Lecture 6. 23.10.2015

10. Postscript to Ch. I: Systemic issues – "Big-picture stuff" *(not examinable).*

Recall (L0) the three underlying truths about this course (indeed, this subject, at this level): *Anything important enough becomes political. Politics is not an exact science. Mathematics is an exact science.*

Thus, while this course is self-contained as "A22", it is not and cannot be self-contained if we broaden the picture: the big picture is, to some extent, unmathematisable – even in principle. We illustrate this with a few themes.

(i) *Finance and economics.* What ultimately counts – in terms of jobs, people's lives, growing food, making things that people want, etc. – is the *real economy.* Finance is the means (the supply of money) to an end (the production of food, goods, services, creation of jobs, etc.). But, damage to the financial system (the Slump of 1929 and the 1930s, the Crash of 2007/8/...) can be longer-lasting than economic damage. Think of the financial system as the *nervous system* of the economy. A nervous breakdown can disrupt life more than a broken leg, etc. – and have longer-lasting effects (cf. PTSD).

(ii) *Economics: Keynes, Hayek/Friedman, the postwar consensus, the neo-liberal consensus.* The great crisis of world economics 1900-50 was the Slump (or Depression), the misery caused by which was a major cause of WWII. Three factors helped to cure this: F. D. Roosevelt and the New Deal (US, 1930s); J. M. Keynes (books, 1930s); the stimulus to economies of WWII. Keynesian economics (governments should spend their way out of a slump by infrastructure projects etc.) formed the post-war consensus (1945-1979/80). Then came the neo-liberal consensus (politically, Thatcher and Reagan; intellectual underpinning from Hayek and Friedman) – free markets, legal restrictions on trade unions, etc. (and the collapse of communism). The Nobel-prize winning US economist Paul Krugman puts it thus: *big business doesn't like Keynesian economics, as it diminishes its bargaining power.* Do your own digging here, and form your own view. But note that in science, liking something or not has zero effect on whether it is right, or whether it works: a scientist should like what he sees, rather than see what he likes. But in economics this is less true, because big issues involve politics, and so perception, and so psychology. Ordinary members of the public do not even know the names of the economists mentioned above, still less what they did; this limits their ability to make informed decisions about matters of current political controversy, where they see things through the 'distorting prism' of media coverage, partisan political debate, etc. Three comments:

(a) The above underlines the difference in mission between the Imperial College of Science, Technology and Medicine and (say) the London School of Economics and Political Science;
(b) Anyone intending to work in, say, the financial services industry should know *at least as much* about such things as a well-informed member of the public. This cannot be done overnight! One obvious way is to read a decent newspaper regularly over a period of time (or online equivalent).
(c) The profound importance of the issues here can be seen in, e.g., the damage to the life prospects of the young. Even discounting Greece here, youth unemployment has been over 50% in some European countries for long periods since the Crash (and much higher in disadvantaged parts of the community, with predictable consequences for social cohesion, crime etc.).
(iii) *The Euro.* By common consent, the euro was *political* in aim and in origin. It is *financial* and *economic* in substance and in content. There is a tension between these two. This is seen dramatically in the events concerning Greece (Grexit? Brexit?). What is your view on all this?
(iv) *Asset-price bubbles.* The Greenspan years were one long asset-price bubble, which ended in tears. The Chinese economy now has another one ... .
(v) *Quantitative easing (QE).* Since the Crash, governments moved to rescue a banking system at risk of collapse by QE. The idea was to 'create electronic money' for banks to lend to businesses, to free up and kick-start the real economy, so that life could get back to normal. What tended to happen instead was that banks – also under pressure to rebuild their balance sheets – did so by holding onto this new money, rather than lending it as intended. At the same time, interest rates have been at historic lows for long periods – indeed, real (as distinct from nominal) interest rates have often been *negative*, which would have been dismissed as laughable a decade ago. But keeping interest rates at near-zero for long periods has itself had distorting effects – such as fuelling a new asset-price bubble, and so sewing the seeds for the next Crash. How should policy-makers proceed here?
(vi) *Banks: too big to fail? separate investment banking from retail banking?* As Keynes said: if you owe the bank a thousand pounds, it's your problem; if you own the bank a million pounds, it's the bank's problem. Seeming 'too big to fail' (without risking a collapse of confidence as with Lehman Brothers) helped the banks in their dealings with governments post-Crash. The Glass-Steagall Act was passed in the US in 1932 to separate investment (UK: merchant; "casino") banks from retail ("high-street") banks, but this was repealed in 1999. What is the right approach to regulation and legislation here?

Prelude to Ch. II: Integration and area (cf. PfS Lecture 1, SP L1)

We shall mainly deal with area, as this is two-dimensional. We can draw pictures in two dimensions, and our senses respond to this; paper, whiteboards and computer screens are two-dimensional. By contrast, one-dimensional pictures are much less vivid, while three-dimensional ones are harder (they need the mathematics of perspective) – and dimensions higher than four are harder still.

Area.

1. *Rectangles*, base b, height h: area $A := bh$.
2. *Triangles*. $A = \frac{1}{2}bh$.

Proof: Drop a perpendicular from vertex to base; then extend each of the two triangles formed to a rectangle and use 1. above.

3. *Polygons*. Triangulate: choose a point in the interior and connect it to the vertices. This reduces the area A to the sum of areas of triangles; use 2. above.
4. *Circles*. We have a choice:

(a) Without calculus. Decompose the circle into a large number of equi-angular sectors. Each is approximately a triangle; use 2. above [the approximation boils down to $\sin\theta \sim \theta$ for θ small: Archimedes].

(b) With calculus and plane polar coordinates. Use $dA = dr.rd\theta = rdrd\theta$: $A = \int_0^r \int_0^{2\pi} rdrd\theta = \int_0^r rdr. \int_0^{2\pi} d\theta = \frac{1}{2}r^2.2\pi = \pi r^2$.

Note. The ancient Greeks essentially knew integral calculus – they could do this, and harder similar calculations [volume of a sphere $V = \frac{4}{3}\pi r^3$; surface area of a sphere $S = 4\pi r^2 dr$, etc.; note $dV = Sdr$].

What the ancient Greeks did not have is differential calculus [which we all learned first!] With this, they would have had the idea of velocity, and differentiating again, acceleration. Then they might well have got Newton's Law of Motion, Force = mass × acceleration. This triggered the Scientific Revolution. Had this happened in antiquity, the world would have been spared the Dark Ages and world history would have been completely different!

5. *Ellipses*, semi-axes a, b. Area $A = \pi ab$ (w.l.o.g., $a > b$) [Archimedes].

Proof: cartesian coordinates: $dA = dx.dy$.

Reduce to the circle case: compress ['squash'] the x-axis in the ratio b/a [so $dx \mapsto dx.b/a$, $dA \mapsto dA.b/a$]. Now the area is $A = \pi b^2$, by 4. above. Now 'unsquash': dilate the x-axis in the ration a/b. So $A \mapsto A.a/b = \pi b^2.a/b = \pi ab$.

Fine – what next? We have already used *both* the coordinate systems to hand. There is no general way to continue this list. Indeed, I don't know another example of comparable neatness and importance to the above.

The only general procedure is to superimpose finer and finer sheets of graph paper on our region, and count squares ('interior squares' and 'edge squares'). This yields numerical approximations – which is all we can hope for, and all we need, in general.

The question is whether this procedure always works. Where it is clearly most likely to fail is with highly irregular regions: 'all edge and no middle'.

It turns out that this procedure does *not* always work; it works for *some but not all* sets – those whose structure is 'nice enough'. This goes back to the 1902 thesis of Henri LEBESGUE (1875-1941):
H. Lebesgue: Intégrale, longueur, aire. *Annali di Mat.* **7** (1902), 231-259.
Similarly in other dimensions. So: some but not all sets have a length/area/volume. Those which do are called *(Lebesgue) measurable*; length/area/volume is called *(Lebesgue) measure*; this subject is called Measure Theory.

We first meet integration in just this context – finding areas under curves (say). The 'Sixth Form integral' proceeds by dividing up the range of integration on the x-axis into a large number of small subintervals, $[x, x + dx]$ say. This divides the required area up into a large number of thin strips, each of which is approximately rectangular; we sum the areas of these rectangles to approximate the area.

This informal procedure can be formalised, as the *Riemann integral* (G. F. B. RIEMANN (1826-66) in 1854). This (basically, the Sixth From integral formalised in the language of epsilons and deltas) is part of the undergraduate Mathematics curriculum.

We see here the essence of calculus (the most powerful single weapon in mathematics, and indeed in science). If something is reasonably smooth, and we break it up finely enough, curves look straight, so we can handle them. We make an error by this approximation, but when calculus applies, this error can be made arbitrarily small, so the approximation is effectively exact.
Example: We do this sort of thing automatically. If in a discussion of global warming we hear an estimate of polar ice lost, this will translate into an estimate of increase in sea level (neglecting the earth's curvature).
Note. The 'squashing' argument above was deliberately presented informally. It can be made quite precise – but this needs the mathematics of *Haar measure*, a fusion of Measure Theory and Topological Groups.