Self-organized Criticality and Absorbing States: *Lessons from the Ising model*

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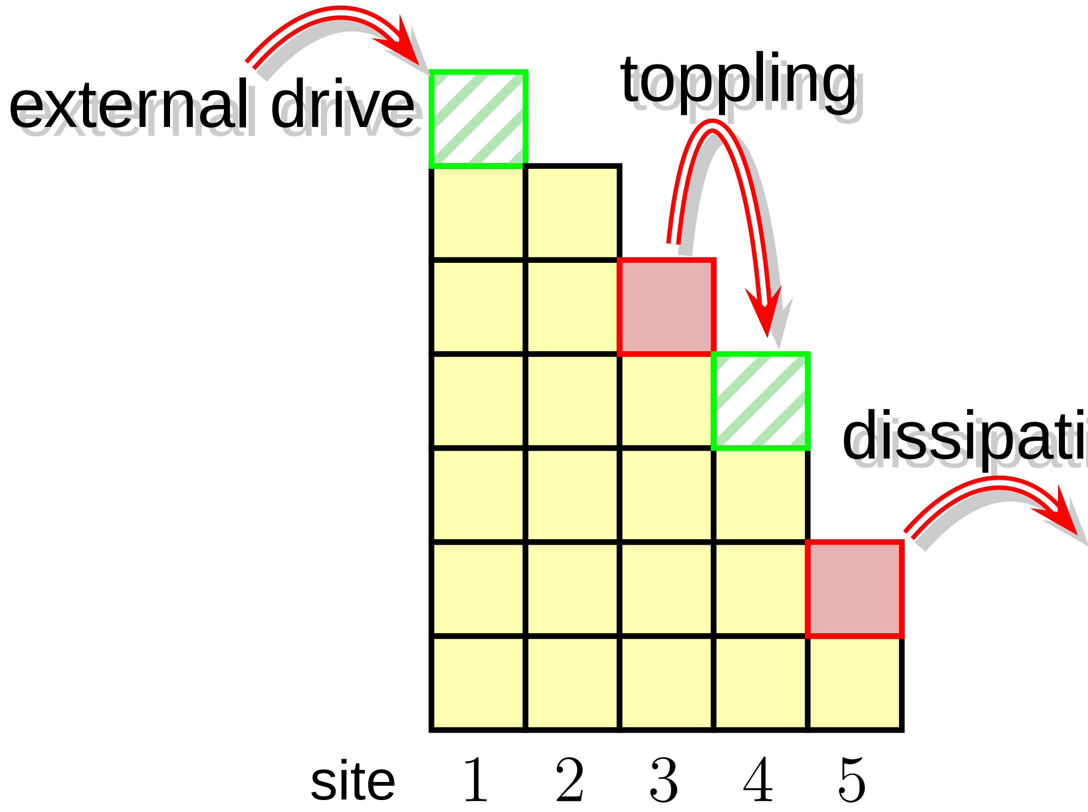
Structure of the talk

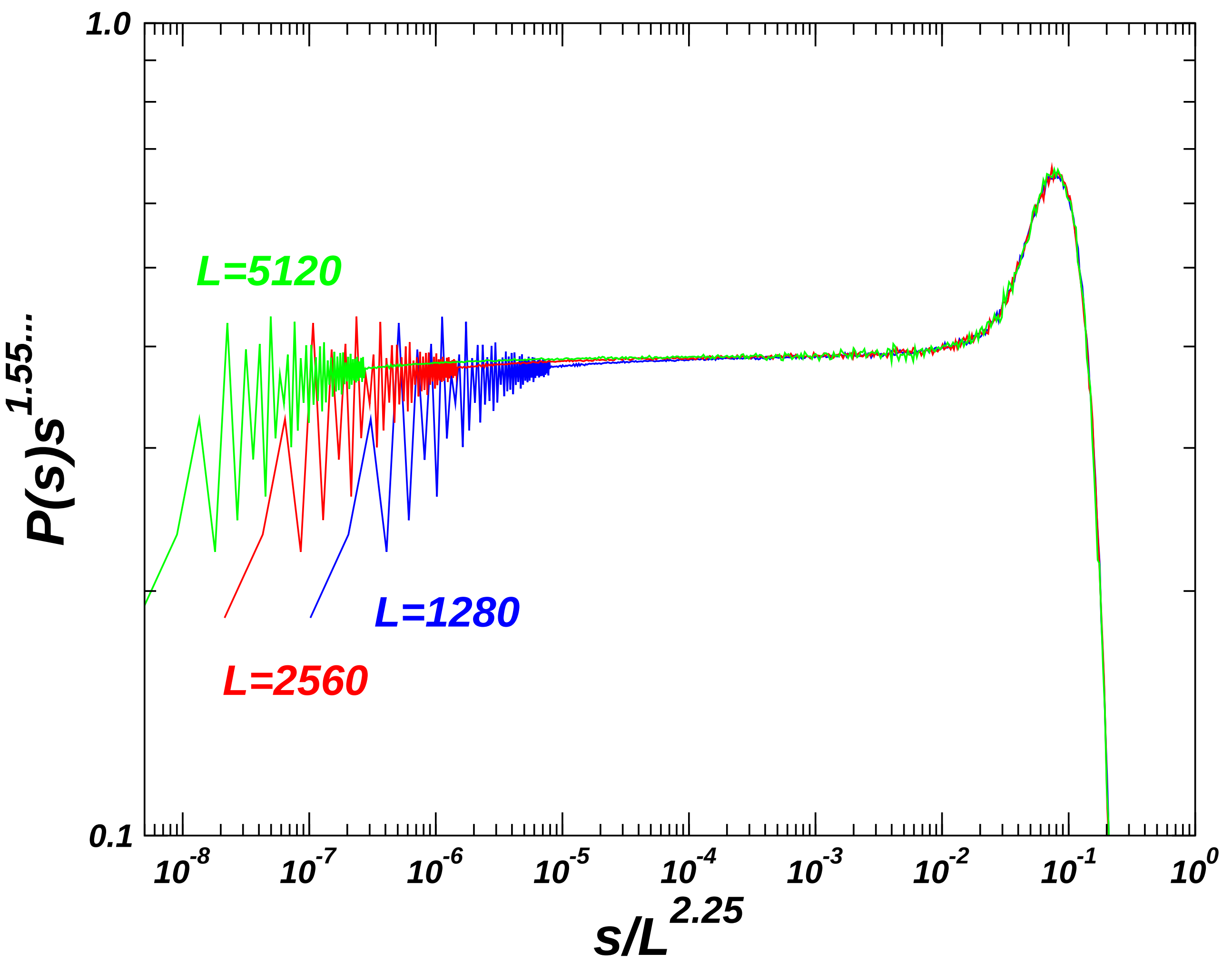
- Overview: SOC and AS
- The AS approach
- Application to the Ising model
- Conclusions

Overview: Self Organised Criticality



- toppling $\longrightarrow$ avalanches
- dissipation at boundaries
- *slow* external drive

Overview: Self Organised Criticality

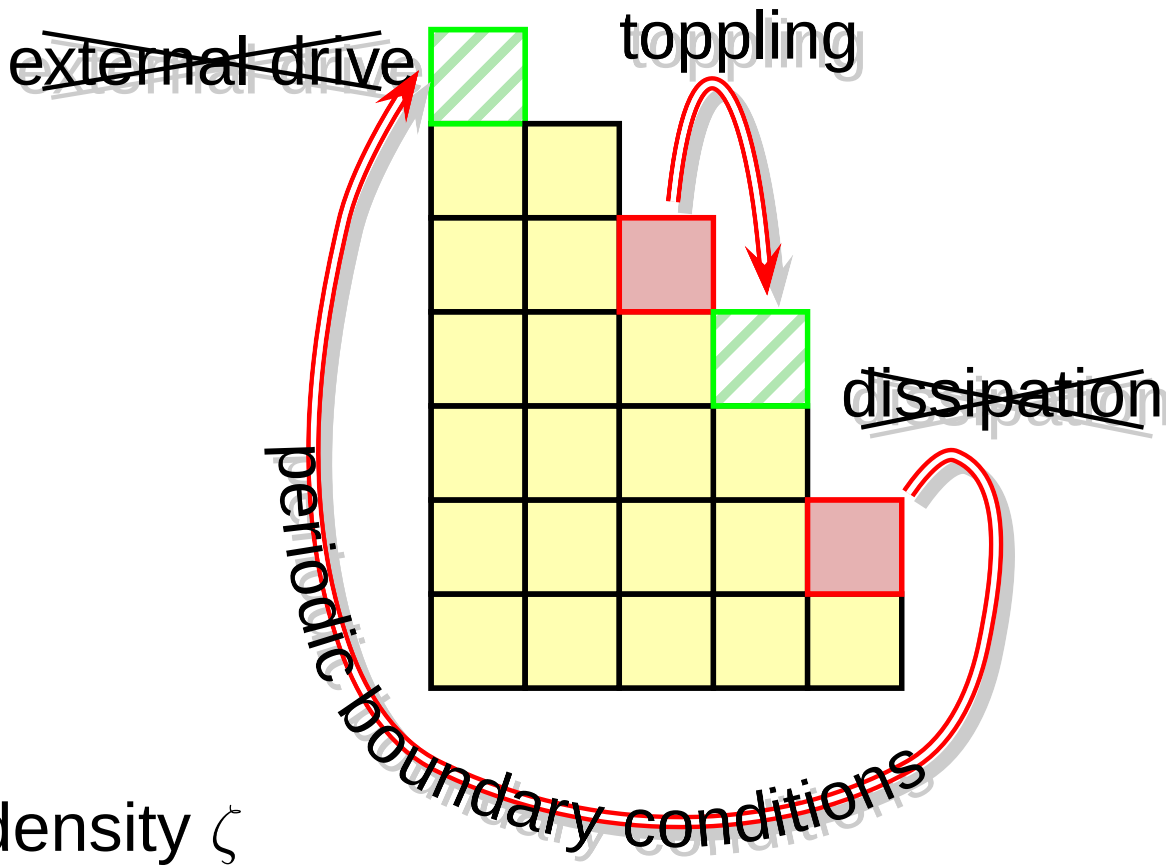


Avalanche size distribution obeys simple scaling

$\longrightarrow$ scale invariance

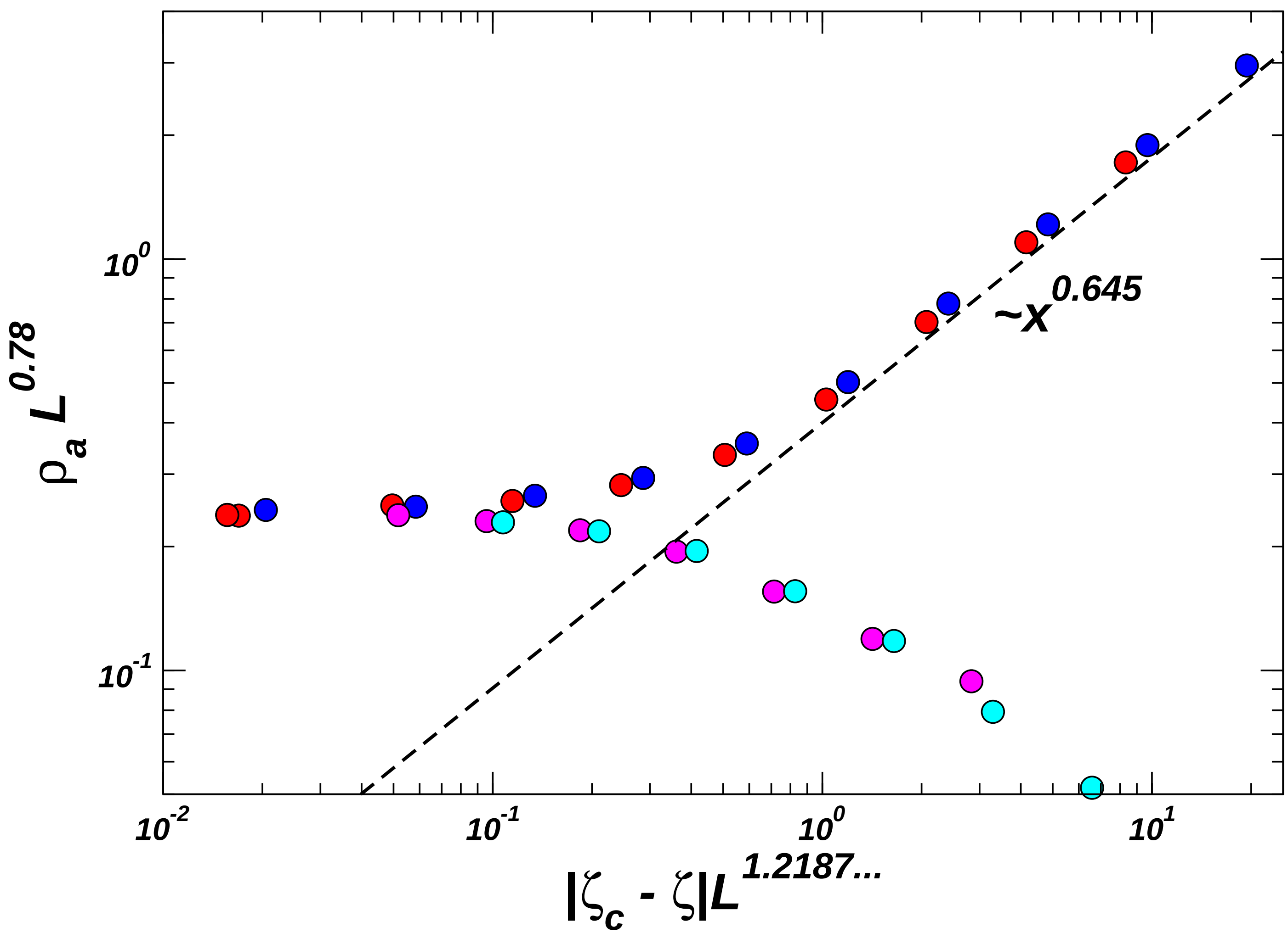
Overview: Absorbing States

(Dickman, Muñoz, Vespignani, Zapperi 2000)



- fixed density ζ
- order parameter: activity
- Fixed energy sandpile: Turning SOC into AS

Overview: Self Organised Criticality



Absorbing state order parameter obeys simple scaling

Understanding SOC in terms of AS

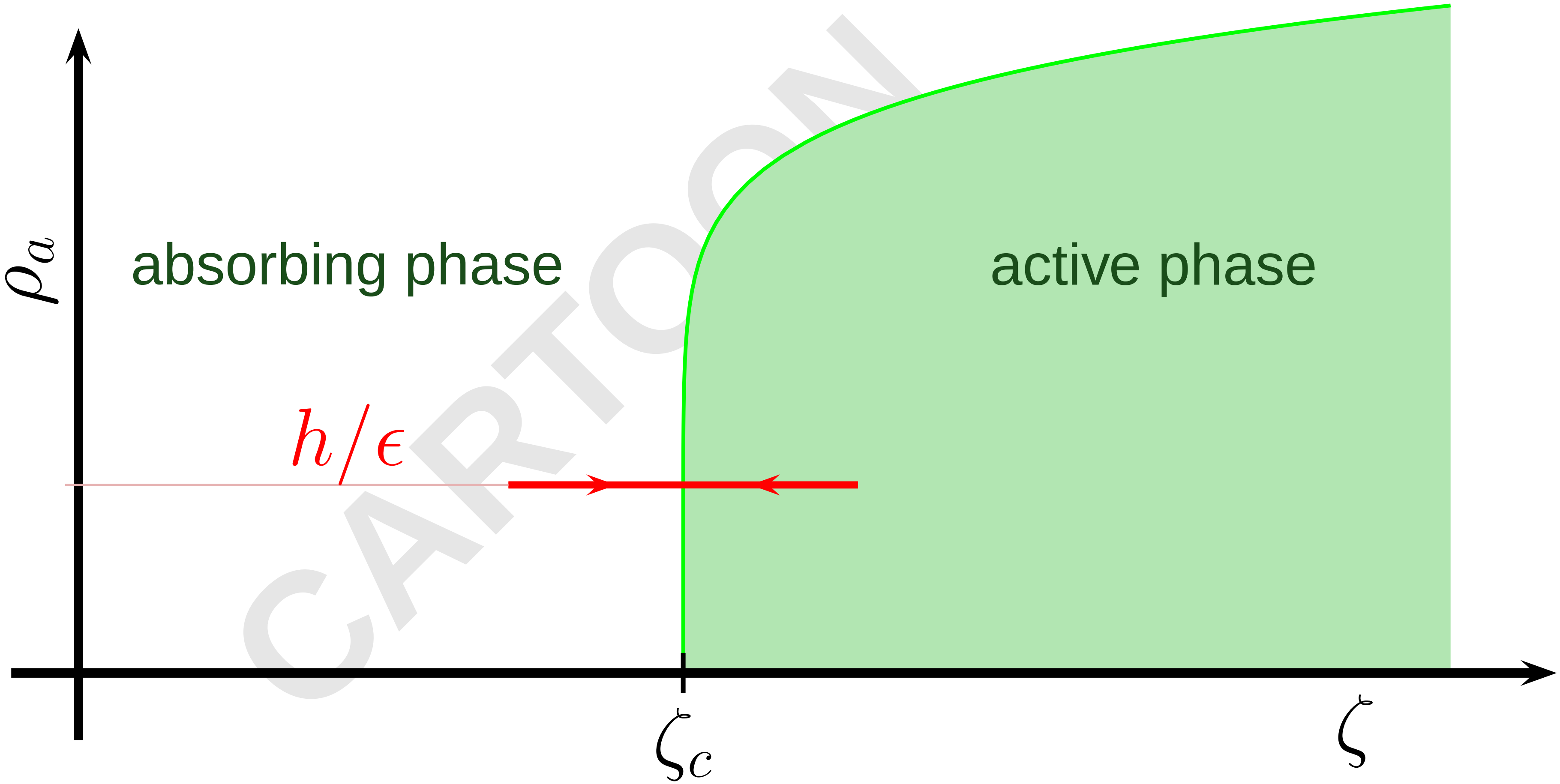
(Dickman, Vespignani, Zapperi 1998)

- SOC model: activity ρ_a leads to dissipation
- dissipation reduces particle density ζ
- density is reduced until system is inactive
  $\longrightarrow$ absorbing phase
- external drive increases particle density
  $\longrightarrow$ back to active phase

An SOC model can be seen as an AS model that drives itself into the inactive phase by dissipation ϵ and is pushed back into the active phase by external drive h.

$$\dot{\zeta} = h - \epsilon \rho_a \xrightarrow{\text{stationarity}} \rho_a = h/\epsilon$$

Understanding SOC in terms of AS



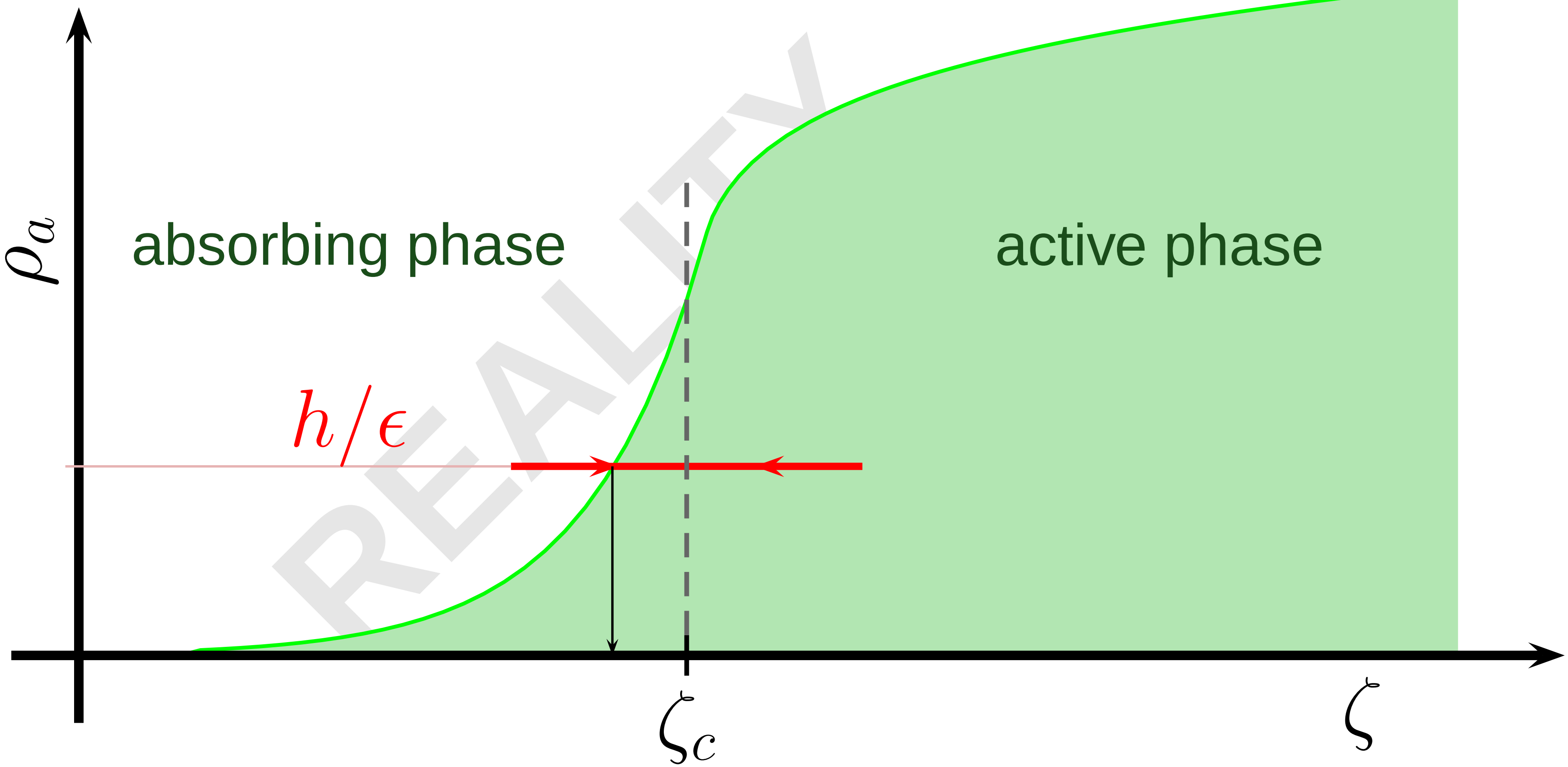
Idea: SOC drives $h/\epsilon = \rho_a$ to 0 as $L \to \infty$

Leading orders: $h(L) = h_0 L^{-\omega}$ and $\epsilon(L) = \epsilon_0 L^{-\kappa}$

Understanding SOC in terms of AS

- External drive h is slow to create distinct avalanches
- Dissipation ϵ is weak for scale invariance
- $\longrightarrow$ double limit: $h, \epsilon, h/\epsilon \to 0$
- Leading orders in finite systems:
  - $h(L) = h_0 L^{-\omega}$
  - $\epsilon(L) = \epsilon_0 L^{-\kappa}$

Understanding SOC in terms of AS



Problem:
Finite size changes the position of effective density.

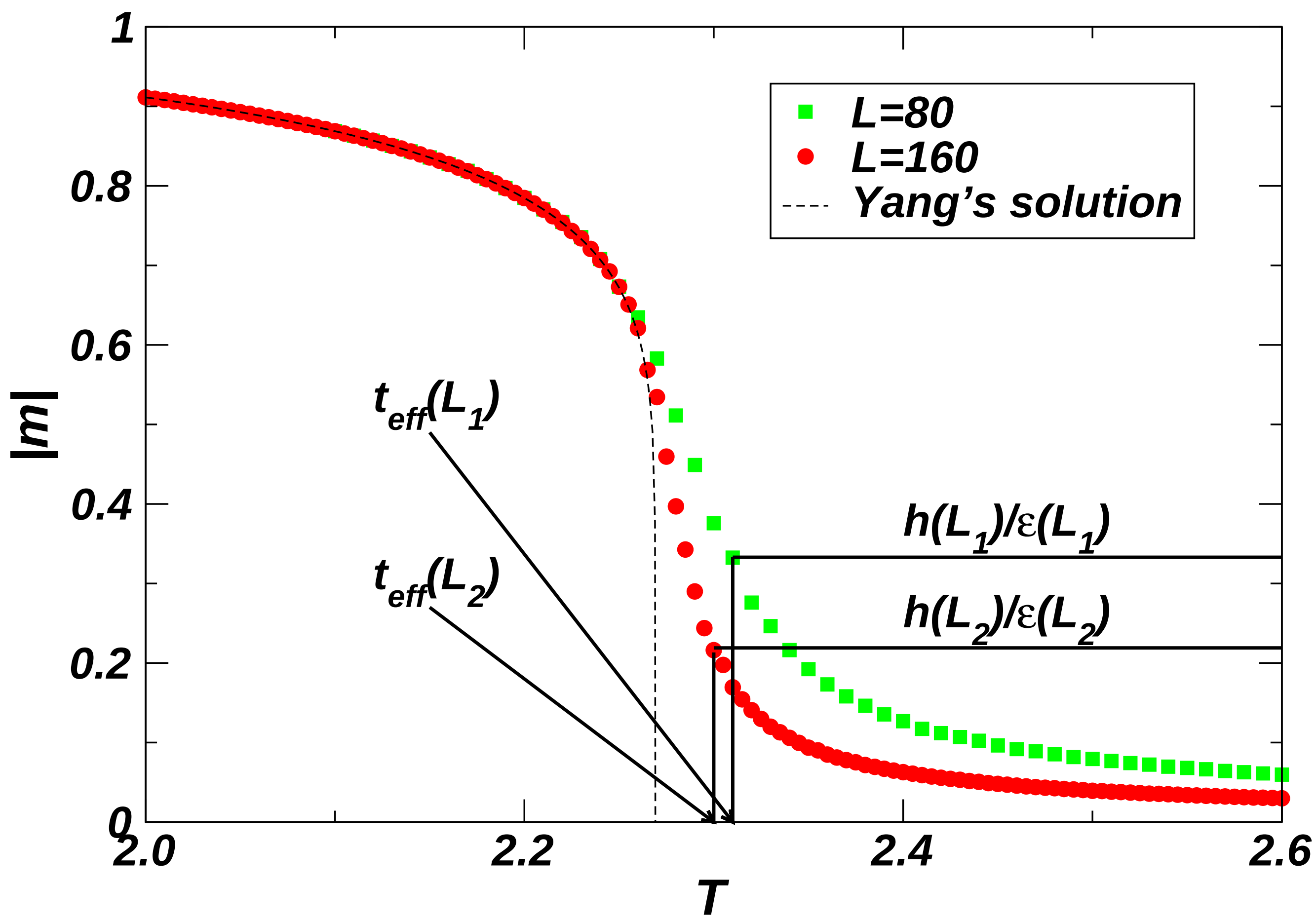
Translation to the Ising model

- Activity ρ_a $\longrightarrow$ magnetisation m
- Density ζ $\longrightarrow$ temperature T
- Driving h $\longrightarrow$ cooling h
- Dissipation $\epsilon\rho_a$ $\longrightarrow$ heating ϵm
- Equation of motion: $\dot{T} = -h + \epsilon m$

A choice of h and ϵ imposes a particular $m = h/\epsilon$ and results in a particular effective average temperature t_{eff}.

Question: What is $t_{\text{eff}}(L)$ if $m(t_{\text{eff}}) = h/\epsilon \propto L^{\kappa-\omega}$?

The Ising model



How the effective temperature changes with system size.

Analysis in the Ising model

Question: What is $t_{\text{eff}}(L)$ if $m(t_{\text{eff}}) = h/\epsilon \propto L^{\kappa-\omega}$?

- Low temperature phase:

$$m \propto t_{\text{eff}}^{\beta} \quad \Rightarrow t_{\text{eff}} \propto L^{\frac{\kappa-\omega}{\beta}} \quad -1/\mu$$

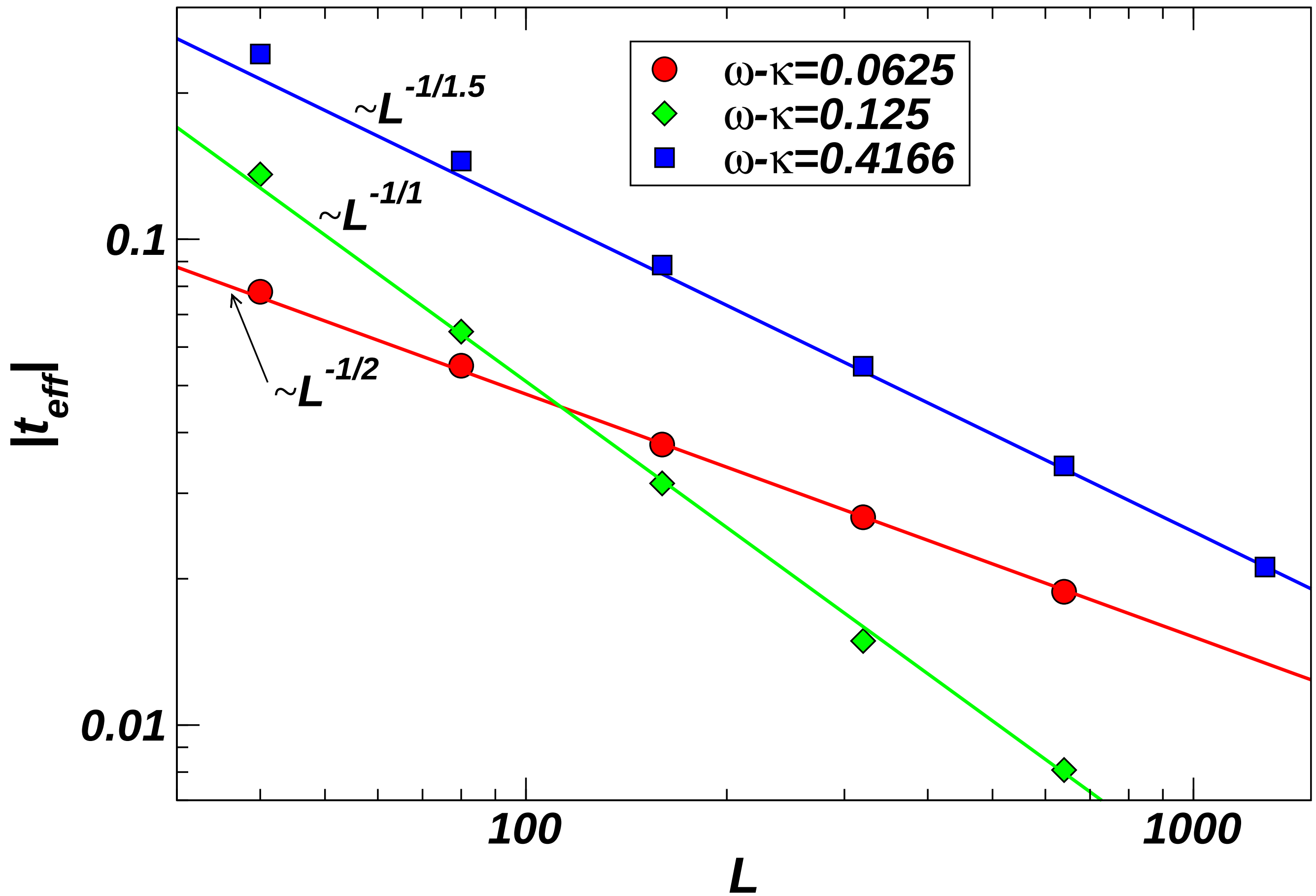
- High temperature phase:

$$m \propto \sqrt{L^{-d}\, t_{\text{eff}}^{-\gamma}} \quad \Rightarrow t_{\text{eff}} \propto L^{(\frac{d}{2}+\kappa-\omega)/(\frac{\gamma}{2})} \quad -1/\mu$$

$$t_{\text{eff}} \propto L^{-1/\mu}$$

In finite size scaling ν is replaced by $\mu > \nu$, a function of $\kappa - \omega$. Standard finite size scaling only for $\omega - \kappa = \beta/\nu$.

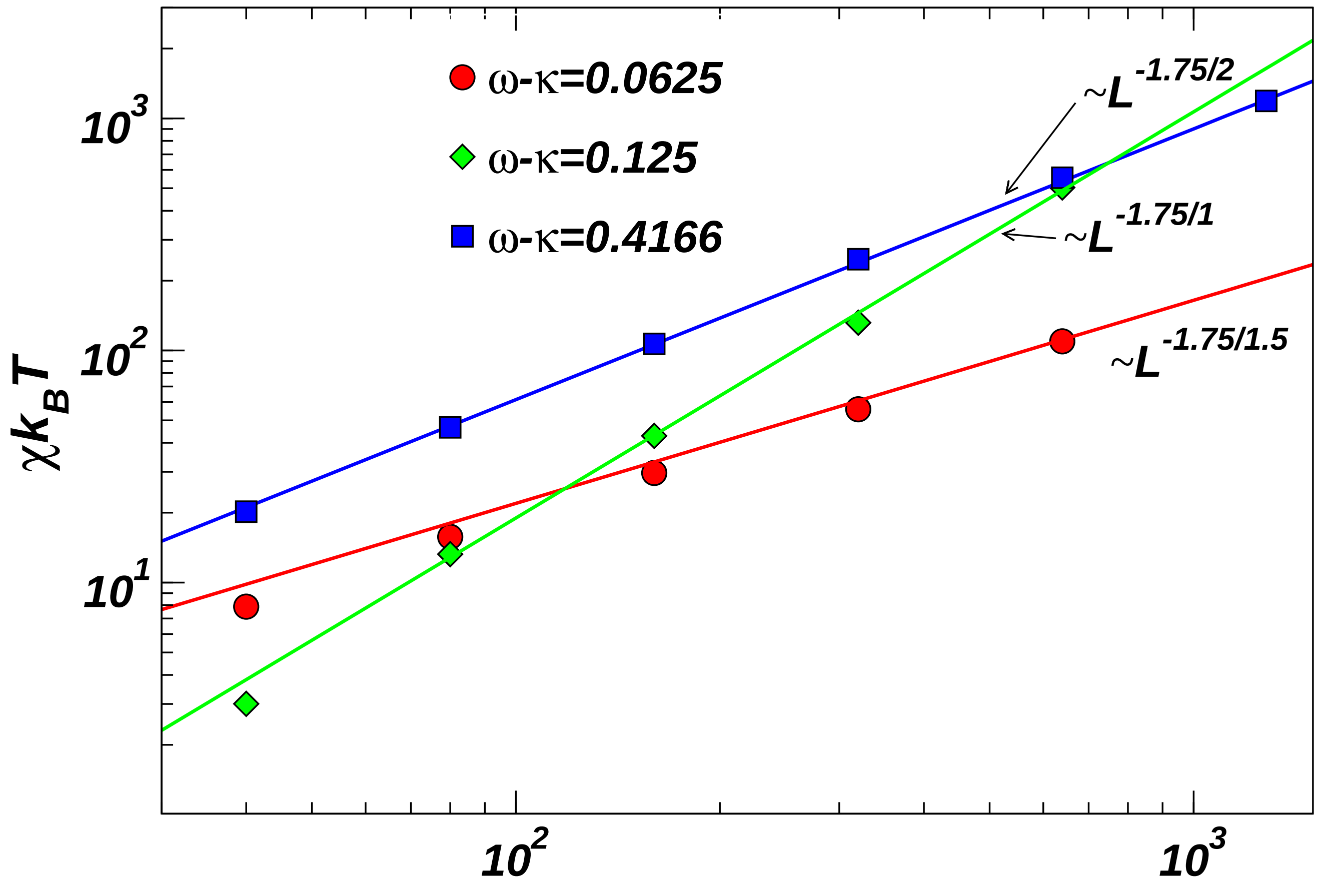
The Ising model: Results I



Scaling of the effective temperature in the Ising model:

$$t_{\text{eff}} \propto L^{-1/\mu}$$

The Ising model: Results II



Scaling of the susceptibility in the Ising model:

$$\chi \propto L^{\gamma/\mu}$$

Conclusions

- The AS mechanism drives the model to the critical point
- The AS mechanism does not reproduce standard exponents
- Key questions:

> Why are AS and SOC so consistent?
>
> Why is SOC universal?

Acknowledgements

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