m3a22cwsoln(14).tex

**M3A22 SOLUTIONS TO ASSESSED COURSEWORK. 8.12.2014**

Q1. (i) The value $C(K)$ of the right to buy an asset worth $S$ for $K$ is $(S-K)_+$. This is non-increasing in $K$. // **[2]**
(ii) $C(K_1) - C(K_2)$ is the advantage the right to buy at $K_1$ gives over the right to buy at $K_2$. This is worth $K_2 - K_1$ if both are exercised, and less otherwise. (One can also do Q1 by arbitrage arguments.) **[2]**

Q2. Write $\lambda := (K_3 - K_2)/(K_3 - K_1) \in (0,1)$. We have to show that $C(K)$ is *convex* in $K$, i.e. $C(K_2) = C(\lambda K_1 + (1-\lambda)K_3) \leq \lambda C(K_1) + (1-\lambda)C(K_3)$ ('curve lies below chord'). This is clear from the graph of $C(K) = (S-K)_+$. More formally: $S - K$ and 0 are linear in $K$; linear functions are convex; the maximum of two convex functions is convex (check! – the set of convex functions forms a *lattice*).

Most people constructed an arbitrage (technically unnecessary but good practice!), and/or used Q1, and/or broke the payoff into four cases:
(i) $S < K_1$: all options worthless: LHS = RHS = 0. **[2]**
(ii) $K_1 \leq S < K_2$: only $C(K_1)$ has value, $S - K_1$; LHS = 0, RHS = $\lambda(S - K_1) \geq 0$. **[2]**
(iii) $K_2 \leq S < K_3$: LHS worth $S - K_2$, RHS worth $\lambda(S - K_1)$.

$$\begin{aligned}
RHS - LHS &= \lambda(S-K_1) - (S-K_2) \\
&= \frac{K_3 - K_2}{K_3 - K_1}.(S-K_1) - (S-K_2) \\
&= K_2 - K_1.\frac{K_3 - K_2}{K_3 - K_1} - S[1 - \frac{K_3 - K_2}{K_3 - K_1}] \\
&= K_3.\frac{K_2 - K_1}{K_3 - K_1} - S.\frac{K_2 - K_1}{K_3 - K_1} \\
&= (K_3 - S).\frac{K_2 - K_1}{K_3 - K_1} \geq 0.
\end{aligned}$$ **[10]**

(iv) LHS = RHS: for, LHS worth $S - K_2$; RHS worth
$\lambda(S - K_1) + (1-\lambda)(S - K_3) = S - \lambda K_1 - (1-\lambda)K_3 = S - K_2$. // **[2]**

NHB