1 These errata.

When Serre heard that the London Mathematical Society were going to reprint this book, he urged them to make a correction to his Ugly Lemma (a correction already indicated in his collected works). I was independently motivated to try and compile a list of errata for the book (despite having read the claims in the original preface of the source of the inaccuracies!), because I had once spent two hours stuck with a non-commutative commutative diagram, the error in which I finally traced back to a slip in the first printing of the book. It occurred to me that several of my colleagues the world over may well know of other errata, and that perhaps one could use the internet to try and gather slips known to various other people. I posted a request for errata on the number theory mailing list `NMBRTHRY@listserv.nodak.edu`, at the website `mathoverflow.net`, and I also individually approached (via email) several individuals whom I knew to be careful readers. Ultimately I would like to thank J.-P. Serre, R. Schoof, K. Conrad, B. Conrad, H. Lenstra, J. Oesterlé and R. Bellovin, each of whom provided a list which was either substantial or contained at least one gem, and I would like to thank the many other people who provided me with smaller lists. All are documented below, and, of course, well over 95 percent of them are utterly trivial.

As far as I know, hardback version of the book (published in 1967 and again in 1969) and the paperback versions (1986, 1987, 1990) are the same (that is, no corrections to the text were made in the mean time). The errata below hence apply to both editions. Notation: "px ly" means page x, line y from the top (not counting running headers) and "px l-y" means y lines up from the bottom of page x (counting lines in footnotes, and counting equations as one line).

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2 Corrections.

This section consists of mathematical slips that may confuse or mislead the reader (if they are learning the theory from the book).

p3 Proposition 1: The proof of the proposition is in Chap. II §5, not §10, and furthermore what is proved there is the uniqueness of the completion, rather than the uniqueness of the extension (which is also true, but is never used in the book).

p45 l5: The right hand side of the equation should be $n+1$ not n.

p52 l-15: K should be V.

p52 l-12: the subscripts should be outside the norm signs in the second equation.

p53 two lines above section 9: the last subscript n should be an N in the first norm on the right hand side of the equation.

p56 first line of Theorem: second k should be $||$.

p69 l-15: it should say "(i) S contains all the v with $|\alpha_v|_v \neq 1$" (note that the footnote can be used to check that this set is finite).

p69 l-14: $|\xi_v|_v < \frac{1}{2}C$ should be $|\xi_v|_v < \frac{1}{2C}$.

p75 l1: (19.9) should be (19.10).

p78 l6: b_{ij} should be b_{1j}.

p78 equation (A.24): the second $N_{V/P}$ should be $N_{V/R}$, and similarly two lines later.

p79, line -8: change "$\beta_1, \beta_2, \ldots, \beta_m$" to "$\beta_1\beta_2\ldots\beta_m$".

p83 equation (C.3): $|f(\alpha)|/|f'(\alpha)|$ should be $|f(\alpha_0)|/|f'(\alpha_0)|$.

p88 l-11: $(1-\zeta^i)$ should be $(1-\zeta)^i$.

p88, l-10: change "$i = 1, \ldots$" to "$i = 0, 1, \ldots$"

p88 Lemma 6: the discriminant of a number field is typically viewed in the book as an ideal, rather than an integer, so statements about discriminants of cyclotomic fields here should be viewed "up to sign".

p90 Lemma 2: we should assume that $[L : K] = n$, and the last clause in the statement of the lemma should say "(b must have order n in $K^\times/(K^\times)^n$)".

p90 Lemma 3: the cyclic extensions should be assumed to have degree n.

p91 l7: the ν should be above the arrow in $L^\times \to L^\times$.

p92 l-6: replace $\mathfrak{o}$ by $\mathfrak{o}_\mathfrak{p}$.

p92 l-2: f_j^* should be g_j^*.

p95 l1: Here the G-action on $\mathrm{Hom}(\Lambda, X)$ is defined by $(g.f)(\lambda) = f(\lambda g)$ (and in particular note that we are not using the definition of the G-action on $\mathrm{Hom}(A, B)$ for A and B left G-modules given earlier in the section).

p98 Replace the second sentence of section 4 by "If A' is a G'-module, we can form the G-module $A = \mathrm{Hom}_{G'}(\Lambda, A')$. We give A the following G-module structure: if $\phi \in A$ then $g.\phi$ is the homomorphism $h \mapsto \phi(hg)$." (the given "left action" is not a left action).

p99 l-17 should be $\mathrm{Inf} : H^q(G/H, A^H) \to H^q(G/A)$.

p99 Equation (4.2) should be $H^q(G, A^t) \to H^q(G, A)$.

p101 l13 the Hom should be $\mathrm{Hom}(\mathbf{Z}[G/H], A^*)$.

p104 both horizontal maps in the displayed commutative diagram should be labelled $\hat{\delta}$.

p106 The left hand side of of equation (7.3) should be $d(f.g)$

p118 Corollary 2, and p119 Corollary 3: the limits here are unwritten but should be over U.

p124 displayed argument at bottom of page: $f(y)^{-1}$ would be a clearer thing to say than $f^{-1}(y)$ (three times).

p126 l-13: UNRESOLVED ISSUE in Proposition 4 proof, first line. The condition $H^1(H, A) = 0$ is equivalent to $H^1(H/(H \cap U_i), A^{H \cap U_i}) = 0$ for all open normal U_i of G. But is it equivalent to what the author claims, which is the vanishing of $H^1(H/(H \cap U_i), A^{U_i})$ for all U_i?

p129, l10: insert ")" before ".".

p130 l1, replace §2.7 by §2.8.

p130 l14, replace §2.5 by §1.5.

p131 last line before Corollary 1, replace §2.7 by §2.8.

p131 Corollary 2 should start "Let L/K be a Galois extension...".

p131 l-10: $H^2(\hat{\mathbf{Z}}, K_{nr})$ should be $H^2(\hat{\mathbf{Z}}, K_{nr}^\times)$.

p133 Proof of Corollary 1, first sentence should be "Note that if L/K is Galois then a less...".

p135, part b) of Lemma 4. Replace $H^q(H, M)$ by $H^q(K/H, M^H)$ and replace $\hat{H}^0(H, M)$ by $\hat{H}^0(K/H, M^H)$.

p135 l-6. Replace "to G/H" by "to H".

p138 Application: one should appeal to Cor. 1 rather than Cor. 2.

p140, line 3 of section 2.3: replace $H^2(G, \mathbf{Z})$ by $H^1(G, \mathbf{Q}/\mathbf{Z})$.

p140 l-8: replace s by s_α.

p140 l-2: replace "Prop. 2" by "Prop. 1". Also, the proof seems a little confused. One defines s'_α to be $(\alpha, L'/K)$, and then checks that $s_\alpha \mapsto s'_\alpha$ follows from the equality of character values (rather than equality of the character values following from Prop. 2).

p141, first line after the first displayed diagram: replace K'^{ab}/K by K'^{ab}/K'.

p144 l-4: change "generated" to "topologically generated".

p145: $G = G_{L/K}$.

p145: In the statement of Proposition 6, replace "of Proposition 3" by "of Proposition 5".

p148 l7: $(X) = \ldots$ should be $f(X) = \ldots$.

p150 l-17: left hand side should be $f \circ \phi^{(p)} - \phi^{(p)} \circ g$.

p151 l13: the last $[a]$ should be $[b]$.

p154 l17: replace the first (or both) r_π by $r_\pi(\omega)$.

p154 l19: strictly speaking one now has to check that the K_π from section 3.6 is the same as the K_π from section 2.8 (but this follows relatively easily from what we have shown already).

p154 line 2 of section 3.8: replace "2.3" by "2.7".

p154 l-8: replace I_K by I'_K.

p156 l3: replace "3.3" by "3.4".

p156 l10: replace β_j by β^j.

p157 first line of section 4.2: L/K should be assumed abelian.

p157 l-2: (χ) should be $f(\chi)$.

p. 162, footnote: the Artin–Tate notes "Class Field Theory" are available from AMS–Chelsea (2008).

p170 l-18: replace "make U^S an arbitrarily small. . ." by "ensure U^S is contained in an arbitrarily small. . ."

p175 displayed commutative diagram: the left hand objects should be $\mathcal{M}^\times = M_w^\times$ and $K_v^\times$.

p175, two lines after the commutative diagram: replace $N_{M/K}$ by $N_{M/K}(J_M)$.

p183 l1: change "Proposition 2" to "Proposition 2.3".

p183 equation (7): replace the second K by K^*.

p190 l-12: the source of ψ_p should be $\mathbf{Q}_p^*$.

p195 line 14: $\beta_2(\mathrm{infl}\, b)$ should be $\beta_2(\varepsilon_2(\mathrm{infl}\, b))$.

p196 l5: the equation should be $\mathrm{Im}(\beta_1) \supset \mathrm{Im}(\mathrm{inv}_1)$.

p207, 2-3 lines below displayed formula for $\psi_{\mathfrak{p}}(x)$: replace "determined by" by "trivial on".

p209 ll11–19: most of the statements about indexes seem to be wrong or at least ambiguous, but the conclusion (the final formula) is correct.

p211 l14: insert "primitive" before "Dirichlet".

p211 l-13: replace "0 or 1" by "1 or 0".

p212 l14: $+c_{\mathfrak{p}}$ should be $-c_{\mathfrak{p}}$.

p214 l15: the definition of c should be $c = [k : \mathbf{Q}]$, and furthermore $g(s)$ should not be taken to be the function given, but rather the function $-L'(s,\chi)/L(s,\chi) - g(s,\chi)$, with $g(s,\chi)$ defined on page 213.

p221 l-4: replace $\mathfrak{P}$ with $\mathfrak{q}$.

p222 l-9: left hand side should be $\chi^*(\mu_0^m)$.

p225 l11: after "can be taken to be rational integers" add "(provided one does not insist that the extensions K/Ω_l be cyclic)".

p236 l5: replace "2.5" by "1.2, Prop. 1".

p242: in the second big displayed diagram, the E_K in the top left should be an E_k.

p353: equation (∗∗) should read $\left(\frac{\lambda}{b}\right) = \prod_{v \in S}(b, \lambda)_v$.

p357 l-14: the left hand side of the equation should be $f(tX + Y)$.

p360, last line of exercise 5.1: replace "4.3" by "4.4".

p366: under "Tchebotarev, N.", also list pages 165 and 227.

3 Typesetting issues and typos.

These are just slips which are unlikely to derail the reader.

p26 l-4: the first *Hom* should not be in italics.

p30 l-16: "the L_1" should be "that L_1"

p33 l18: change $0(h^2)$ to $O(h^2)$.

p33, lines -11 and -14: change Z to $\mathbf{Z}$.

p45 l-8: change $b \in \mathbf{Z}$ to $b \in \mathbf{Z}_{\geq 1}$.

p52 l12: $(-)^{n-1}$ is now usually written $(-1)^{n-1}$.

p52 l-1: the absolute values signs are misprinted.

p53 l2: the absolute value sign on the right is misprinted.

p54 l-5: italicize A (twice).

p55 l7: change $\oplus$ to $\bigoplus$.

p57 l-11: change $\oplus$ to $\bigoplus$.

p60 l2: change $\oplus$ to $\bigoplus$.

p61 l8: the subscript in β_p is too low.

p73 l6: replace "vica" by "vice".

p74 equation (19.14): change $\oplus$ to $\bigoplus$ (twice).

p76 l-6: $(-)^n$ is now usually written $(-1)^n$.

p78 l4: do not italicize *End.* (twice)

p84 l-1: italicize X and put a comma after α.

p85 l-4: Q_p should be $\mathbf{Q}_p$.

p86 l7: Q should be $\mathbf{Q}$.

p86 l16: "numb$f$ering" $\rightarrow$ "numbering", and the f should be moved one line lower onto line 17.

p86 second line of statement of Corollary 2: Q should be $\mathbf{Q}$.

p87 l5: Q should be $\mathbf{Q}$.

p87 l-6: Q_p should be $\mathbf{Q}_p$ twice.

p87 l-4 and l-1: Q_p should be $\mathbf{Q}_p$.

p88 l17: $Z[\zeta]$ should be $\mathbf{Z}[\zeta]$.

p89 l-5: an "automorph" is usually called an "automorphism" nowadays.

p91 l9: the arrow below ν is missing.

p91 l-3: "Chapter, II Section 16" should be "Chapter II, Section 16", but it's not clear what relevance this section has anyway.

p106 l14: $f^o g$ should be $f.g$.

p115 l-3: "follows from (iii)" should be "follows from (ii)".

p118 l-15: change $g \rightarrow (gH_i)$ to $g \mapsto (gH_i)$ (see p. xviii)

p119 l16: change $g \rightarrow (gH_i)$ to $g \mapsto (gH_i)$ (see p. xviii)

p123, last line of section 2.4: C should be C.

p123 lines -8,-9,-10: change ,.., to ,...,

p129 l10: insert another) after $[k : \mathbf{F}_p]$.

p133, Corollary 1, line 2: do not italicize *Br*.

p135, line 6 of lemma 4: replace the second "M)" by "M))".

p139 l14: put ")" before the final ".".

p141 l-15: the q here is (clearly) not the same q as on line 9.

p141 l-8: Z should be $\mathbf{Z}$.

p143, lines -3 to -2: change "Lubin Tate's" to "Lubin-Tate's".

p145 l-1: the subscript in $\mathbf{R}_+^\times$ is cut off a bit.

p147, first line after definition: insert ")" before ".".

p147 l-17: now K is a local field.

p150, proof of Proposition 1: strictly speaking (c) is not proved that way (but it follows from (a), (b), (d)).

p150 l-10: replace "left-and" by "left- and"

p155 l18: target of i_G should be $\mathbf{Z} \cup \{\infty\}$.

p155 l-11: change G^v to G^v.

p157 l9: replace "intertia" by "inertia".

p158 l-4: replace "$K'|K$" by "K'/K".

p168 l5: replace F by F.

p168 l-16: replace C by C.

p173: Title of section 5.6 should be "Number Field Case".

p177 Last symbol on page should be J_L.

p179 l12: insert ")" before the second "=".

p189 l-14: replace "$= 0$" by "$= 1$".

p192 l-11: replace "infiinte" by "infinite".

p197, 6th line of "case $r = -2$": H^{-2} should be $\hat{H}^{-2}$.

p198 l-10: a reference for this "fundamental duality theorem" is Theorem 6.6 in Chapter XII of Cartan-Eilenberg.

p201 l-17: ψ_k should be ψ_K.

p211 definition of $\Phi(s,\chi)$: this definition is out by a non-zero constant from the now standard definition of the completed L-function attached to χ (but of course this does not affect the functional equation).

p211 l-7: replace "*seq*" by "*seq.*"

p214 l13: if $\eta = 0$ then this equation should be interpreted as saying that the sum is $o(x)$.

p215 l6: the right side is missing an l.

p215 l-1: insert "relative to k" after "absolute first degree".

p224 l-4: this is not really "another way of arriving at a contradiction" because because this sort of argument is precisely how one proves linear independence of characters.

p225 lines 1–3: the proposed procedure seems to involve inverting a submatrix that depends on the choice of p (but the Brauer argument mentioned later on makes this remark moot).

p229 l9: the f on the left side is cut off a bit.

p230 l7: The author is using the Prime Ideal Theorem in k as well (and could use the Cebotarev density theorem instead).

p240, lines -2 and -4: change $\to$ to $\mapsto$.

pp. 248–249: change "*math.*" to "*Math.*" in the references.

p249 l5: "*Nauk.*" should be "*Nauk*".

p252 l1: change "sem -simple" to "semi-simple".

p284, lines -8, -11, -12: change $\mathbf{Q}(i)$ to $\mathbf{Z}[i]$.

p290 l8: change "Wood's Hole" to "Woods Hole".

p292 l-16: insert colon after "field".

p293 l17: change "inversible" to "invertible".

p298, Lemma 2: the field diagram is hard to read in parts. The bottom line is from k to k' and is labelled with the letter E.

p303 l14: remove comma after "enough".

p303 l25: remove comma after "difficult".

p303, line -2,-3: the Brumer reference appeared in Michigan Math. J. **13** (1966), 33–40.

p304, lines 9, 10: the abbreviation for the journal should match Shafarevich's second paper on p. 249.

p312 l-7: $L(k^+ - 0)$ should be $L(k^+ - \{0\})$.

p322 l8: in the integral, the bold d is slightly too high.

p333 l14: change $|\ \widehat{f}(\mathfrak{a}\xi)|$ to $|\widehat{f}(\mathfrak{a}\xi)|$.