Lie algebras

Test 1

February 14, 2017

In this test the ground field is the field of complex numbers $\mathbb{C}$.

1. Determine the centre of the Lie algebra of upper triangular matrices $\mathfrak{t}(2)$.

2. How many non-isomorphic Lie algebras of dimension 2 are there?
(a) one, (b) two, (c) three, (d) infinitely many
(Justify your answer.)

3. Let $\mathfrak{g}$ be a Lie algebra of dimension 3 such that the derived algebra $\mathfrak{g}'$ has dimension 2. Prove that $\mathfrak{g}'$ is abelian.
(Hint: By Question 1 of Problem Sheet 2 the elements of $\mathrm{ad}(\mathfrak{g}')$ are linear transformations with zero trace.)

4. What can you say about the Killing form of a nilpotent Lie algebra?
(Justify your answer.)