

**M3F22 PROBLEMS 6. 17.11.2017**

Q1 (*The doubling strategy*).

Analyze the ‘doubling strategy’ (“the original martingale”): when betting on tossing a fair coin, respond to losing by doubling the stakes.

- (i) Show that an eventual win is certain.
- (ii) Show that this certain gain is 1.
- (iii) Show that the mean time to this win is 1.
- (iv) Explain why this is an impossible (or at least suicidal) strategy to follow in practice.

*Note.* 1. It ought not to be possible to ‘make something out of nothing’ in this way – and it isn’t.

2. There are those who respond to confrontation by ‘upping the ante’ in a similar way. You might care to ponder the implications of this as a life strategy.

Q2. *Simple random walk (SRW)*.

We bet on independent tosses of a fair coin. Our strategy is to bet until we are first ahead, and then quit. Let  $T$  be the time we quit (a stopping time). Show that:

- (i)  $T$  has PGF

$$E[s^T] = \frac{1 - \sqrt{1 - s^2}}{s}$$

(hint: look at what happens if we play until we are 2 ahead for the first time);

- (i)  $T < \infty$  a.s.: we are certain of achieving a net gain of +1 eventually;
- (ii)  $ET = +\infty$ : the mean waiting time till this happens is infinite;
- (iii) this strategy too is impossible in practice.

*Recommended Reading:* Grimmett & Stirzaker [GS], §5.2 (Bingham & Kiesel Ex. 3.4).

NHB