1. In each of the following cases, find whether the integral exists, and if so find its value.

(i) \( \int_{3}^{\infty} x^{-3/2} \, dx \). 
(ii) \( \int_{1}^{2} (x-1)^{-2/3} \, dx \). 
(iii) \( \int_{0}^{1} \ln x \, dx \). 
(iv) \( \int_{1}^{\infty} \ln x \, dx \). 
(v) \( \int_{0}^{\pi/2} \tan x \, dx \).

2. Evaluate 
(i) \( \int_{-2}^{2} \sinh(x^3) \, dx \). 
(ii) \( \int_{-3}^{3} e^{-5|x|} \, dx \).

3. Determine the length of the curve \( y = \cosh x \) which lies between the lines \( x = 0 \) and \( x = 1 \). 
(This curve, called a catenary, is the shape of a heavy cable hanging under gravity.)

4. Find the length of the curve \( y = \frac{x^2}{4} - \frac{1}{2} \ln x \) between \( x = 1 \) and \( x = 3 \).

5. The curve described by the cable of a suspension bridge is given by 
\[ y = \frac{H}{L^2} x^2 - 2 \frac{H}{L} x + H, \]
where \( x \) is the distance from one end of the bridge of length \( 2L \) and \( H \) is the height of the top of the cable above its lowest point. 
Show that the length of the cable is 
\[ (L^2 + 4H^2)^{1/2} + \frac{1}{2} \frac{L^2}{H} \sinh^{-1} \frac{2H}{L}. \]
If \( H \) is much smaller than \( L \), show that the length of the cable is approximately 
\[ 2L \left\{ 1 + \frac{2}{3} \left( \frac{H}{L} \right)^2 \right\}. \]
[Hint: put \( \varepsilon = \frac{2H}{L} \) and note that \( \sinh^{-1} \varepsilon = \varepsilon - \frac{\varepsilon^3}{6} + \ldots \).]

6. Find the length of the curve \( x = \theta + \sin \theta, \ y = 1 + \cos \theta \) between \( \theta = 0 \) and \( \theta = \pi \).

### Answers for Problems 7

1. (i) \( \frac{1}{2} \ln \left| \frac{x}{2-3x} \right| + c \). 
 (ii) \( \frac{1}{2} \ln \left| \frac{x^2}{x+1} \right| + c \). 
 (iii) \( \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c \). 
 (iv) \( 2x^{1/2} - 2 \tan^{-1} x^{1/2} + c \). 
 (v) \( \ln |x^3 + 2x^2 + x| + c \). 
 (vi) \( \frac{1}{2} (\tan^{-1} x^2) \). 
 (vii) \( \ln |\ln x| + c \). 
 (viii) \( \tan x - x + c \). 
 (ix) \( \frac{1}{2} \ln |\sin x + \cos x| + \frac{\pi}{2} + c \). 
 (x) \( \left( \frac{2}{\sqrt{3}} \right) \tan^{-1} \left( \left( \frac{2}{\sqrt{3}} \right) \tan \frac{x}{2} + \frac{1}{\sqrt{3}} \right) + c \). 
 (xi) \( x^2 \sin x + 2x \cos x - 2 \sin x + c \). 
 (xii) \( \frac{1}{2} e^x (\sin x + \cos x) + c \). 
 (xiii) \( \ln |\tan \frac{x}{2}| - x \cosecx + c \). 
 (xiv) \( \frac{x^{k+1}}{k+1} \left( \ln |x| - \frac{1}{k+1} \right) + c \). 
 (xv) \( \frac{\pi}{8} \). 
 (xvi) \( \frac{2}{3} \). 
 (xvii) \( \ln |(x^2 + 4x + 13)^{1/2} + x + 2| + c \).

2. 1. \( -x^4 \cos x + 4x^3 \sin x + 12x^2 \cos x - 24x \sin x - 24 \cos x + c \). 
 4. \( \frac{1}{2} \ln 2 - \frac{1}{4} \).