## M3F22/M4F22/M5F22 EXAMINATION 2016-17

Q1. Discuss the concept of limited liability, with reference to, e.g., bankruptcy and moral hazard.

Q2. In a two-period binary model, at each node the stock goes up by a factor of 5/4 or down by a factor of 4/5, each with positive probability. The payoff is $(S-8)_+$, with $S$ the final stock-price; the initial stock-price is 8. Neglect interest.
(i) Find the martingale probability $p^*$ that the stock goes up.
(ii) Working down the tree, find the value of the option at each node.
(iii) Working up the tree, find the hedging portfolio at the time-0 and time-1 nodes.

Q3. (i) Define Brownian motion $B = (B_t)$.
(ii) Find its covariance.
(iii) For $c \in (0, \infty)$, show that the scaled process $B_c$, where

$$B_c(t) := B(c^2 t)/c \qquad (t \geq 0),$$

is again Brownian motion.
(iv) Discuss the limitations this imposes on the suitability of Brownian motion as a model for driving noise (or uncertainty) in financial modelling.

Q4. (i) Define volatility.
(ii) Comment briefly on: historic volatility; implied volatility; the volatility surface.
(iii) How do option prices depend on volatility, and why?
(iv) Discuss the effect of trading volume on volatility, and its implications for market stability.

Q5 (Mastery question). Write rho ($\rho$) for the 'Greek' giving the sensitivity of option prices to the (riskless) interest rate $r$. You may quote the Black-Scholes formula for a call price,

$$C_t := S_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2), \qquad (BS)$$

where

$$\phi(x) := e^{-\frac{1}{2}x^2}/\sqrt{2\pi}, \qquad \Phi(x) := \int_{-\infty}^{x} \phi(u) du,$$

$\tau := T - t$ the time to expiry,

$$d_1 := \frac{\log(S/K) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \qquad d_2 := \frac{\log(S/K) + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}.$$

(i) Show that for European calls, $\rho > 0$.
(ii) Give the financial interpretation of this.
(iii) Show that for European puts, $\rho < 0$.
(iv) Again, give the financial interpretation of this.
(v) Do these results extend to American options?

N. H. Bingham