# Asynchronously Parallelised Percolation on Distributed Machines

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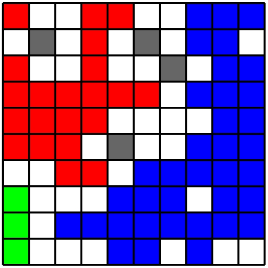
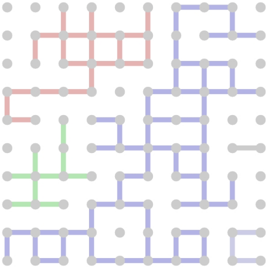
# Outline

# Illustration of the model



Sites occupied with probability $p_s$



Bonds active with probability $p_b$

**Definition of a cluster**

sites connected through occupied sites and active bonds

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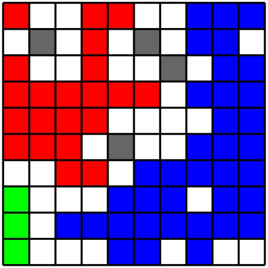

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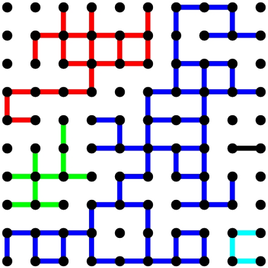

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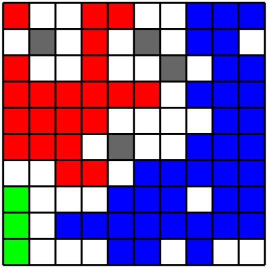

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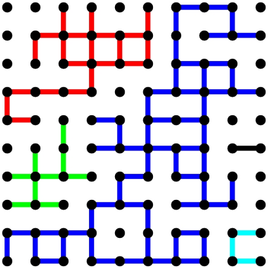

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## Definition of a cluster

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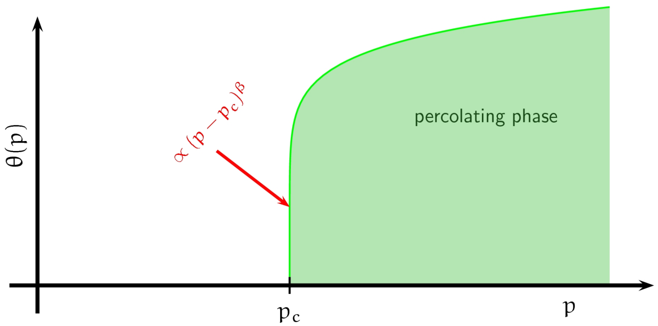
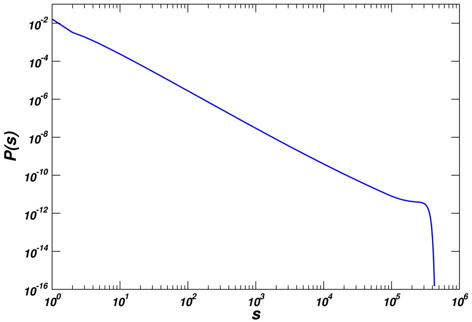
# Key features I



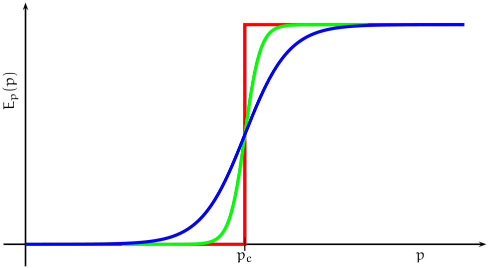

Order parameter $\theta$: fraction in the "infinite" cluster
In 2D: $\beta = 5/36$

# Key features II




Cluster size distribution (density of *s*-clusters per site):
$P(s) = as^{-\tau}\mathcal{G}(b(p-p_c)s^{\sigma})$ where $\tau = 187/91$ and $\sigma = 36/91$.

# Key features III




Crossing probability for different system sizes.

# Why percolation?

- Long history (Flory 1941)
- Renaissance because of Conformal Field Theory (Langlands *et al.* 1992, Cardy 1992)
- Numerics as a guide
  - Study leading to Conformal Field Theory
  - Multiple spanning clusters
- Open questions
  - Higher dimensions
  - universality
  - relation lattice $\longleftrightarrow$ Conformal Field Theory

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# A brief history I

- Three dimensional polymers: Flory 1941
- Mathematics: Hammersley and Broadbent 1954
- $p_c = 1/2$ in 2D bond percolation conjectured in 1955
- $\theta(1/2) = 0$ by Harris, 1960
- $p_c = 1/2$ tackled by Sykes and Essam, 1963
- "Dormant state"

Details: Grimmet, *Percolation*, 2000

# A brief history II

- Back on stage: Russo, and Seymour and Welsh, 1978
- Kesten: $p_c = 1/2$, 1980
- Uniqueness of infinite cluster: Newman and Schulman, 1981
- Renaissance because of Conformal Field Theory for crossing probabilities: Langlands *et al.* 1992, Cardy 1992
- Multiple spanning clusters: Hu and Lin 1996, Aizenman 1997, Cardy 1998
- Percolation is SLE with $\kappa = 6$, Smirnov 2001

# Outline



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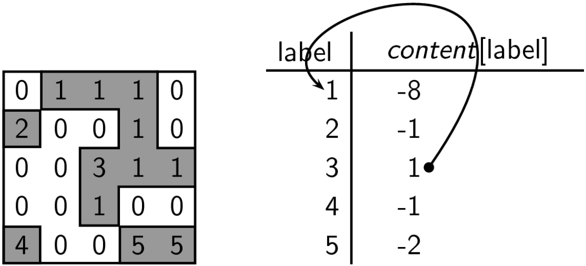
# The Algorithm: Hoshen-Kopelman

## Overview

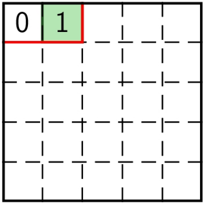



- scan row by row
- label clusters using list of labels
- remember configuration of “active” sites

(Hoshen and Kopelman, 1976)

# The Algorithm: Hoshen-Kopelman

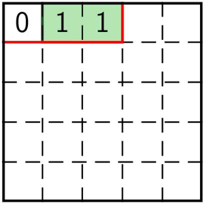
## Step by step




| label | *content*[label] |
|---|---|
| 1 | -1 |

# The Algorithm: Hoshen-Kopelman

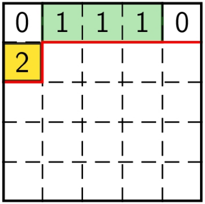
## Step by step




| label | *content*[label] |
|---|---|
| 1 | -2 |

# The Algorithm: Hoshen-Kopelman

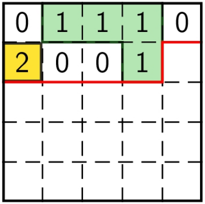
## Step by step




| label | *content*[label] |
|---|---|
| 1 | -3 |
| 2 | -1 |

# The Algorithm: Hoshen-Kopelman

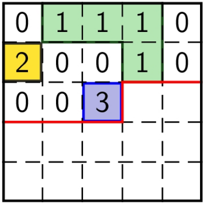
## Step by step




| label | *content*[label] |
|---|---|
| 1 | -4 |
| 2 | -1 |

# The Algorithm: Hoshen-Kopelman

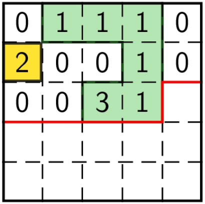
## Step by step




| label | *content*[label] |
|---|---|
| 1 | -4 |
| 2 | -1 |
| 3 | -1 |

# The Algorithm: Hoshen-Kopelman

## Step by step



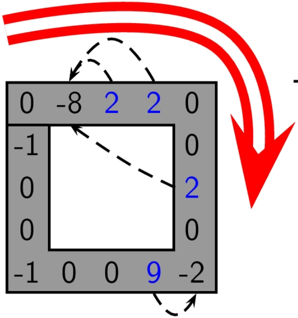

| label | *content*[label] |
|---|---|
| 1 | -6 |
| 2 | -1 |
| 3 | 1 |

# Outline

# The parallel algorithm

## Border Preparation

(N. R. Moloney and G.P. 2003)



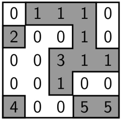

| label | *content*[label] |
|---|---|
| 1 | 2 |
| 2 | 16 |
| 3 | 2 |
| 4 | 13 |
| 5 | 9 |

- scan around the boundary
- move roots into boundary

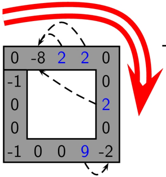
# The parallel algorithm

## Border Preparation – Comparison

(N. R. Moloney and G.P. 2003)




| label | *content*[label] |
|---|---|
| 1 | -8 |
| 2 | -1 |
| 3 | 1 |
| 4 | -1 |
| 5 | -2 |




| label | *content*[label] |
|---|---|
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- scan around the boundary
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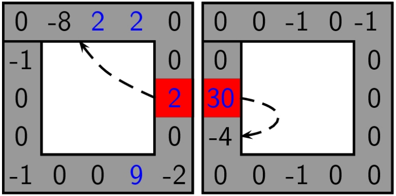
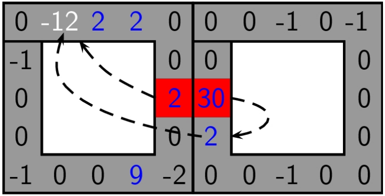
# The Algorithm

## Gluing




Before

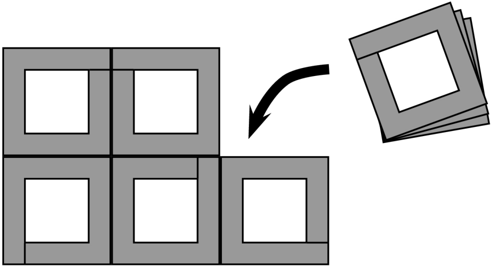



After

- Small patches produced at slave nodes (asynchronous)
- Assembly at master nodes:
  - Shift labels for uniqueness
  - Redirect roots — larger cluster prevails

related: (Rapaport, 1992)

# The Algorithm
## Features



- Huge lattices: $10^6$ realisations of $30\,000 \times 30\,000$ or a single lattice $22.2 \cdot 10^6 \times 22.2 \cdot 10^6$ (hierarchical nodes).
- Flexibility (boundary conditions, aspect ratios)
  Reduced correlations by rotating, mirroring and permuting
- Asynchronous
- Minimal hardware (CPU, memory, network)

# Outline



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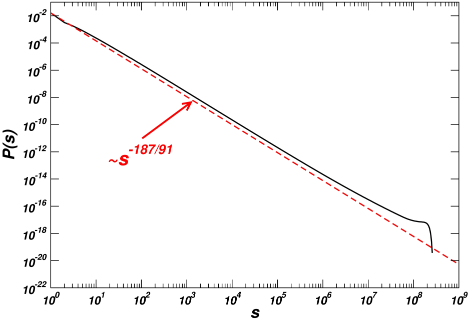
# Results

## Cluster Size Distribution




$p_s = 0.59274621$, free boundaries, $30\,000 \times 30\,000$ sites

# Results

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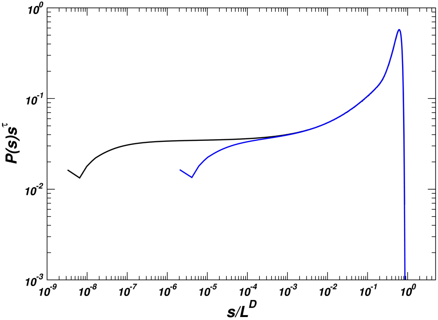
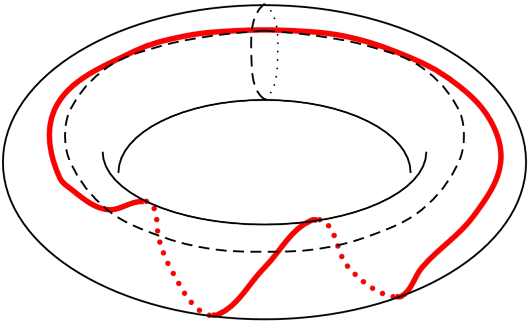



Site percolation, histogram normalised, shifted and binned
$p_s = 0.59274621$, free boundaries, $L = 1\,000$ and $30\,000$

# Results

## Winding on a Torus



Probability of winding clusters
with particular winding numbers

# Results

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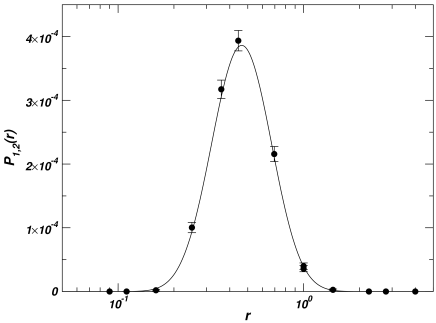


bond percolation, probability of $(1, 2)$ winding
circles numerical, line analytical (Pinson, 1994)

(G. P. and N. R. Moloney 2004)

# Summary

- Very large systems
- Very flexible (reduced correlations)
- Minimal hardware
- Asynchronous
- Numerical test of CFT results
- New open questions (exotic clusters, universality)

# Acknowledgements


- This work has been carried out at Imperial College London
- Supported partly by EPSRC (NRM + GP), the Beit fellowship (NRM), the NSF (GP) and the Humboldt Foundation (GP)
- Thanks to A. Thomas, K. Christensen, P. Anderson, M. Kaulke, O. Kilian, D. Moore, B. Maguire, and D. Erickson for their help