

Lecture 5. 20.10.2015

The example above is highly instructive. First, it clearly represents the simplest possible non-trivial case: only two time-points (with one time-period between them, hence the ‘single-period’ of the title), and only two possible outcomes (hence the ‘binary’ of the title). Secondly, it shows that there is a theory hidden here, which gives us a definite prescription to follow (and some surprises, such as not involving the ‘real’ probability p above). This prescription is simple to implement, and can be justified by explicitly constructing an arbitrage to exploit doing anything else [if the option is offered for sale too cheaply, buy it, if too dearly, write it]. This theory is the Black-Scholes theory, which we consider in detail in Chapters IV and VI. The technical key to the Black-Scholes prescription is the introduction of p^* and its associated expectation operator E^* . In technical language, this is the *equivalent martingale measure*. Now each of these three terms needs full introduction. We shall talk about measures in II.1 below, about equivalent measures in II.4, and martingales in III.3 and V.2. We stress: the Black-Scholes theory – that is, rational option pricing – cannot be done without all these concepts. This is why we need Chapter II on the necessary background on measure theory, and Chapters III and V on the necessary background on stochastic processes.

There are basically three options open to those teaching, and learning, how to price options etc.

1. One can avoid measure theory altogether (cf. [CR]). This is technically possible rigorously in the discrete-time setting of Ch. III – though at greater length, because the key concepts cannot be addressed explicitly. It is also possible non-rigorously in continuous time; cf. [WHD], who base their approach on partial differential equations (PDE).
2. One can learn measure theory first – say, from the excellent book [W]. This, however, puts the subject beyond the reach of most people who need it and use it in practice – and beyond reach of most of this audience.
3. One can do as we shall do (and as [BK] and a number of other books do): state what we need from measure theory, and use its language, concepts, viewpoint and results, without proving anything. This makes good sense: the constructions and proofs of measure theory are quite hard (say, final year undergraduate or first-year postgraduate level for good mathematics students with a bent for analysis – quite a select group!). Using measure theory taking its results for granted, however, is quite easy, as we shall see.

9. Complements

1. *Types of risk.*

Institutions encounter risks of various types. These include:

Market risk.

This is the risk that one's current market position (the aggregate of risky assets one holds) goes down in value (things one is long on get cheaper, and/or things one is short on get dearer).

Credit risk.

This is the risk that counter-parties to one's financial transactions may default on their obligations.

When this happens, debts cannot be (or are not) paid in full. Usually, payment is made in part, by negotiation between the parties (it may be cheaper to agree a partial repayment than to force the other party into bankruptcy), or by the administrators or liquidators in the case of companies. This raises issues of *moral hazard*, below.

Operational risk.

This is risk arising from the internal procedures of an institution: failure of computer systems for implementing transactions (the failure of the Taurus clearing system on the London Stock Exchange was one example); fraudulent or unauthorised trading made possible by inadequate supervision; etc.

Liquidity risk.

This is the risk that one will be unable to implement a planned or agreed transaction because of lack of cash-in-hand to trade with, and/or willingness to trade. The Credit Crunch of 2007/8 on was caused by banks realising they had piles of toxic debt on their hands (see below), and so did not know what their balance sheets were worth; that other banks were similarly placed; hence that banks no longer trusted themselves or each other, and so refused to lend to each other. So the financial system froze up; so the real economy froze up.

Model risk.

To handle real-world phenomena of any complexity, one needs to model them mathematically. To quote Box's Dictum: All models are wrong; some models are useful.¹ Use of an inappropriate model to set the prices at which one buys and sells exposes the institution to open-ended losses, to competitors with better models.

2. *Risk management.* The problems of 2007/8 on have made the importance

¹George E. Box, 1919-. British statistician

of risk management obvious. For an excellent book-length treatment, see e.g. [MFE] A. J. McNEIL, R. FREY & P. EMBRECHTS: *Quantitative risk management: Concepts, techniques, tools*. Princeton UP, 2005.

We know from Markowitz that we should have a balanced portfolio, with lots of negative correlation. The danger is *large* losses. These are quantified by the *tails* of the distributions – the joint distribution of our portfolio. The point of diversifying is so that what we lose on the swings we gain on the roundabouts. Two comments:

(a) Whether this works for large losses depends on the tail properties of the joint distribution. It does *not* work if this is normal – as it is in the benchmark Black-Scholes model.

(b) When the whole market is falling – as in a financial crisis – none of the risk-management techniques useful under normal market conditions work.

3. *Moral hazard*. Before the limited liability company, if one defaulted, one was liable to the *whole* of the loss incurred by one's counter-party. This made trading very dangerous (the early traders were called merchant adventurers) – all the more as insurance had not developed by then.²

Limited liability was what made ordinary people willing to undertake the risks of trading, and so paved the way for the development of modern business, commerce, capitalism etc.

The moral hazard here is the possibility of gambling with other people's money (see John Kay's excellent new book, *Other people's money*, L0). If it works, fine. If not, walk away (writing off one's limited liability) and leave them to bear the loss. "Playing Russian roulette with someone else's head" is a rather brutal description of this.

Bankruptcy law varies from country to country, and is too complicated to pursue here. But one sees moral hazard where it concerns us in, e.g.:

(a) start-ups of hedge funds (or, dot-com companies);

(b) aggressive traders – who (for the sake of their bonuses) gamble with their careers – but with other people's money;

(c) credit rating – where the credit rating agencies had a financial incentive to pass as AAA some highly questionable financial asset, etc.

4. *Securitization*. This term covers the drive in recent years to seek out new financial markets by identifying risks that people might want to cover

²Lloyds of London predates limited liability. The Lloyds participants – "names" – had unlimited liability. Many were driven into personal bankruptcy in the Lloyds scandals of the 90s. See Google for the ghastly details.

themselves against, and creating new financial derivatives that can be sold to address this perceived need. These derivatives too could be traded, etc. The upshot was an explosion of trade in increasingly artificial financial products, developed by the R&D departments of the financial institutions. By 2007/8, the leaders of these institutions did not understand these products – could not price them, and could not value their holdings of them (above).

One specific trigger of the US crash in 2007 was the explosive growth in sub-prime mortgages. These were granted to people who would not have qualified as financially sound enough to get a mortgage previously, but who wanted to buy their own house. This new and profitable market proved irresistible to US banks – leading to a great house-price bubble, which burst (as bubbles do) in 2007. The knock-on effects hit the UK in 2008 (Northern Rock, etc.). The real damage of this failure of the financial sector has been its devastating and ongoing consequences on the real economy.

5. *Macro-prudential issues.* As the above illustrates, financial matters are too important to be left to financiers. Proper regulation is vital.

6. *Forwards and futures.* Forwards are agreements between buyer and seller made now, but concerning delivery in the future. They are not traded. Futures are options on things that will come to market in the future (next year's grain crop, for example), and these are traded (extensively). There are good accounts in Hull's books, [H1], [H2].

7. *OTC and exchange-traded contracts.* OTC – "over-the-counter" – denotes a transaction made between an individual buyer and an individual seller. As options on standard transactions develop, these are assets themselves that can be traded in exchanges (e.g., the CBOE, which opened in 1973: I.3).

8. *Marking to market.* This is a system whereby the exchanges cover themselves and their clients against the risk of large losses. If one party to a trade is, on current market prices, exposed to a potentially heavy loss, a *margin call* will be required by the exchange. Margin calls actually trigger many financial failures (but limit the losses of the counter-parties).

9. *Forex.* Forex is an abbreviation for foreign exchange. International trade involves more than one currency' currencies move against each other. There is a vast market in derivatives to cover the risks involved.

10. *Swaps.* From Hull [H2] Ch. 5: "Swaps are private agreements between two companies to exchange cash flows in the future ... The first swap contracts were negotiated in 1981. Since then the market has grown very rapidly. ..." There are even options on swaps – *swaptions* – etc.