m3a22prob10.tex

PROBLEMS 10. 18.12.2015

Q1 *Gaussian distributions.*

With the multivariate normal (multinormal, multivariate Gaussian) distribution as in V.2 W8, show that all linear combinations $\sum_1^n a_i X_i$ of a multinormal random vector are normal (in one dimension). [This is actually the best way to *define* the multinormal.]

Deduce that Itô integrals $\int_0^t f(s) dB_s$ with f continuous and deterministic are normally (Gaussian) distributed.

Q2 *Ornstein-Uhlenbeck process.*

For $V = (V_t)$ the solution to the Ornstein-Uhlenbeck SDE (OU)

$$dV = -\beta V dt + c dB : \qquad (OU)$$

(i) By using the Itô isometry, or otherwise, show that V_t has distribution

$$V_t \sim N(0, \sigma^2 (1 - e^{-2\beta t})/(2\beta)).$$

(ii) By (i) and independence of Brownian increments, or otherwise, show that the covariance is

$$cov(V_t, V_{t+u}) = \sigma^2 e^{-2\beta u}(1 - 2e^{-2\beta t}) \qquad (u \geq 0).$$

(iii) Show that V is Gaussian and Markov.
(iv) Show that V_t converges in distribution as $t \to \infty$, and find the limit distribution. //

NHB