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Brexit (continued)
5. Will there be a second Referendum (called a 'people's vote' by those in favour of one)? This could be very divisive politically; see below.
6. What will the effects on UK politics be? There could well be crises in several areas:
a. The Conservative Party. This could split, if differences of opinion between 'hard Brexiteers' (led by Rees-Mogg, Johnson etc.), 'soft Brexiteers' (led by the PM) and Remainers prove too great.
b. The Labour Party. This too faces severe political tensions. The great majority of Party members and of Party MPs are Remainers, and so (following the landslide defeat of the Government's Brexit bill in Parliament on Tuesday 15 January 2019, and the defeat of the resulting Labour motion of no confidence in the Government, and so for a general election) tend to favour a second referendum. But:
(i) The North of England was economically devastated by the de-industrialisation under Thatcher in the 1980s, and has not recovered. It is the poorest part of the UK, and has the most to lose from Brexit. But, opinion there has tended to blame the EU for its plight (specifically, competition from EU workers from Eastern Europe, willing to work for lower wages than UK workers). So resentment against a second referendum might be most intense there, in natural Labour country. Most Labour MPs from the area accordingly oppose a second referendum.
(ii) The Labour leader, Jeremy Corbyn, is a life-long critic of the EU (presumably on 'socialist/capitalist' grounds; he is not alone here; so too is e.g. Larry Elliott, the financial correspondent of *The Guardian*), and wants a general election rather than a second referendum. But in the light of the two recent votes and the 'clock running down', most of his senior colleagues, e.g. (Sir) Keir Starmer, want a second referendum, and want the Labour Party to say so and campaign for one. So there are risks of splits for Labour too.
7. The UK dimension. Scotland and Wales voted Remain. Most NI voters did too, but their political representatives in Parliament, the DUP, support the Conservative Government and Brexit. Opinion in England is split: the young and the south favour Remain, the north and the old favour Brexit. The resulting political tensions risk long-term damage to the UK itself.

Stocks v. bonds v. other assets.

Business hates uncertainty. But (above), we all have to live with an unusual and damaging amount of uncertainty at present. Most big UK firms are having to spend money and management effort to 'insure' against damaging effects of a possible Brexit.

The global financial/economic outlook is complicated, not just by Brexit-related issues, but also by the US-China trade war. The US is the largest economy in the world, the 'home of capitalism', and is now led by a President whose mottos are 'Put America first' and 'Make America great again (MAGA)'. China is the second-largest economy, is 'capitalist economically, but Communist politically'; it remains to be seen how stable this combination proves over time. The rate of expansion of the Chinese economy in recent decades is not sustainable (exponential growth is unsustainable, if only on environmental grounds). A recession in China would have damaging effects world-wide. The UK economy would be particularly vulnerable, because of the uncertainties above.

The outlook for *stock* is depressed by the political uncertainties above. The outlook for *bonds* is depressed by the historically low interest rates in the decade since the Crash of 2008. So, investors may turn to other assets, such as *gold* and *oil*.

Gold is a traditional 'hedge asset' in times of uncertainty such as the present. Its value lies in being rare, and chemically inert (so does not degrade), rather than any objective use-value.

Oil is a particularly complicated case, as oil prices are governed by several different kinds of pressures. (i) Limited supply. The world cannot replace its fossil fuels, which have a finite supply; the danger of exhaustion is a standing threat. (ii) Technological innovation and change. Recent years have seen a move away from petrol/diesel-powered vehicles to electrically powered ones. Against this, the US has increased production of oil from oil shale (at alarming environmental cost). (iii) Geo-politics. The tensions between the US, Saudi Arabia (and other oil-producing countries in the Arab world) and Russia introduce the complexities of a 'triangular game', with a major impact on world economics.

Globalisation.

Globalisation as we know it began to develop in the 1980s and 90s, partly under the influence of the collapse of communism (and so the end of the Cold War) and the development of the Internet (enabling information, and

money, to be transferred anywhere in the world, in bulk and nearly instantaneously). As with all great changes, there are advantages and disadvantages, and winners and losers. One consequence, of course, is the increasing internationalisation of UK universities, both in students and in staff (I like to play the game of 'Spot the Englishman' in Mathematics seminars here in London).

Prominent losers from globalisation in the UK, US and elsewhere (Germany, France, Italy etc.) have been workers in traditional industries. The UK pioneered the Scientific Revolution (Newton, etc.) in the 17th C., the Agricultural Revolution (Jethro Tull and the seed drill, etc.) and the Industrial Revolution (James Watt and the steam engine, etc.) in the 18th C. The UK pioneered the De-industrialisation Revolution under Thatcher in the 1980s. Similar changes took place in the US under Reagan (Thatcher, elected 1979, and Reagan, elected 1980, were 'political bed-mates'). The result has been the emergence of the *rust belt*: areas of formerly industrial areas, which have lost their industry; factories decay and rust, etc. Lost manufacturing jobs have largely not been replaced (except by unskilled, temporary, low-paid jobs in call centres and the like). The result has been growing political resentment by those who have felt 'left behind'. While Thatcher and Reagan were on the right, political leaders of the left and centre who succeeded them tended to 'buy into' their legacy. For example, Blair (UK, Labour, in power 1997-2007), Clinton (US, Democrats, 1992-2000) and Schröder (Germany, SDP, 1998-2005) were all enthusiastic globalisers. This left them and their parties vulnerable to being outflanked by those (on the left or right) who promised to protect them and their jobs from globalisation — usually by imposing tariff barriers on foreign imports, and/or restricting immigration (to reduce competition for jobs).

It is clear with hindsight that this was the crucial factor in the victory of Trump (Republican) over Hilary Clinton (Democrat) in the 2016 US presidential election (though recall that Clinton had over 3 million more votes than Trump; Trump was elected because of the US Electoral College system). This has had many negative effects — but, it has forced a re-think by conventional politicians of all persuasions (Trump is unconventional) on the issues concerning globalisation that led to the present political situation.

The Environment and Climate Change.

Thirty-odd years ago, the environment and climate change were not considered major political issues; now they are. One turning-point was the

launch in 1992 (at the Rio Earth Summit) of the United Nations Framework Convention on Climate Change (UNFCCC). The scientific consensus nowadays is that climate change is real, and that emission of greenhouse gases (such as carbon dioxide, from burning coal in power stations, etc.) must be reduced. Global temperatures are known to have risen since the Industrial Revolution; whereas such things as Ice Ages occur naturally, the consensus is that much or most of this increase is the result of human activity. The scientific consensus is that major changes in how we live are necessary if the Earth is to remain habitable in the future. There are respectable scientists who are sceptical about this (not many; check out the Internet if you want more here). More disturbingly, some politicians deny the existence of climate change (whether through scientific ignorance, political advantage, etc.) The most disturbing example is the policy of the Trump administration in the US. Federal employees are forbidden to use the phrase 'climate change'; instead, they must refer to 'extreme weather events'.

These things matter to us in this course! Think for example of the devastating forest fires recently in California. Trump's first reaction was to deny that climate change was responsible, blame 'poor forest management' instead, and threaten to cut off Federal aid. Now that the death toll is high and rising (over a thousand people are missing), this has changed (California has been designated a disaster area; the President has visited the devastated town of Paradise CA and expressed sympathy, etc.) Recall (MATL480, Ch. VII) Insurance). You are invited to consider the effect on future insurance in California. There are serious questions on insurability (fire insurance, house insurance), and habitability. These could (and probably will) be enormously expensive. There may well be great changes, in the insurance and re-insurance industries here, and in the population of California and its distribution. In addition to that, think of the housing market in California, the associated mortgage market, and the effect on mortgage interest rates there.

4. History

History: Interest rates

There is a good account of the history of interest, from antiquity to modern times, in James & Webber [JW, Ch. 2]. As part of their conclusions, these authors, writing in 2000, conclude (p.37) that 'there is no such thing as a risk-free [interest] rate'. This conclusion is confirmed by more recent events, and particularly the Crash of 2007/8 and its aftermath; see below.

Nevertheless, we shall study *risk-free interest rates* for much of this course. One has to learn to walk before one learns to run $\cdots$.

History: Options

Options go back to antiquity. The first mathematician for whom we have a result named after him was the ancient Greek Thales (theorem of Thales: an angle in a semi-circle is a right-angle). Thales is also the 'father of options': around 580 BC, Thales bought options on the future use of olive presses (for making olive oil). When there was an abundant olive crop (as he had predicted), and presses were in high demand, he made a fortune. (Thales is also considered the father of the sciences and of western philosophy.)

Moving forward to the modern world:
Louis Bachelier (1870 - 1946) was the first to introduce Brownian motion into finance, to study option prices (hence the name Bachelier Society for one of the main societies in mathematical finance).

More recently, subject to the assumptions of an idealized market (no arbitrage, etc.), Fischer Black (1938-1995) and Myron Scholes (1941-) derived their formula of 1973 by showing that the option price satisfied a partial differential equation (PDE), of parabolic type (a variant of the *heat equation*). In 1973 Robert Merton (1944-) gave a more direct approach. Meanwhile, 1973 was also the year when the first exchange for buying and selling options opened, the Chicago Board Options Exchange (CBOE).

The Black-Scholes formula led to an explosive growth in financial derivatives, used nowadays by banks and companies world-wide. This mathematical result has contributed to creating a vast new market: the derivatives market worldwide has reached 708 trillion dollars (US GDP: 15 trillion).

Subsequent events:
1997, Nobel Prize for Scholes and Merton (Black, the genius, had died);
1998, Long Term Capital Management, a US hedge fund, had to be bailed out, with huge losses (Scholes and Merton were both on the board of LTCM at the time);
2007/2008 crisis (ongoing): within a month in 2008: Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landbanki, Glitnir and Kaupthing, Merrill Lynch (and in the UK, Northern Rock).

Following all this, the bond market broke up, and interest rates that used to be very close to each other and were used to model risk-free rates for different maturities started to diverge.

History: The Business Cycle

The traditional view here is that when the economy was expanding – 'boom', with demand and activity increasing – firms would compete for labour, wages would rise, costs would rise, prices would rise, inflation would rise. The central bank – Bank of England (BoE) in UK – would *increase* interest rates – Bank rate – to make borrowing money more expensive. This would decrease the demand for borrowing by business, and the economy would contract. By contrast, when the economy was contracting – 'bust', or 'slump' – the Bank would reduce interest rates, to make it cheaper for business to borrow. This would have the effect of making business expansion cheaper; businesses would tend to expand. The expansion, once under way, would tend to overshoot the natural mean position, leading to the next expansionary phase and the next business cycle. There is a good deal of theory on such business cycles. However, since 2007/08 the economy has been consistently flat. In an effort to promote growth, the authorities have held interest rates at historically low levels for long periods. In the UK, bank rate is now 0.75%, up from 0.25%, unprecedentedly low. The authorities have also resorted to unconventional monetary measures, such as *quantitative easing (QE)*, usually described informally as creating electronic money. This has had the desired effect of moving the economy back towards normal, from the crisis of the Crash and its immediate aftermath. But, QE has had undesirable and unpredicted effects. In particular, it has led to a large increase in asset prices. This had benefited those who hold assets – principally, the already affluent. This has widened the gap between the rich and the poor, decreasing social mobility and increasing social and political tensions. In addition, low interest rates have penalised savers. This is both unfair to them, and undesirable nationally: we suffer from an excess of consumer indebtedness, so saving should be encouraged.

History: The Crash of 2007/08 and after – and before $\cdots$

The Crash has changed our view on all sorts of things – economic, financial, political, and indeed *interest rates*, the subject of this course! Some of my views were published in my paper (available in the journal, and on my homepage, under the link to Papers):
[B] N. H. BINGHAM, The Crash of 2008: A mathematician's view. *Significance* **5** (2008), 173-5, MR2654655.

Persistent depression

The major western economies have been very slow to recover from the Crash of 07/08. This is not unprecedented: the Japanese economy has had similar – and worse – experiences. Japan was devastated during World War II. After it, and American occupation, the Japanese economy experienced an 'economic miracle', similar to that in Germany (the *Wirtschaftswunder*). From the late 50s to around 1990, Japan had a dominant position in several areas of manufacturing: ship-building (especially oil tankers and supertankers), steel, cars, electronics (from transistor radios on), etc. There was then a financial crisis – with hindsight, perhaps a precursor of the western Crisis in 2007/08, which involved an asset-price bubble – bubbles burst! The economy was stagnant throughout the 90s, which were described as Japan's lost decade. But things have been little better later (lost decade*s*). Another major factor has been *globalisation*. How to cope with such persistent depressions is the source of ongoing economic and political debate, and controversy (Brexit, Trump, etc.)

Over-reaction, and 'getting stuck'.

Markets typically over-react, once they start to react. This is (at least in part) a reflection of the two things that, as is well known, really get markets moving: *fear* and *greed*. These are both quite natural; neither brings out the best, in people or in institutions.

As for 'getting stuck': it is common for a serious slump to take a very long time to recover from. We have our own post-Crash experience; we have the Japanese experience in the 1990s and 2000s (above). Further back, we have the US experience of the Slump (or Depression), following the Wall Street Crash of Tuesday 29 October 1929. This scarred the American psyche (and our own) so badly that it is still remembered:
books – read John Steinbeck, *The Grapes of Wrath* (if you haven't read it already – if you have, re-read it);
songs – listen to *Buddy, can you spare a dime* (Bing Crosby, 1932), on YouTube), etc.
It led to the election of Franklin D. Roosevelt as US President in 1932; his New Deal helped the US economy to recover partially. But what really kick-started the US economy, and cured the Slump, was World War II, and its massive demands for munitions etc. – for which we have to thank (if that is the word) the Japanese attack on Pearl Harbor (7.12.1941).

The US economy had its Crash in 2007-8, and has not fully recovered. Hence the resentment by those who feel excluded, which led to the 2016 elec-

tion of Trump as President.

5. Assumptions

Multiple curves
LIBOR

This is the *London Inter-Bank Offer Rate* – the rate at which banks lend to each other (at various maturities). This is set by taking the average of quotes from the participating leading banks, and used to be considered reliable. However, there has been illegal market manipulation (price-fixing – the so-called *Lie-bor scandal*). See e.g., in addition to [VF1] (1a)
[VF2] Liam VAUGHAN and Gavin FRENCH, How bankers fixed the world's most important number. The long read, *The Guardian*, 18 January 2017.
Overnight Indexed Swaps (OIS).

These were introduced in the mid-90s. Maturities range from 1 week to 2 years or longer. They are based on the *overnight rates*, used by banks to lend to each other for a day or two. These are harder to manipulate than LIBOR (some are quoted by central banks), and as the loan period is short there is little credit risk.

Nowadays – still in the aftermath of the Crisis of 2007/8 – it is no longer realistic to ignore credit risk and liquidity effects in interest-rate modelling – in effect, pretending that there *is* a risk-free rate governing the LIBOR and inter-bank markets. OIS is a partial solution, as it is the best proxy for a (non-existent) default- and liquidity-free interest rate. But some credit-risk and liquidity effects remain, and show up especially in stress-testing under strong stress scenarios.
SONIA.

This is the Sterling Overnight Index Average. See I.2. Since April 2018, it is now used by the Bank of England, and preferred by them to LIBOR.

We will return to these problems – credit-risk and liquidity effects – later (VI). But next, we need to consider the classical theory.

We can think of multiple curves in interest-rate theory as analogous to multiple prices – bid-ask spread – in incomplete markets. Although real markets are incomplete, so real prices are not unique, we deal with complete markets first (we learn to walk before we learn to run!), as we did in MATL480.

So until further notice, we assume:
no credit risk; no liquidity risk; no multiple curves.

Terminology.

We are going to assume that there is *one* interest-rate curve (no multiple curves – the assumption above). But, there are competitors! Which shall we choose? Which shall we use?

In V.3, we talk about the LIBOR Market Model (LMM). In V.4, we talk about the Swap Market Model (SMM), and *Black's swaption formula*of 1977 (the analogue here of the Black-Scholes formula of MATL480). As usage has not standardised yet, we shall stick to these names here. If practitioners start to use 'SONIA Market Model' instead, we will switch, and talk about the SONIA and Swap Market Models, SOMM and SWAM.

Note on terminology. It sometimes happens that things no longer in use survive in ordinary speech. For example, 'to turn on a sixpence' means to turn (a car, say) in a very small space. Before sterling decimalised in 1971, money was "LSD" (Latin: libra, solidus, denarius): it had pounds (librae), shillings (solidi, 20 to the pound) and pence (denarii, 12 to the shilling), $£m/n/0$. The smallest silver coin was the sixpence, hence the expression.

6. Are interest rates positive?

There are two good reasons *not* to lend money:

(a) it deprives one of the use of one's own money (for the duration of the loan);

(b) there is the possibility that one may not get it back – of default (Ch. VI).

Of course, it is common to lend to a family member, close friend etc. – human beings are social animals. But to lend to an unknown stranger, in a business environment, is quite a different matter. One will need some *inducement* to do so – and this is where *interest*, and *interest rates*, come in.

In an extended slump, business activity is flat, and dangerous. So it is dangerous for banks to lend to business (which is what is needed to kick-start things), because of the risk of default (VI). It is much safer for banks to lend to the central bank (Bank of England here), effectively, to the Government – as this will not default. So it is perfectly reasonable for the central bank to *charge banks for looking after their money* – to punish them for not lending it to business, or to induce them to do so.

Inflation and deflation

With Bank Rate so low – at 0.75 %, after years at 0.25 %, ridiculously low by historical standards – and in view of frictional costs, real interest rates

are effectively negative. This is potentially very dangerous, because of the risk of *deflation*: if prices are falling, people may defer buying till later, to get things cheaper; the economy will then freeze up even worse, exacerbating the whole problem ... . Negative interest rates have actually happened ...

The damaging effects of *inflation* are well-known (e.g., the hyper-inflation in Germany and Austria post-WWI devastated their economies, and so their societies and political systems, and paved the way to the rise of Nazism, so to WWII). Governments and central banks need to steer a middle course between these two! – *media via tutissima* (the middle of the road is the safest).

7. Econometrics; macroeconomic policy

Economic data typically arrive at regular time-intervals – monthly, quarterly or annually. Statistical data of this kind, where *time* is crucially relevant, are known as *time series (TS)*, widely studied in Statistics (see e.g. the SMF (Statistical Methods for Finance) link on my homepage, Ch. V). They are the principal data sources used by government (Treasury) and central banks (BoE) to determine macroeconomic policy – e.g., to steer a middle course between the opposing dangers of inflation and deflation (above). One of the key concepts here is *cointegration*. Cointegrated series are series that move together, and commonly occur in economics. These concepts arose in econometrics, in the work of R. F. ENGLE (1942-) and C. W. J. (Sir Clive) GRANGER (1934-2009) in 1987. Engle and Granger gave (in 1991) an illustrative example – the price of tomatoes in North Carolina and South Carolina. These states are close enough for a significant price differential between the two to encourage sellers to transfer tomatoes to the state with currently higher prices to cash in; this movement would increase supply there and reduce it in the other state, so supply and demand would move the prices towards each other.

Engle and Granger received the Nobel Prize in Economics in 2003. The citation included the following: "Most macroecomomic time series follow a stochastic trend, so that a temporary disturbance in, say, GDP has a long-lasting effect. These time-series are called non-stationary; they differ from stationary series which do not grow over time, but fluctuate around a given value. Clive Granger demonstrated that the statistical methods used for stationary time series could yield wholly misleading results when applied to the analysis of nonstationary data. His significant discovery was that specific

combinations of nonstationary time series may exhibit stationarity, thereby allowing for correct statistical inference. Granger called this phenomenon cointegration. He developed methods that have become invaluable in systems where short-run dynamics are affected by large random disturbances and long-run dynamics are restricted to economic equilibrium relationships. Examples include the relations between wealth and consumption, exchange rates and price levels, and short- and long-term interest rates."

Spurious regression.

Standard least-squares method work perfectly well if they are applied to *stationary* time series. But if they are applied to *non-stationary* time series, they can lead to spurious or nonsensical results. One can give examples of two time series that clearly have nothing to do with each other, because they come from quite unrelated contexts, but nevertheless have a high value of R^2. This would normally suggest that a correspondingly high propertion of the variability in one is accounted for by variability in the other – while in fact *none* of the variability is accounted for. This is the phenomenon of *spurious regression*, first identified by G. U. YULE (1871-1851) in 1927, and later studied by Granger and Newbold in 1974. We can largely avoid such pitfalls by restricting attention to stationary time series, as above.

From Granger's obituary (*The Times*, 1.6.2009): "Following Granger's arrival at UCSD in La Jolla, he began the work with his colleague Robert F. Engle for which he is most famous, and for which they received the Bank of Sweden Nobel Memorial Prize in Economic Sciences in 2003. They developed in 1987 the concept of cointegration. Cointegrated series are series that tend to move together, and commonly occur in economics. Engle and Granger gave the example of the price of tomatoes in North and South Carolina .... Cointegration may be used to reduce non-stationary situations to stationary ones, which are much easier to handle statistically and so to make predictions for. This is a matter of great economic importance, as most macroeconomic time series are non-stationary, so temporary disturbances in, say, GDP may have a long-lasting effect, and so a permanent economic cost. The Engle-Granger approach helps to separate out short-term effects, which are random and unpredictable, from long-term effects, which reflect the underlying economics. This is invaluable for macroeconomic policy formulation, on matters such as interest rates, exchange rates, and the relationship between incomes and consumption."

Endogenous and exogenous variables.

The term 'endogenous' means 'generated within'. The $ARCH$ and $GARCH$ models used by Engle and Granger show how variable variance (or volatility) can arise in such a way. By contrast, 'exogenous' means 'generated outside'. Exogenous variables might be the effect in a national economy of international factors, or of the national economy on a specific firm or industrial sector, for example. Often, one has a vector autoregressive (VAR) model, where the vector of variables is partitioned into two components, representing the endogenous and exogenous variables.

Discrete and continuous time.

While econometric data arrives discretely (monthly trade figures, daily closing prices for stocks, etc.), continuous time is more convenient for dynamic models of the economy. See e.g.
A. R. BERGSTROM: *Continuous-time econometric modelling*, OUP, 1990.

Our most powerful weapon, in mathematics, and so in science, is *calculus*. Calculus is about *limits*, and so involves a *continuous* variable, drawn from the *real numbers*, $\mathbb{R}$ (or d-space $\mathbb{R}^d$, or the complex numbers $\mathbb{C}$). But when we *calculate* numerically (at least on any scale), we use *computers*. Computers work *discretely* (they deal with real numbers in binary form, truncating the fractional part, as we truncate decimals and work only to so many decimal places). So we constantly go back and forth between these two settings, discrete and continuous, as we pass between computing and (e.g.) calculus. This is why in MATL480 we had two chapters each on discrete time and continuous time — we need both, both in practice and conceptually.

The development of the Internet and fibre-optic cables (which, using photons rather than electrons, transmit information at the speed of light — the theoretical maximum, by Relativity) has pushed the distinction between the discrete and continuous to the limit, in *high-frequency trading*. This is an area in its own right, and there is now a specialist journal, *High Frequency*. Of course, trading done at such high speed has to be automated, and *algorithmic trading* is very important nowadays. The relevant academic area is *Machine Learning*, of great and growing importance.