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M3A22/4A22/5A22 MATHEMATICAL FINANCE

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Imperial College London, 12 October – 18 December 2015

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Course website: My homepage.

Clore, Mon 12-1, Tue 12-1, Fri 4-5, + 5-6 [MSc in Math. Finance students]

Books.

Course text: Ch. 1-6 of

[BK] N. H. BINGHAM and Rüdiger KIESEL: *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed., CUP, 2004 (1st ed. 1998).

Alternatives:

S. E. SHREVE: *Stochastic calculus for finance. Vol. I: The binomial asset pricing model; Vol. II: Continuous-time models*, Springer, 2004.

T. MIKOSCH: *Elementary stochastic calculus, with finance in view*, World Scientific, 1998.

Course Website: M3A22 link on my home-page (Imperial College > Mathematics Department > Staff > Staff List > Bingham > Homepage: favouritize this to get it in one click).

Other relevant links (on my home-page):

[SP] Stochastic Processes [30 hours, MSc, Mathematial Finance];

[SA] Stochastic Analysis [20 hours, MSc].

[LTCC] Measure-theoretic probability theory [10 hours; MSc].

For background:

[PfS] Probability for Statistics;

[SMF] Statistical Methods for Finance;

[Math482] – a course along these lines I gave at Liverpool.

We shall make systematic use of *conditioning* (informally: using what we know). For background, see e.g.

[BF] N. H. BINGHAM and J. M. FRY: *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS), Springer, 2010.

Books for reference include:
[CR] John C. COX and Mark RUBINSTEIN: *Options markets*. Prentice Hall, 1985.
[H1] HULL, J. (1995): *Introduction to futures and options markets* (2nd ed), Prentice-Hall, ('baby Hull'), or
[H2] HULL, J. (1993): *Options, futures and other derivative securities* (2nd ed.), Prentice-Hall ('Hull').
Background and general interest
[K1] John KAY, *The Truth about Markets: Their Genius, their Limits, their Follies*. Penguin/Allen Lane, 2003.
[K2] John KAY, *Other People's Money: Finance: Masters of the Universe or Servants of the People?* Profile Books, 2015, £16.99).
[AH] Anat R. ADMATI & Martin HELLWIG, *The Bankers' New Clothes: What's Wrong with Banking and What to Do about it*, Princeton UP, 2013.
[G] Alan GREENSPAN, *The age of turbulence*. Penguin, 2007.

I think the Kay books are essential reading for anyone thinking of working in the financial services industry – maybe [AH] too. I thoroughly recommend [G] – but get the latest edition of it that you can. The author was Chairman of the US Federal Reserve (Fed) 1987-2006. His views up to 2007 were largely Panglossian optimism (markets know best, and are self-correcting, etc.). The ongoing problems since have forced a re-think; see the epilogues to later editions, his evidence to the House Committee, etc.
Mathematics, for reference
[D] J. L. DOOB: *Stochastic processes*, Wiley, 1953.
[N] J. NEVEU: *Discrete-parameter martingales*, North-Holland, 1975.
[KS] KARATZAS, I. & SHREVE, S. (1988): *Brownian motion and stochastic calculus*. Graduate Texts in Math. **113**, Springer.
[RY] REVUZ, D. & YOR, M. (1999): *Continuous martingales and Brownian motion*. Grundlehren der math. Wiss. **293**, Springer, 3rd ed. (1st ed. 1991, 2nd ed. 1994,).
[RW1] ROGERS, L. C. G. & WILLIAMS, D. (1994): *Diffusions, Markov processes and martingales, Volume 1: Foundation*, 2nd ed.
[RW2] ROGERS, L. C. G. & WILLIAMS, D. (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*. Wiley.

Exam: Standard format; syllabus and lecturer as last year.
Assessed Coursework: One assignment, 10% credit, Week 6 (due Week 7).

CONTENTS

§6. Continuous-time limit of the binomial model [L17-18]
§7. More on European options [L18-19]
§8. American options [L19-20]
V. STOCHASTIC PROCESSES IN CONTINUOUS TIME [5 h: L20-25].
§1. Filtrations; finite-dimensional distributions [L21]
§2. Classes of processes [L21]
§3. Brownian motion [L21-22]
§4. Quadratic variation (QV) of Brownian motion; Itô's Lemma [L22-23]
§5. Stochastic integrals; Itô calculus [L23-24]
§6. Stochastic differential equations (SDEs); Itô's Lemma [L24-25]
VI. MATH. FINANCE IN CONTINUOUS TIME [$5\frac{1}{2}$ h: L25-30].
§1. Geometric Brownian motion and asset prices [L25]
§2. The Black-Scholes model and the Black-Scholes PDE [L26-27]
§3. The Feynman-Kac formula and the Black-Scholes formula [L27-28]
§4. Girsanov's theorem and change of measure [L29]
§5. Real options (Investment options); Postscript [L30]

About the course: Content.

There are two difficult areas here, one pure mathematical, one non-mathematical.

(a) *Measure Theory.*

'Grown-up probability', such as is needed for this course (Itô calculus, etc.) is measure-theoretic. Ideal preparation would be a full course in Measure Theory (such as M3P19 Measure and Integral, Autumn Term), and then Probability (M3P6, Spring Term). But only a minority of students attending this course will have had these. So, we deal with necessary measure-theoretic preliminaries in Chapter II. In the time available, one cannot prove the guts of technical measure theory – the key approximation arguments. So we quote these, confining proofs to what goes before and what comes afterwards (both much easier).

(b) *Economics.*

Finance is a small and specialised part of Economics. Ideal preparation would also include a good grounding in Economics. For Mathematics students, there would not be room for this in the curriculum here. Some may have an Economics qualification from school – but these will be a small minority. So again, we have to take a lot for granted, and write into the record the necessary economic and financial background, in Chapter I.

In this regard, please bear three things in mind:
1. Anything important enough becomes political (M. Maurice Couve de Murville). This stuff is certainly important.
2. Politics in not an exact science (Bismarck). But,
3. Mathematics is an exact science.
We will be doing lots of mathematics – in particular, we derive the Black-Scholes formula. We will extend calculus, the most powerful single weapon we have, to become probabilistic (Itô calculus) and apply it to these problems. But, there are limits to which finance, economics, or anything involving human psychology, is mathematicisable. As always in Applied Mathematics, we have to be on guard: if we don't simplify enough, we can't do anything; if we over-simplify, we can do things, but can't trust our conclusions.

Just as important as the technical mathematics, you need to think about the systemic faults at the geofinancial/economic/political level thrown up by the crisis of 2007 on (Credit Crunch, etc.). Any prospective employer in the financial services industry should ask you questions about this, and your views on it, in interview. We spend half of one lecture on such things (I.10, L6 – not examinable). The rest is down to you.

For a range of views here, see e.g.
Quantitative Finance 15 no. 4 (2015), Special Issue on Interlinkages & Systemic Risk, esp. Dempster's review of the Admati-Hellwig book, 579-582;
N. H. BINGHAM: The Crash of 2008: A mathematician's view. *Significance* **5** no. 4 (2008), 173-175 [on my home-page, under Papers].

About the course: Format.

The course has two kinds of student: those who take it as an elective (M3A22 Year 3 BSc; M4A22 Year 4 MSci; M5A22 MScs other than Math. Finance; exam in summer), and those who take it as a core course (MSc in Math. Finance; exam in January). For the core MF students, this is simply one course among others of a similar style – no need to say more, so they can skip the rest of this.

For elective students, there are several things to say (prompted by SOLE feedback, 2014-15).
(i) Use of PC/OHP, rather than whiteboard.

There is far too much material to cover in 'whiteboard mode', which is why 'PC/OHP mode' will be used. (For background: I typically cover 2 pages per lecture, in LaTeX or longhand, in whiteboard mode and 4, LaTeX, in OHP mode; see my homepage for links to all my courses here since 2006 –

there are several examples of each.) If this is going to be a problem for you, perhaps this course is not for you.
(ii) Handouts.

I have always provided these before (in addition to the notes on the screen – the "belt and braces" approach), but will rely this time on the screen only in lectures, and will put the material up on the website before the lectures. I have been asked to do this to save paper – spare the environment. Also, lap-top users may prefer not to have to lug paper around, etc.
(iii) Attendance at lectures.

There are two good reasons to attend lectures: it gives a reasonably painless exposure to the material, and it provides insight into the mind-set of the examiner. I advise you to attend lectures, on both grounds. If you choose not to (be it on your own head, and good luck to you): not only will this year's teaching material go up on the course website as it is delivered, but all of last year's material is already up there. There are a number of changes in detail, but the overlap between this year's material and last year's will be over 90%. But don't not come to lectures 'to save time'. You would save only the 30 contact hours, which is piffling compared to the time needed – an eighth of an academic year – to make a serious commitment to this course or any other course. And if you're not serious about a course, don't do it.
(iv) Worked examples on the whiteboard.

I was asked to include worked examples on the whiteboard. There really isn't time to do much of this. But you can find your own worked examples in the recommended texts, and besides, you have Problems and Solutions 1-10. If this is going to be a problem for you, perhaps this course is not for you.
(v) English prose rather than mathematical symbolism.

Some students have apparently never encountered use of prose rather than symbols in a maths course here. Symbolism is an extension of ordinary language, to be used when ordinary language will not do the job. We will use lots of symbolism – when we need to; we will also use lots of ordinary language – English prose – when we don't. Again, if this is going to be a problem for you, perhaps this course is not for you. But there is really nothing to be frightened of – the ability to write decent prose is a valuable transferable skill, and one that will stand you in good stead in the job market.

NHB