STATISTICAL METHODS IN FINANCE, ASSIGNMENT 3

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Exercise 1 (Confidence interval)**.** Let $X_1, \ldots, X_n$ denote a random sample from a Gaussian random distribution with unknown mean μ and variance σ^2. Assume that observations yield $\sum_{i=1}^n x_i = b$.

(i) For any $\alpha \in (0,1)$ determine the confidence interval forμ.
(ii) Taking $\alpha = 5\%$, $n = 20$ and $b = 200$, plot the confidence interval as a function of σ. What are the values for $\sigma = 25\%$?

Exercise 2 (Neyman-Pearson)**.** [Exercise 25 in the Lecture notes]
Consider $\mathcal{F} = \{\mathcal{N}(\mu, \theta^2), \theta > 0\}$, with $\mu \in \mathbb{R}$ known, and the hypotheses $\Theta_0 = (0, \sigma_0]$ and $\Theta_1 = (\sigma_0, +\infty)$, for some $\sigma_0 > 0$. Analyse the test defined by

$$\mathcal{R} := \left\{ \frac{\mathcal{L}_\theta(\mathcal{X}_n)}{\mathcal{L}_{\sigma_0}(\mathcal{X}_n)} > c \right\},$$

for some constant $c > 0$ to be determined, where $\mathcal{L}$ denote as usual the likelihood function.

Exercise 3 (Hypothesis testing 1)**.** Consider an iid sample $X_1, \ldots, X_n$ distributed as an Exponential random variable with parameter $\lambda > 0$, and consider the following test:

$$\mathcal{H}_0 : \lambda = \lambda_0 \qquad \text{vs} \qquad \mathcal{H}_1 : \lambda = \lambda_1,$$

for some $\lambda_0, \lambda_1 > 0$ and some level $\alpha \in (0,1)$.

(1) Determine the distribution of $\sum_{i=1}^n X_i$.
(2) Determine the joint distribution of $(X_1, \ldots, X_n)$.
(3) Using Neyman-Pearson's lemma, determine the optimal level c^*.

Exercise 4 (Hypothesis testing 2)**.** Consider an iid sample $X_1, \ldots, X_n$ distributed as a Uniform random variable on $[0, \theta]$ and consider the test

$$\mathcal{H}_0 : \theta = \theta_0 \qquad \text{vs} \qquad \mathcal{H}_1 : \theta = \theta_1,$$

for some $\theta_0, \theta_1 > 0$ and some level $\alpha \in (0,1)$.

(i) Determine the joint distribution of $(X_1, \ldots, X_n)$.

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(ii) In the setting of Neyman-Pearson's lemma, determine the optimal level c^* as well as the power of the test. What do you conclude?

(iii) Consider now a rejection region of the form $\mathcal{R}_k := \{\max\{X_1, \ldots, X_n\} \leq k\}$ for some k. Determine the optimal value of k and the power of the test. Can we conclude that this test is the most powerful for some level?