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M3A22/4A22/5A22 MATHEMATICAL FINANCE

N. H. BINGHAM

Imperial College London, 13 October – 19 December 2014

6M47; 020-7594 2085; n.bingham@ic.ac.uk; Office hour Fri 4-5

Course website: My homepage.

Clore, Mon 11-12, Tue 5-6, Fri 3-4

Books. Our main text will be Ch. 1-6 of
[BK] N. H. BINGHAM and Rüdiger KIESEL: *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed., CUP, 2004.
Course Website: M3A22 link on my home-page (Imperial College > Mathematics Department > Staff > Staff List > Bingham > Homepage: favouritize this to get it in one click). Other relevant links:
[SP] Stochastic Processes [30 hours, MSc, Mathematial Finance];
[SA] Stochastic Analysis [20 hours, MSc].
[LTCC] Measure-theoretic probability theory [10 hours; MSc].

For background:
[PfS] Probability for Statistics;
[SMF] Statistical Methods for Finance;
[Math482] – a course along these lines I gave at Liveerpool.

We shall make systematic use of *conditioning* (informally: using what we know). For background, see e.g.
[BF] N. H. BINGHAM and J. M. FRY: *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS), Springer, 2010.

Books for reference include:
[DP] Avinash K. DIXIT and Robert S. PINDYCK: *Investment under uncertainty*. Princeton University press, 1994.
[CR] John C. COX and Mark RUBINSTEIN: *Options markets*. Prentice Hall, 1985.
[PS] G. PESKIR and A. N. SHIRYAEV: *Optimal stopping and free-boundary problems*. Birkhäuser, 2006.

[E] Alison ETHERIDGE: *A course in financial calculus*, CUP, 2002.
[WHD] WILMOTT, P., HOWISON, S. & DEWYNNE, J. (1995): *The mathematics of financial derivatives: A student introduction*, Oxford Financial Press ['PDE with everything'];
[H1] HULL, J. (1995): *Introduction to futures and options markets* (2nd ed), Prentice-Hall, ('baby Hull'), or
[H2] HULL, J. (1993): *Options, futures and other derivative securities* (2nd ed.), Prentice-Hall ('Hull').
Background and general interest
[B1] Peter L. BERNSTEIN: *Capital ideas: The improbable origins of modern Wall Street*. New York: The Free Press, 1992.
[B2] Peter L. BERNSTEIN: *Against the Gods: The remarkable story of risk*. Wiley, 1996.
[S] Robert L. SHILLER: *Irrational exuberance*. Princeton Univ. Press, 2000.
[G] Alan GREENSPAN, *The age of turbulence*. Penguin, 2007.

I thoroughly recommend [G] – but get the latest edition of it that you can. The author was Chairman of the US Federal Reserve (Fed) 1987-2006. His views up to 2007 were largely Panglossian optimism (markets know best, and are self-correcting, etc.). The ongoing problems since have forced a re-think; see the epilogues to later editions, his evidence to the House Committee, etc.

Mathematics, for reference
[D] J. L. DOOB: *Stochastic processes*, Wiley, 1953.
[N] J. NEVEU: *Discrete-parameter martingales*, North-Holland, 1975.
[KS] KARATZAS, I. & SHREVE, S. (1988): *Brownian motion and stochastic calculus*. Graduate Texts in Math. **113**, Springer.
[RY] REVUZ, D. & YOR, M. (1999): *Continuous martingales and Brownian motion*. Grundlehren der math. Wiss. **293**, Springer, 3rd ed. (1st ed. 1991, 2nd ed. 1994,).
[RW1] ROGERS, L. C. G. & WILLIAMS, D. (1994): *Diffusions, Markov processes and martingales, Volume 1: Foundation*, 2nd ed.
[RW2] ROGERS, L. C. G. & WILLIAMS, D. (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*. Wiley.

Exam: Standard format. The syllabus is the same, but as the lecturer, and so the style, is new, I will set a Mock Exam (+ Solutions). I will examine on what I have taught – no surprises.
Assessed Coursework: One assignment, 10% credit, Week 6.

CONTENTS

§5. European options. The discrete Black-Scholes formula [L17]
§6. Continuous-time limit of the binomial model [L17-18]
§7. More on European options [L19]
§8. American options [L19-20]

V. STOCHASTIC PROCESSES IN CONTINUOUS TIME [5 h: L21-25].

§1. Filtrations; finite-dimensional distributions [L21]
§2. Classes of processes [L21-22]
§3. Brownian motion [L22-23]
§4. Quadratic variation (QV) of Brownian motion; Itô's Lemma [L23]
§5. Stochastic integrals; Itô calculus [L24-25]
§6. Stochastic differential equations (SDEs); Itô's Lemma [L25]

VI. MATH. FINANCE IN CONTINUOUS TIME [5 h: L26-30].

§1. Geometric Brownian motion and asset prices [L26]
§2. The Black-Scholes model and the Black-Scholes PDE [L26-28]
§3. The Feynman-Kac formula and the Black-Scholes formula [L28-29]
§4. Girsanov's theorem and change of measure [L29-30]
§5. Extensions; Postscript [L30]

Note. Please bear three things in mind in this course:
1. Anything important enough becomes political (M. Maurice Couve de Murville). This stuff is certainly important.
2. Politics in not an exact science (Bismark). But,
3. Mathematics is an exact science.

We will be doing lots of mathematics – in particular, we derive the Black-Scholes formula. We will extend calculus, the most powerful single weapon we have, to become probabilistic (Itô calculus) and apply it to these problems. But, there are limits to which finance, economics, or anything involving human psychology, is mathematicisable. As always in Applied Mathematics, we have to be on guard: if we don't simplify enough, we can't do anything; if we over-simplify, we can do things, but can't trust our conclusions.

Just as important as the technical mathematics, you need to think about the systemic faults at the geofinancial/economic/political level thrown up by the crisis of 2007 on (Credit Crunch, etc.). Any prospective employer in the financial services industry should ask you questions about this, and your views on it, in interview. Some of my views are in
N. H. BINGHAM: The Crash of 2008: A mathematician's view. *Significance* **5** no. 4 (2008), 173-175 [on my home-page, under Papers]. NHB