M5F22 Mathematical Finance MSc: EXAMINATION 2016-17

Six questions, do five; 20 marks per question

Q1. In the context of insurance, comment briefly on:
(i) limited liability;
(ii) reinsurance;
(iii) regulation;
(iv) lender of last resort.

Q2. (a) Define volatility.
(b) Comment briefly on: historic volatility; implied volatility; the volatility surface.
(c) How do option prices depend on volatility, and why?
(d) Discuss the effect of trading volume on volatility, and its implications for market stability.

Q3. In a (Cox-Ross-Rubinstein) binomial-tree model with discount rate $1+\rho$ per period, 'up' and 'down' factors $1+u$, $1+d$ and 'up' and 'down' probabilities q, $1-q$, find the condition for q to be the risk-neutral probability.

Describe how to price an American put with strike K in an N-period binomial-tree model.

What is the connection here with the Snell envelope?

Q4. (i) State without proof the optional stopping theorem.
(ii) In what circumstances does the conclusion of the theorem not hold?
(iii) In what circumstances does one encounter uniformly intregrable martingales?
(iv) Describe briefly optimal stopping applied to American puts with finite time-horizon.
(v) What happens if the time-horizon is infinite?

Q5. (i) If $B = (B_t)$ is Brownian motion and θ is a parameter, show that $M = (M_t)$, with

$$M_t := \exp\{\theta B_t - \frac{1}{2}\theta^2 t\},$$

is a martingale. (You may quote that the moment-generating function of $N(\mu, \sigma^2)$ is $\exp\{\mu t + \frac{1}{2}\sigma^2 t\}$.)

(ii) If Y has distribution $N(\mu, \sigma^2)$ and $X = e^Y$, X has the *log-normal distribution* $LN(\mu, \sigma^2)$. Find $E[X]$.
(iii) Show that in the Black-Scholes model, prices are log-normally distributed.
(iv) What is the relevance of (i) to the use of Girsanov's theorem when deriving the Black-Scholes formula in continuous time?

Q6. (i) Define the Poisson process $N = (N_t)$ with rate λ, and the compound Poisson process $S = (S_t)$ with rate λ and jump-distribution F, $CP(\lambda, F)$.
(ii) For $CP(\lambda, F)$, find the characteristic function of S_t.
(iii) Find the mean and variance of S_t, when F has mean μ and variance σ^2.
(iv) Show that with λt large, S_t is approximately normally distributed.

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