MIS PROBABILITY AND STATISTICS I

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M1S: PROBABILITY AND STATISTICS I

INTRODUCTION

This course is concerned with developing mathematical concepts and techniques for modelling and analyzing situations involving uncertainty.

No previous exposure to ideas of Probability or Statistics will be assumed.

Assessment of uncertainty in such real-life problems is a complex issue which requires a rigorous mathematical treatment. This course will develop the probability framework in which questions of practical interest can be posed and resolved.

Course Objectives

- to develop a mathematical framework in which to handle UNCERTAINTY and VARIABILITY
- to introduce the logic and mathematical properties of PROBABILITY
- to introduce techniques useful in probability calculations
- to identify specific modelling environments that correspond to common experimental situations

Recommended Reading:

There are a large number of introductory texts on Probability and Statistics.

Particularly recommended for this course are:

Elementary Probability, by D. Stirzaker.

A First Course in Probability, by S. Ross.

Introduction to probability and mathematical statistics, by L. J. Bain and M. Engelhardt.

Course WWW page:

http://stats.ma.ic.ac.uk/~das01/M1S/

which will contain sketch lecture summaries, links to course handouts, and other links.

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M1S PROBABILITY AND STATISTICS I

SYLLABUS

CHAPTER 1 : Sample Spaces and Events

1.1 Representing uncertainty in experimental contexts

1.2 Manipulating Collections of Sample Outcomes
Sample outcomes
Sample spaces (discrete/continuous, finite/countable/uncountable
Events - occurrence, the Certain Event, the Impossible Event
Set theory notation

1.3 Operations of Set Theory
Binary Operations - Complement/Union/Intersection
Exhaustive/Exclusive Events
Elementary results - De Morgan's Laws
Extensions of union/intersection ideas to more than two events - finite/countable unions
Associative/Distributive Laws
Representations of Complex Systems - component networks

MOTIVATION

Many real-life situations involve "UNCERTAINTY"

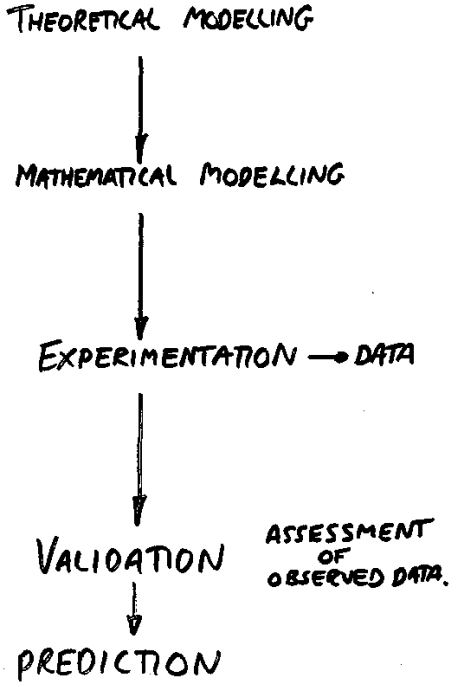
UNCERTAINTY - absence of perfect / complete knowledge

(unpredictability, variability...)

PROBABILITY THEORY IS A FRAMEWORK DERIVED TO HANDLE UNCERTAINTY.

Many scientific procedures involve observation of the real-world with the goal of

- assessing the validity of some theoretical model
- comparing different procedures
- predicting future behaviour
- optmal decision making



THE ROLE OF PROBABILITY AND STATISTICS

PROBABILITY THEORY

- gives the framework for
  (i) explaining the variability in observed data
  (ii) predicting future behaviour

STATISTICS

- allows the formal validation of theoretical models in the light of observed data.

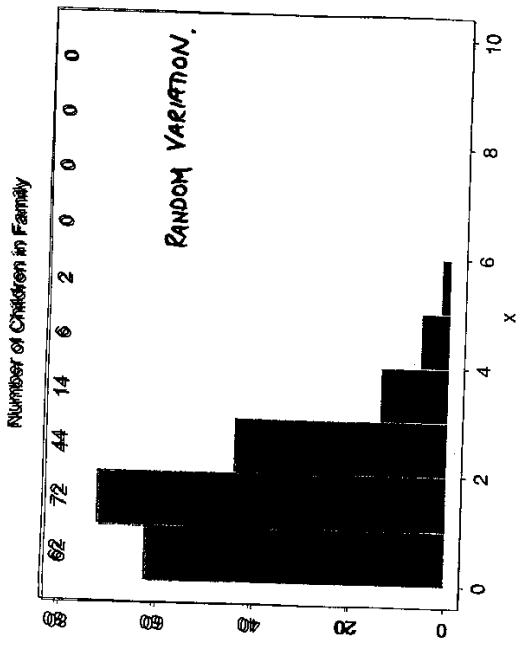
Important distinction :

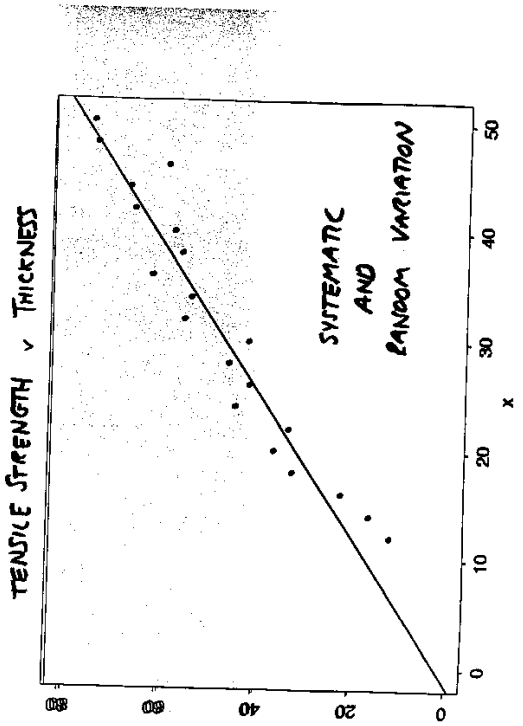
"SYSTEMATIC" — DETERMINISTIC, PREDICTABLE

OR

"RANDOM" — STOCHASTIC, UNPREDICTABLE

variation ?





We will use probability for the following purposes:

SUMMARY

COMPARISON

VALIDATION

PREDICTION

EXAMPLE

COMPARISON OF TWO DRUG TREATMENTS

	CURED	NOT CURED	TOT.
DRUG A	19	11	30
DRUG B	270	183	453

IS DRUG A BETTER THAN DRUG B?

Table 10.14 Spectacle wearing among juvenile delinquents and non-delinquents who failed a vision test (Weindling *et al.*, 1986)

		Juvenile delinquents	Non-delinquents	Total
Spectacle wearers	Yes	1	5	6
	No	8	2	10
	Total	9	7	16

EXAMPLE

SURVIVAL AFTER HEART SURGERY

Heart transplant patients survive for time T after surgery

Can T be predicted from

PATIENT AGE / SEX
EMERGENCY STATUS
SURGICAL PRACTICE ?

Is it possible to optimize these factors?

EXAMPLE

RELIABILITY

'Structural Integrity"

- The resistance to failure of a component

Objective : to predict a "safe" working lifetime under different operating conditions.

[e.g. material composition, temperature exposure to corrosive elements]

RECENT "REAL-WORLD" EXAMPLES.

- Health / Education League Tables
- Comparison of Heart Surgery Techniques [BRISTOL ROYAL INFIRMARY INQUIRY]
- Cancer screening
- Risk Assessment [BSE/CJD, Mobile Phones MMR]

EXAMPLE

PREVALENCE OF LUNG CANCER

1982-1988 ≈ 900,000 patients in a US study

Attempt to relate

CHANCE OF DYING FROM LUNG CANCER

to factors

AGE, SEX, SOCIAL STATUS, NUMBER OF YEARS SMOKED, AGE STARTED SMOKING.

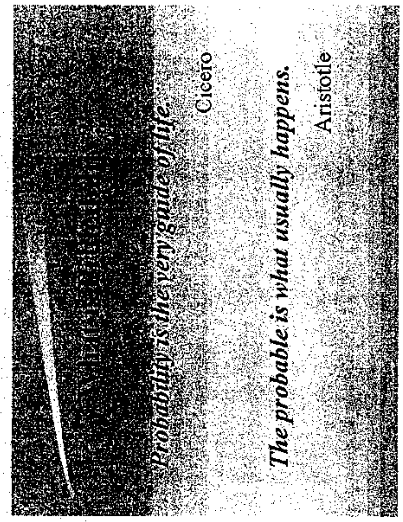
VERY IMPORTANT QUESTION

WHAT IS PROBABILITY?

- not asking "What is the probability of...?"

but rather

"Can you describe what 'probability' is?"



Welcome to the Home Page of J. Orlin Grabbe

. . . inspecting the global underbelly: privacy, money laundering, espionage.

Probability is a One-Pound Jar of Jelly

Probability is a one-pound jar of jelly. You take the jelly. . .

Adolphe Quetelet

Adolphe Quetelet

What is Probability

Few desires have proved as strong as humanity's wish to know the future course of events, so as to avoid or minimise risk. Modern European cultural history is in large measure the story of how the concept of probability has been employed with the aim of increasing the sum of human happiness. [By probability, we generally mean the likelihood of a particular event occurring, given a particular set of circumstances.]

[The probability of an event is generally expressed as a quantitative measurement. In other words, when we talk about the probability of an event occurring, we use a number to signify how likely it is the event will occur.]

Theory of Probability

A critical introductory treatment

Volume 1

BRUNO DE FINETTI

Professor of the Theory of Probability at the University of Rome

PROBABILITY DOES NOT EXIST.

We will study the mathematical properties of probability

(with illustrations using simple mechanical examples
COINS, DICE, CARDS, LOTTERY...)

rather than be concerned with precisely how it is defined.