

**PROBLEMS 3. 31.10.2014**

- Q1. (i) Show that the volume of a sphere of radius  $r$  is  $V = 4\pi r^3/3$ .  
(ii) Show that the surface area of a sphere of radius  $r$  is  $S = 4\pi r^2$ .  
(iii) Derive each from the other.

Q2. Show that the volume of the ellipsoid

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$$

is  $V = 4\pi abc/3$ .

- Q3. (i) Show that the volume of a tetrahedron of base area  $A$  and height  $h$  is  $V = Ah/3$ .  
(ii) Show that this holds also for ‘generalised tetrahedrons’, obtained by taking any plane shape with area  $A$  and boundary curve  $C$ , and joining the points of  $C$  to some vertex  $V$  a height  $h$  above  $C$ .

- Q4. (i) Show that the volume of revolution obtained by rotating a curve  $y = f(x)$  about the  $x$ -axis between  $a$  and  $b$  is  $V = \pi \int_a^b f(x)^2 dx$ .  
(ii) Hence re-derive the volume of a sphere.

Q5 (*Generalised Pythagoras theorem*). A right-angled triangle has sides 1 (the hypotenuse), 2 and 3. A semicircle (or any other plane shape) of area  $A_1$  is drawn with base side 1; similar copies of this are drawn with bases sides 2 and 3, with areas  $A_2$ ,  $A_3$ . Show that

$$A_1 = A_2 + A_3.$$

Deduce Pythagoras’ theorem on taking these shapes to be squares.

NHB