Schedule and references

Monday

First talk : Introduction to the topic by the mentors.

Second talk : Basics on Riemann surfaces : topological surfaces (their classification, their cohomology and their fundamental group), Riemann surfaces (basic facts, analytic and algebraic points of view, Hodge theory on H^1, line bundles, Jacobian, non-linear description of Hodge theory on H^1 i.e. "nonabelian" Hodge theory for GL_1).

References :

— S. Donaldson, Riemann surfaces, notes available at http://wwwf.imperial.ac.uk/ skdona/RSPREF.PDF

Third talk : Vector bundles on Riemann surfaces : topological classification, holomorphic and algebraic vector bundles, hermitian metrics, connections, curvature.

References :

— Montserrat Teixidor I Bigas, Vector bundles on curves, notes available at http://emerald.tufts.edu/ mteixido/files/vectbund.pdf

Fourth talk : GIT versus symplectic reduction : finite dimensional case, basic examples, infinite dimensional illustration : Narasimhan-Seshadri theorem, stability for vector bundles, moduli space of (semi)stable bundles.

References :

— V. Hoskins, Geometric invariant theory and symplectic quotients, notes available at http://userpage.fu-berlin.de/hoskins/GITnotes.pdf

— R. P. Thomas, Notes on GIT and symplectic reduction for bundles and varieties, arXiv :math.AG/0512411

— S. Donaldson. A new proof of a theorem of Narasimhan and Seshadri. J. Diff. Geom. 18 (1983), 269-277

Tuesday

First talk : Hitchin's equations : four dimensional motivation from Yang-Mills theory, reduction to two dimensions.

References :

— Section 1 of N. Hitchin. The self-duality equations on a Riemann surface. Proc. London Math. Soc. (3) 55 (1987), 59-126.

Second talk : Examples of Hitchin's equations : explicit simple examples, connection with classical hyperbolic geometry, uniformization, Teichmüller theory.

References :

— N. Hitchin. Lie groups and Teichmüller space. Topology 31.3 (1992), 449-473.

Third talk : Higgs bundles : definition, stability, moduli space of (semi)stable Higgs bundles, relation with the moduli space of vector bundles.

References :

— Sections 2 and 3 of N. Hitchin. The self-duality equations on a Riemann surface. Proc. London Math. Soc. (3) 55 (1987), 59-126.

Fourth talk : From Hitchin's equations to Higgs bundles and viceversa.

References :

— Sections 2 and 4 of N. Hitchin. The self-duality equations on a Riemann surface. Proc. London Math. Soc. (3) 55 (1987), 59-126.
— C. Simpson. Higgs bundles and local systems. Publ. Math. I.H.E.S. 75 (1992), 5-95.

Wednesday

First talk : Character varieties : definition, basic properties, Riemann-Hilbert correspondence, relation with Hitchin's equations (harmonic metrics).

References :

— Section 9 of N. Hitchin. The self-duality equations on a Riemann surface. Proc. London Math. Soc. (3) 55 (1987), 59-126.
— K. Corlette. Flat G-bundles with canonical metrics. J. Diff. Geom. 28 (1988), 361-382.
— C. Simpson. Higgs bundles and local systems. Publ. Math. I.H.E.S. 75 (1992), 5-95.

Second talk : Examples of character varieties : explicit examples, e.g. the affine cubic surface.

References :

— W. Goldman, D. Toledo. Affine cubic surfaces and relative $SL(2)$-character varieties of compact surfaces. Preprint arXiv :1006.3838 (2010).

Thursday

First talk : Infinite dimensional gauge theoretic point of view on Hitchin's equations, hyperkähler quotient construction, hyperkähler metric, twistor family, comparison of the various complex structures.

References :

- Section 9 of N. Hitchin. The self-duality equations on a Riemann surface. Proc. London Math. Soc. (3) 55 (1987), 59-126.
- C. Simpson. The Hodge filtration on nonabelian cohomology. arXiv preprint alg-geom/9604005 (1996).

Second talk : Spectral curves, Hitchin fibration, structure of completely integrable system, explicit examples.

References :

- Sections 5 and 6 of N. Hitchin. The self-duality equations on a Riemann surface. Proc. London Math. Soc. (3) 55 (1987), 59-126.
- N. Hitchin. Stable bundles and integrable systems. Duke Math. J. 54 (1987), 91-114.
- N. Hitchin. Langlands duality and G2 spectral curves. The Quarterly Journal of Mathematics 58.3 (2007), 319-344.

Third talk : Nonabelian Hodge theory formulation.

References :

- C. Simpson. The Hodge filtration on nonabelian cohomology. arXiv preprint alg-geom/9604005 (1996).

Fourth talk : Non-compact curves (tame/wild), singular curves.

References :

- C. Simpson. Harmonic bundles on noncompact curves. J. Amer. Math. Soc. 3 (1990), 713-770.
- H. Boden and K. Yokogawa. Moduli spaces of parabolic Higgs bundles and parabolic K (D) pairs over smooth curves : I. International Journal of Mathematics 7.05 (1996), 573-598.
- O. Biquard and P. Boalch. Wild non-abelian Hodge theory on curves. Compositio Mathematica 140.01 (2004), 179-204.
- U. Bhosle. Generalised parabolic bundles and applications to torsionfree sheaves on nodal curves. Arkiv för matematik 30.1 (1992), 187-215.

Friday

First talk : Betti numbers : rank two case, other examples, HLRV conjecture, P=W conjecture.

References :

- Sections 7 and 9 of N. Hitchin. The self-duality equations on a Riemann surface. Proc. London Math. Soc. (3) 55 (1987), 59-126.
- T. Hausel, and F. Rodriguez-Villegas. Mixed Hodge polynomials of character varieties. Inventiones mathematicae 174.3 (2008), 555-624.
- M.A. de Cataldo, T. Hausel, and L. Migliorini. Topology of Hitchin systems and Hodge theory of character varieties : the case A_1. arXiv preprint arXiv :1004.1420 (2010).

Second talk : Higher dimensional story : higher dimensional versions of nonabelian Hodge theory and of Hitchin's moduli space, applications to the topology of algebraic varieties (e.g. Simpson's constraint on the fundamental group).

References :

- C. Simpson. Higgs bundles and local systems. Publ. Math. I.H.E.S. 75 (1992), 5-95.
- A. Fujiki. Hyperkähler structure on the moduli space of flat bundles. Prospects in Complex Geometry, L.N.M. 1468 (1991), 1-83.
- C. Simpson. Moduli of representations of the fundamental group of a smooth projective variety, II. I.H.E.S. Publ. Math. 80 (1995), 5-79.

Third talk : Relation with Donaldson-Thomas theory.

References :

- B. Davison. Cohomological Hall algebras and character varieties. arXiv preprint arXiv :1504.00352 (2015).
- W. Chuang, DE. Diaconescu, and G. Pan. Wallcrossing and cohomology of the moduli space of Hitchin pairs. arXiv preprint arXiv :1004.4195 (2010).

Fourth talk : Future directions talk by the mentors.