M3A22 MATHEMATICAL FINANCE: MOCK EXAM 2014-15

Q1. (a) Discuss the types of risk facing financial institutions, including: market risk; credit risk; operational risk; liquidity risk; model risk.
(b) Comment briefly on stress testing by financial regulators.

Q2. What is meant by saying that a process $C = (C_n)$ is *previsible* (or *predictable*)?

Define the martingale transform $C \bullet X$ of a process $X = (X_n)$ by a previsible process C, and give its financial interpretation.

Show that if X is a martingale and C is bounded and previsible, $C \bullet X$ is a martingale null at zero.

Q3. (i) Define the *vega* of an option. Given the Black-Scholes formula, show that vega is positive for European call options.
(ii) Show that vega is also positive for European put options.
(iii) Does this extend to American options?

Q4. (i) Give the stochastic differential equation of geometric Brownian motion, and its interpretation in terms of the stock-price dynamics of the Black-Scholes model.
(ii) Show how this stochastic differential equation changes when we discount by the riskless interest rate r.
(iii) Show how it changes further when we apply Girsanov's theorem to change to the equivalent martingale (or risk-neutral) measure.
(iv) What is the role of the representation theorem for Brownian martingales here?

N. H. Bingham