceedings of the Cambridge Philosophical Society* (2007), 142, 319.

Source: EurekAlert

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