M5MF22 MATHEMATICAL FINANCE: EXAM 2014-15

Six questions; do five; 20 marks each

Q1. Comment briefly on:
(i) size of market participants and of trades;
(ii) normal v. crisis market conditions;
(iii) continuous v. jump price processes;
(iv) discrete v. continuous time.

Q2. The current price of copper is \$6,720 per tonne. In a year's time, the price will be up to 6820 or down to 6570, each with positive probability. Neglect interest.
(i) Price a call option C for a tonne of copper in a year's time, with strike price K the current price 6720.
(ii) Hedge this option.
(iii) You see C being traded now for \$ 80. What do you do?
(iv) You see C being traded now for \$ 40. What do you do?
(v) Who buys such options, and why?
(vi) Who buys the corresponding put options, and why?

Q3. Analyze the 'doubling strategy': when betting on tossing a fair coin, respond to losing by doubling the stakes.

Show that this leads to an eventual certain gain. Explain why this does not work in practice as a means of making money.

Q4. Define, and discuss briefly,
(i) filtrations;
(ii) uniformly integrable martingales;
(iii) risk-neutral measures;
(iv) risk-neutral valuation;
(v) insider trading.

Q5. (i) State the Ornstein-Uhlenbeck stochastic differential equation for a process $V = (V_t)$, and interpret the terms that appear in it.
(ii) Solve this equation.
(iii) Obtain the limit distribution of V_t as $t \to \infty$.
(iv) Obtain the covariance function, and find its limit as $t \to \infty$.

(v) Show that V is Markov.
(vi) Explain what is meant by saying that V is mean-reverting, and the relevance of this model to interest-rate theory.

Q6. (i) Give the stochastic differential equation for the price $S = (S_t)$ of a risky asset in the continuous-time Black-Scholes model.
(ii) Give also its solution.
(iii) Describe the sample paths of this solution.
(iv) Write down the formula for the price of a European call with strike K, expiry T and riskless interest rate r. Describe without proof how to deduce the Black-Scholes formula.
(v) Is this model complete? Give reasons.
(vi) Why is hedging a portfolio in a continuous-time Black-Scholes model problematic? (You may quote that Brownian motion $W = (W_t)$ has finite quadratic variation t.)
(vii) How might these problems be circumvented?

N. H. Bingham