M2S1 - EXERCISES 5

Covariance And Multivariate Distributions

1. Suppose that X and Y have joint pdf given by

$$f_{X,Y}(x,y) = cxy(1-x-y) \qquad 0 < x < 1, 0 < y < 1, 0 < x+y < 1.$$

for some constant $c > 0$. Find the covariance of X and Y.

2. Suppose that X and Y have joint pdf that is constant on the range $\mathbb{X}^{(2)} \equiv (0,1) \times (0,1)$, and zero otherwise.

(a) Find the marginal pdf of random variables $U = X/Y$ and $V = -\log(XY)$, stating clearly the range of the transformed random variable in each case.

(b) Find the pdf and cdf of $Z = X - Y$.

3. Suppose that continuous random variables X_1, X_2, X_3 are independent, and have marginal pdfs specified by

$$f_{X_i}(x_i) = c_i x_i^i e^{-x_i} \qquad x_i > 0$$

for $i = 1, 2, 3$, where c_1, c_2 and c_3 are normalizing constants. Find the joint pdf of random variables Y_1, Y_2, Y_3 defined by

$$Y_1 = X_1/(X_1 + X_2 + X_3) \qquad Y_2 = X_2/(X_1 + X_2 + X_3) \qquad Y_3 = X_1 + X_2 + X_3$$

and evaluate the marginal expectation of Y_1.

4. Suppose that X and Y are continuous random variables with pdf given by

$$f_{X,Y}(x,y) = \frac{1}{2\pi} \exp\left\{-\frac{1}{2}\left(x^2 + y^2\right)\right\} \qquad x, y \in \mathbb{R}$$

(a) Let random variable U be defined by $U = X/Y$. Find the pdf of U.

(b) Suppose now that S is a random variable, independent of X and Y, with pdf given by

$$f_S(s) = c(\nu) s^{\nu/2-1} e^{-s/2} \qquad s > 0$$

where ν is a positive integer and $c(\nu)$ is a normalizing constant depending on ν. Find the pdf of random variable T defined by

$$T = \frac{X}{\sqrt{S/\nu}}$$

5. Suppose that the joint pdf of random variables X and Y is specified via the conditional density $f_{X|Y}$ and the marginal density f_Y as

$$f_{X|Y}(x|y) = \sqrt{\frac{y}{2\pi}} \exp\left\{-\frac{yx^2}{2}\right\} \qquad x \in \mathbb{R} \qquad\qquad f_Y(y) = c(\nu) y^{\nu/2-1} e^{-\nu y/2} \qquad y > 0$$

where ν is a positive integer. Find the marginal pdf of X.