Combinatorial rigidity of infinitely renormalization unicritical polynomials

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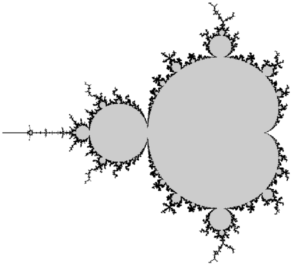
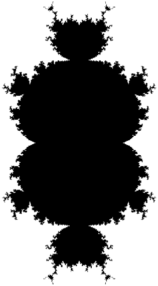
Let $f_c(z) = z^d + c$, $c \in \mathbb{C}$ and $d \geq 2$, with Julia set denoted by $J(f_c)$.

The problem is to show that the dynamics of f_c on $J(f_c)$ is combinatorially rigid! In particular, the geometry of orbits on $J(f_c)$ are uniquely determined by some combinatorial data!

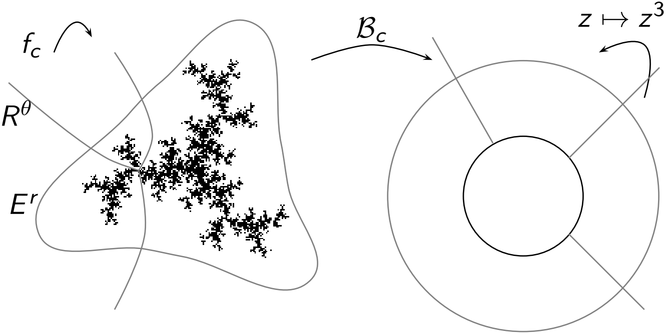
$$\mathcal{M}^{(d)} := \{c \in \mathbb{C} : J(f_c) \text{ is connected}\}$$



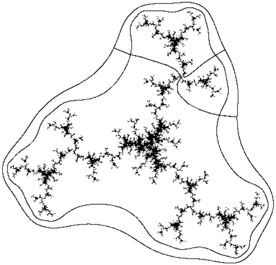


We only consider the third case in

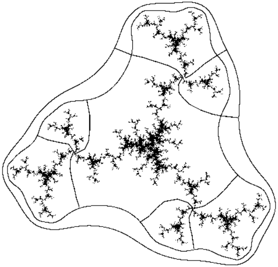
1. f_c has an attracting periodic point
2. f_c has a neutral periodic point
3. all periodic points of f_c are repelling



The puzzle pieces in the cubic case

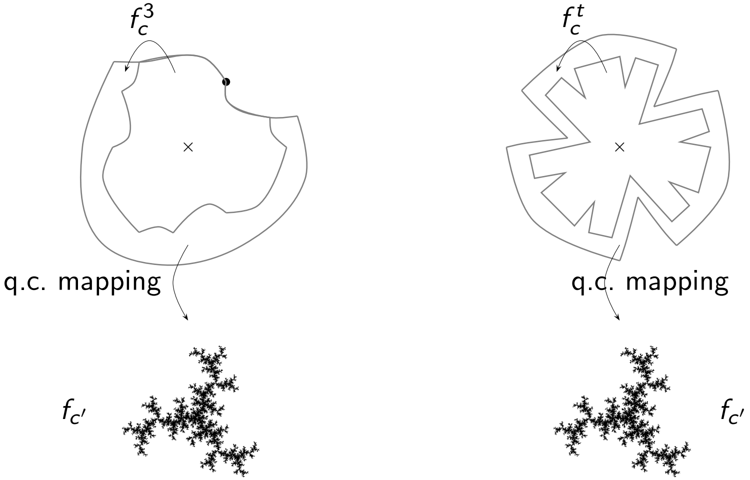


The puzzle pieces in the cubic case

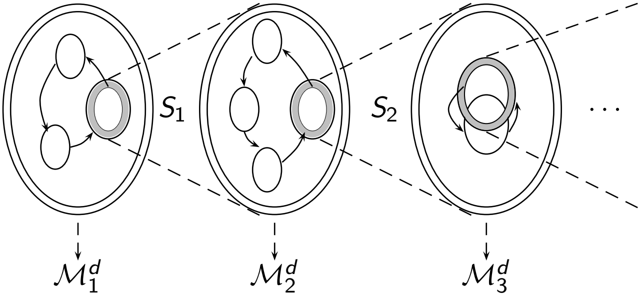


By Yoccoz in 1990 for $d = 2$ and by Kahn-Lyubich in 2005 for $d \geq 2$, the nests of puzzle pieces shrink to points unless the map is renormalizable

renormalization and straightening, repeating the process



The combinatorics of an infinitely renormalizable map

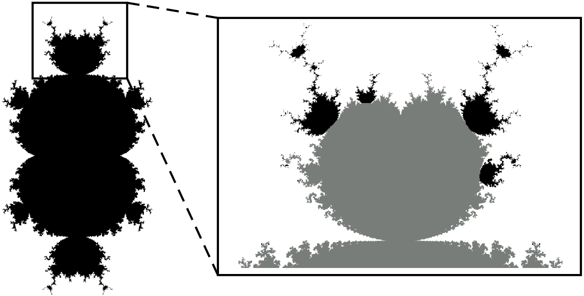


Com(c):=$\langle \mathcal{M}_i^d \rangle_{i=1,2,3,\ldots}$

Conjecture (combinatorial rigidity)

If Com(c)=Com(c'), then $c = \lambda c'$, *for some* λ *with* $\lambda^{d-1} = 1$.

Secondary limbs



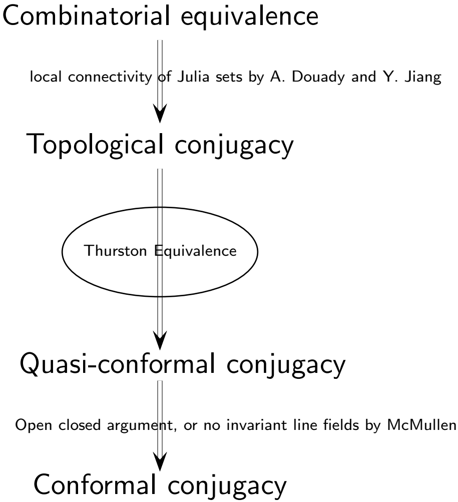
- *Secondary limbs condition*: all $\mathcal{M}_i^d$ belong to a finite number of secondary limbs.
- *A priori bounds*: the moduli of the fundamental annuli are uniformly away from zero.

Theorem (Rigidity)

For every $d \geq 2$, if f_c and $f_{c'}$ satisfy a priori bounds and secondary limbs conditions, then Com (c)=Com(c') implies $c = \lambda c'$ for some $(d-1)$-th root of unity λ.

Earlier results

- for $d = 2$ by Lyubich; His proof uses linear growth of moduli and does not work for arbitrary degree.
- for $d = 2$ and real maps by Graczyk-Swiatek; This also uses linear growth of moduli and, because of symmetry, no homotopy argument is needed.
- for real polynomials by Kozlovski-Shen-van Strien; Arbitrary number of critical points are involved, but the symmetry of the map is used.

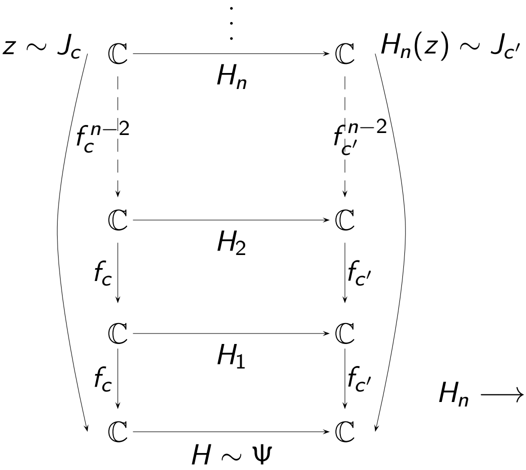


f_c is Thurston equivalent to $f_{c'}$ if there exists a q.c. mapping $H : \mathbb{C} \to \mathbb{C}$ which is homotopic to a topological conjugacy Ψ between f_c and $f_{c'}$, relative the post-critical set of f_c.

Lemma (Thurston-Sullivan?)

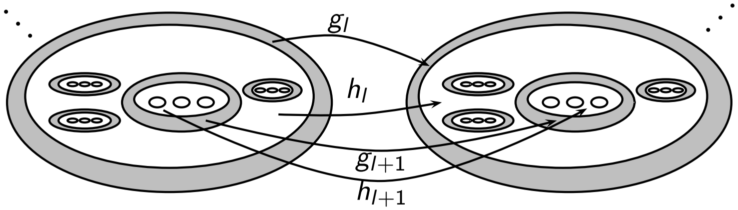
Thurston equivalence implies q.c. equivalence

Proof.



$H_n \longrightarrow G$

The multiply connected regions and buffers for gluing:



Building a Thurston equivalence;
Depending on the type of renromalization on level n and maybe on level $n+1$ as in

- A: on level n primitive type;
- B: on level n satellite type, and on level $n+1$ primitive type;
- C: on level n satellite type, and on level $n+1$ also satellite type.

we define the q.c. mappings between multiply connected regions.

The second case (naturally) imposes the $\mathcal{SL}$ condition on us, as there may be bad scenarios.

The right number of twists for gluings