M1S PROBABILITY AND STATISTICS I

SYLLABUS

CHAPTER 1 : Sample Spaces and Events

1.1 Representing uncertainty in experimental contexts

1.2 Manipulating Collections of Sample Outcomes
Sample outcomes
Sample spaces (discrete/continuous, finite/countable/uncountable
Events - occurrence, the Certain Event, the Impossible Event
Set theory notation

1.3 Operations of Set Theory
Binary Operations - Complement/Union/Intersection
Exhaustive/Exclusive Events
Elementary results - De Morgan's Laws
Extensions of union/intersection ideas to more than two events - finite/countable unions
Associative/Distributive Laws
Representations of Complex Systems - component networks
Disjoint unions/Partitions
Construction of Partitions

CHAPTER 2 : Probability : Definitions, Interpretations, Basic Laws.

2.1 The Meaning of Probability
Interpretations of the Probability Function (relative frequency,classical,subjective)
Simple examples (0/1 experiments e.g. coin/drawing pin)
Specification of probabilities via Odds

2.2 The Mathematical Rules of Probability: The Probability Axioms
The Probability Axioms
Extensions to Axiom III to finite and countable collections of events

2.3 Corollaries to the Axioms
Probabilities for complement events
Probability of a union for non-exclusive events (The General Addition Rule)

CHAPTER 3 : Conditional Probability: Conditioning on New Information.

Conditional Probability - definition, interpretation
Examples of simple calculations
Independence for two events/ Mutual independence for more than two events
The General Multiplication (or Chain) Rule for Events (link to probability trees)

3.1 The Theorem of Total Probability
Statement and Proof of Theorem
Interpretation and Key elements
Application in simple and more complicated examples

3.2 Bayes Theorem
Statement and Proof of Theorem for two events/more than two events
Interpretation : Distinction between $P(A|B)$ and $P(B|A)$
Examples e.g. Medical Diagnosis, Simpson's Paradox
Prior and Posterior Odds
Conditional probability for more than two events
Conditional Independence

CHAPTER 4 : Counting Techniques: Combinatorics.

Enumeration for Equally Likely Outcomes : Combinatorics Problems (Sampling from a Finite Population, Occupancy Problems, Urn Models)

4.1 Counting Operations : Basic Methods and Terminology
Multiplication principle, Factorials
Distinguishable/Indistinguishable Objects;
Sampling with/without replacement;
Ordered/unordered outcomes.
Permutations/Combinations
Binary sequence representations

4.2 Combinatorial Identities

4.3 Partitioning : The Partition formula
Examples of Partitions (e.g. Poker hands)

4.4 Occupancy Problems (Distribution Problems)
Counting Techniques for Occupancy Problems involving
- Distinguishable objects - as a partitioning problem.
- Indistinguishable objects - using a binary sequence representation
Examples e.g. The Birthday Problem

4.5 Urn Models
The Hypergeometric Formula : two alternative justifications
Combinatorial/conditional probability justification
Uses for the Hypergeometric formula in probability calculations (for N, R, n or r varying)
Examples e.g. Fisher's Exact Test

4.6 Generating Functions for Combinatorics Calculations
Generating functions: definitions and uses
Solution of combinatorics problems using generating functions

CHAPTER 5 : Discrete Random Variables and Distributions

5.1 Random Variables
General definition
Discrete case (countable range)

5.2 Probability Mass Function
Definition, Notation, Interpretation
Properties; General Expectation and Variance; Examples.

5.3 Discrete Cumulative Distribution Function
Definition, Notation, Interpretation
Properties
Connection with the probability mass function (summation/differencing)

5.4 Bernoulli Distribution

5.5 Binomial Distribution
Definition, experimental context, interpretation
Mass function Limiting behaviour as $n \to \infty, \theta \to 0$ with $n\theta = \lambda$ constant

5.6 Poisson Distribution
Definition, experimental context, interpretation
Mass function
Connection to the Poisson Process

5.7 Geometric Distribution
Mass function and CDF

5.8 Negative Binomial Distribution
Definition, experimental context, interpretation
Mass function
Connection to Geometric distribution

5.9 Hypergeometric Distribution
Definition, experimental context, interpretation
Mass function
Connection to Binomial distribution

5.10 Probability Generating Functions
Definition
Uses: calculations for sums of independent random variables.
Formulae for PGFs of standard distributions (eg. Geometric, Binomial, Poisson)

CHAPTER 6 : Continuous Random Variables and Distributions

6.1 Continuous Random Variables
General definition
Continuous probability specifications as the limit of discrete specifications

6.2 Continuous Cumulative Distribution Function
Definition, Notation, Interpretation
Properties

6.3 Probability Density Function
Definition, Notation, Interpretation
Properties; Expectation and Variance.
Connection with the continuous CDF function (integration/differentiation)

6.4 Continuous Uniform Distribution

6.5 Exponential Distribution
- Definition, experimental context, interpretation
- PDF and CDF
- Connection to Poisson and Poisson Process
- Connection to Uniform

6.6 Gamma Distribution
- Definition, interpretation
- The GAMMA function and properties
- PDF
- Special case: Chi-squared distribution $(\alpha = n/2, \beta = 1/2)\, \chi^2_n$
- Connection to Exponential
- Connection to Normal

6.7 Normal Distribution
- Definition, experimental context, interpretation
- PDF in standard and non–standard cases
- Linear transformations

CHAPTER 7 : Transformations

- Transformations of random variables $(Y = g(X))$
- General transformation technique
- Transformation Theorem for g 1-1

CHAPTER 8 : Expectation

- Further study of expectations of a discrete/continuous random variable
- Expectations for general functions of random variables
- Properties of Expectations
- Linearity of Expectations
- Probability and Moment Generating Functions as Expectations
- Link between PGF and MGF
- Calculations of PGF/MGF for standard distributions
- Key results and uses of PGFs/MGFs

CHAPTER 9 : Joint Distributions

- General joint discrete/continuous distributions
- Discrete joint/marginal/conditional mass functions
- Continuous joint/marginal/conditional density functions
- Independence for random variables
- Conditional Expectations
- Bivariate Expectations
- The Law of Iterated Expectations
- Covariance/Correlation
- The Convolution Formula
- Expectations (and other calculation techniques) for Sums of Random Variables
- The Central Limit Theorem