

**M3A22/M4A22/M5A22 MATHEMATICAL FINANCE: EXAM
2014-15**

Q1. We assume throughout that our markets are perfect, or frictionless. This entails a number of assumptions: mention the principal ones, and discuss briefly.

Q2. (i) With C the value of an American call option and c the value of the corresponding European call option, prove Merton's theorem $C = c$: that is, that early exercise is never optimal.

(ii) Give the financial interpretation of Merton's theorem.

(iii) Where and why would you see American options exercised early, despite this?

Q3. The current price of wheat is £ 134 per tonne. Next year, the price will be up to 146 or down to 128, each with positive probability. Neglect interest.

(i) Price a call option C for a tonne of wheat next year, with strike price K the current price 134.

(ii) Hedge this option.

(iii) You see C being traded now for £ 3. What do you do?

(iv) You see C being traded now for £ 5. What do you do?

(v) Who buys such options, and why?

(vi) Who buys the corresponding put options, and why?

Q4. (i) In the Black-Scholes model with riskless interest rate r and one risky stock with mean return rate μ and volatility σ , define the Sharpe ratio λ , and discuss how a fund manager would decide the proportion of his funds to be invested in the risky stock.

(ii) Describe briefly, without proofs, how to pass from the dynamics of the risky stock to the Black-Scholes formula for the price of options on it.

(iii) Why does the Black-Scholes formula not involve μ ?

(iv) The Black-Scholes price does depend on σ , but we do not know it: how does one estimate σ ?

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