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Lecture 6. 24.10.2014

2. Risk management. The problems of 2007/8 on have made the importance of risk management obvious. For an excellent book-length treatment, see e.g. [MFE] A. J. McNEIL, R. FREY & P. EMBRECHTS: *Quantitative risk management: Concepts, techniques, tools*. Princeton UP, 2005.

We know from Markowitz that we should have a balanced portfolio, with lots of negative correlation. The danger is *large* losses. These are quantified by the *tails* of the distributions – the joint distribution of our portfolio. The point of diversifying is so that what we lose on the swings we gain on the roundabouts. Two comments:

(a) Whether this works for large losses depends on the tail properties of the joint distribution. It does *not* work if this is normal – as it is in the benchmark Black-Scholes model.

(b) When the whole market is falling – as in a financial crisis – none of the risk-management techniques useful under normal market conditions work.

3. Moral hazard. Before the limited liability company, if one defaulted, one was liable to the *whole* of the loss incurred by one's counter-party. This made trading very dangerous (the early traders were called merchant adventurers) – all the more as insurance had not developed by then.[1]

Limited liability was what made ordinary people willing to undertake the risks of trading, and so paved the way for the development of modern business, commerce, capitalism etc.

The moral hazard here is the possibility of gambling with other people's money. If it works, fine. If not, walk away (writing off one's limited liability) and leave them to bear the loss.

Bankruptcy law varies from country to country, and is too complicated to pursue here. But one sees moral hazard where it concerns us in, e.g.:

(a) start-ups of hedge funds (or, dot-com companies);

(b) aggressive traders – who (for the sake of their bonuses) gamble with their careers – but with other people's money;

(c) credit rating – where the credit rating agencies had a financial incentive to pass as AAA some highly questionable financial asset, etc.

4. Securitization. This term covers the drive in recent years to seek out

[1] Lloyds of London predates limited liability. The Lloyds participants – "names" – had unlimited liability. Many were driven into personal bankruptcy in the Lloyds scandals of the 90s. See Google for the ghastly details.

new financial markets by identifying risks that people might want to cover themselves against, and creating new financial derivatives that can be sold to address this perceived need. These derivatives too could be traded, etc. The upshot was an explosion of trade in increasingly artificial financial products, developed by the R&D departments of the financial institutions. By 2007/8 that the leaders of these institutions did not understand these products – could not price them, and could not value their holdings of them (above).

One specific trigger of the US crash in 2007 was the explosive growth in sub-prime mortgages. These were granted to people who would not have qualified as financially sound enough to get a mortgage previously, but who wanted to buy their own house. This new and profitable market proved irresistible to US banks – leading to a great house-price bubble, which burst (as bubbles do) in 2007. The knock-on effects hit the UK in 2008 (Northern Rock, etc.). The real damage of this failure of the financial sector has been its devastating and ongoing consequences on the real economy.

5. Macro-prudential issues. As the above illustrates, financial matters are too important to be left to financiers. Proper regulation is vital.

6. Forwards and futures. Forwards are agreements between buyer and seller made now, but concerning delivery in the future. They are not traded. Futures are options on things that will come to market in the future (next year's grain crop, for example), and these are traded (extensively). There are good accounts in Hull's books, [H1], [H2].

7. OTC and exchange-traded contracts. OTC – "over-the-counter" – denotes a transaction made between an individual buyer and an individual seller. As options on standard transactions develop, these are assets themselves that can be traded in exchanges (e.g., the CBOE, which opened in 1973: I.3).

8. Marking to market. This is a system whereby the exchanges cover themselves and their clients against the risk of large losses. If one party to a trade is, on current market prices, exposed to a potentially heavy loss, a *margin call* will be required by the exchange. Margin calls that actually triggers many financial failures (but limit the losses of the counter-parties).

9. Forex. Forex is an abbreviation for foreign exchange. International trade involves more than one currency' currencies move against each other. There is a vast market in derivatives to cover the risks involved.

10. Swaps. From Hull [H2] Ch. 5: "Swaps are private agreements between two companies to exchange cash flows in the future ... The first swap contracts were negotiated in 1981. Since then the market has grown very rapidly. ..." There are even options on swaps – *swaptions* – etc.

Prelude to Ch. II: Integration and area (cf. PfS Lecture 1, SP L1)

We shall mainly deal with area, as this is two-dimensional. We can draw pictures in two dimensions, and our senses respond to this; paper, whiteboards and computer screens are two-dimensional. By contrast, one-dimensional pictures are much less vivid, while three-dimensional ones are harder (they need the mathematics of perspective) – and dimensions higher than four are harder still.

Area.

1. *Rectangles*, base b, height h: area $A := bh$.
2. *Triangles.* $A = \frac{1}{2}bh$.

Proof: Drop a perpendicular from vertex to base; then extend each of the two triangles formed to a rectangle and use 1. above.

3. *Polygons.* Triangulate: choose a point in the interior and connect it to the vertices. This reduces the area A to the sum of areas of triangles; use 2. above.
4. *Circles.* We have a choice:

(a) Without calculus. Decompose the circle into a large number of equi-angular sectors. Each is approximately a triangle; use 2. above [the approximation boils down to $\sin\theta \sim \theta$ for θ small].

(b) With calculus and plane polar coordinates. Use $dA = dr.rd\theta = rdrd\theta$: $A = \int_0^r \int_0^{2\pi} rdrd\theta = \int_0^r rdr. \int_0^{2\pi} d\theta = \frac{1}{2}r^2.2\pi = \pi r^2$.

Note. The ancient Greeks essentially knew integral calculus – they could do this, and harder similar calculations [volume of a sphere $V = \frac{4}{3}\pi r^3$; surface area of a sphere $S = 4\pi r^2 dr$, etc.; note $dV = Sdr$].

What the ancient Greeks did not have is differential calculus [which we all learned first!] Had they had this, they would have had the idea of velocity, and differentiating again, acceleration. With this, they might well have got Newton's Law of Motion, Force = mass $\times$ acceleration. This triggered the Scientific Revolution. Had this happened in antiquity, the world would have been spared the Dark Ages and world history would have been completely different!

5. *Ellipses*, semi-axes a, b. Area $A = \pi ab$ (w.l.o.g., $a > b$).

Proof: cartesian coordinates: $dA = dx.dy$.

Reduce to the circle case: compress ['squash'] the x-axis in the ratio b/a [so $dx \mapsto dx.b/a$, $dA \mapsto dA.b/a$]. Now the area is $A = \pi b^2$, by 4. above. Now 'unsquash': dilate the x-axis in the ration a/b. So $A \mapsto A.a/b = \pi b^2.a/b = \pi ab$.

Fine – what next? We have already used *both* the coordinate systems to hand. There is no general way to continue this list.

The only general procedure is to superimpose finer and finer sheets of graph paper on our region, and count squares ('interior squares' and 'edge squares'). This yields numerical approximations – which is all we can hope for, and all we need, in general.

The question is whether this procedure always works. Where it is clearly most likely to fail is with highly irregular regions: 'all edge and no middle'.

It turns out that this procedure does *not* always work; it works for *some but not all* sets – those whose structure is 'nice enough'. This goes back to the 1902 thesis of Henri LEBESGUE (1875-1941):
H. Lebesgue: Intégrale, longueur, aire. *Annali di Mat.* **7** (1902), 231-259.
Similarly in other dimensions. So: some but not all sets have a length/area/volume. Those which do are called *(Lebesgue) measurable*; length/area/volume is called *(Lebesgue) measure*; this subject is called Measure Theory.

We first meet integration in just this context – finding areas under curves (say). The 'Sixth Form integral' proceeds by dividing up the range of integration on the x-axis into a large number of small subintervals, $[x, x + dx]$ say. This divides the required area up into a large number of thin strips, each of which is approximately rectangular; we sum the areas of these rectangles to approximate the area.

This informal procedure can be formalised, as the *Riemann integral* (G. F. B. RIEMANN (1826-66) in 1854). This (basically, the Sixth From integral formalised in the language of epsilons and deltas) is part of the undergraduate Mathematics curriculum.

We see here the essence of calculus (the most powerful single weapon in mathematics, and indeed in science). If something is reasonably smooth, and we break it up finely enough, curves look straight, so we can handle them. We make an error by this approximation, but when calculus applies, this error can be made arbitrarily small, so the approximation is effectively exact.
Example: We do this sort of thing automatically. If in a discussion of global warming we hear an estimate of polar ice lost, this will translate into an estimate of increase in sea level (neglecting the earth's curvature).
Note. The 'squashing' argument above was deliberately presented informally. It can be made quite precise – but this needs the mathematics of *Haar measure*, a fusion of Measure Theory and Topological Groups.