Heteroclinic cycles & dynamics in coupled cell systems

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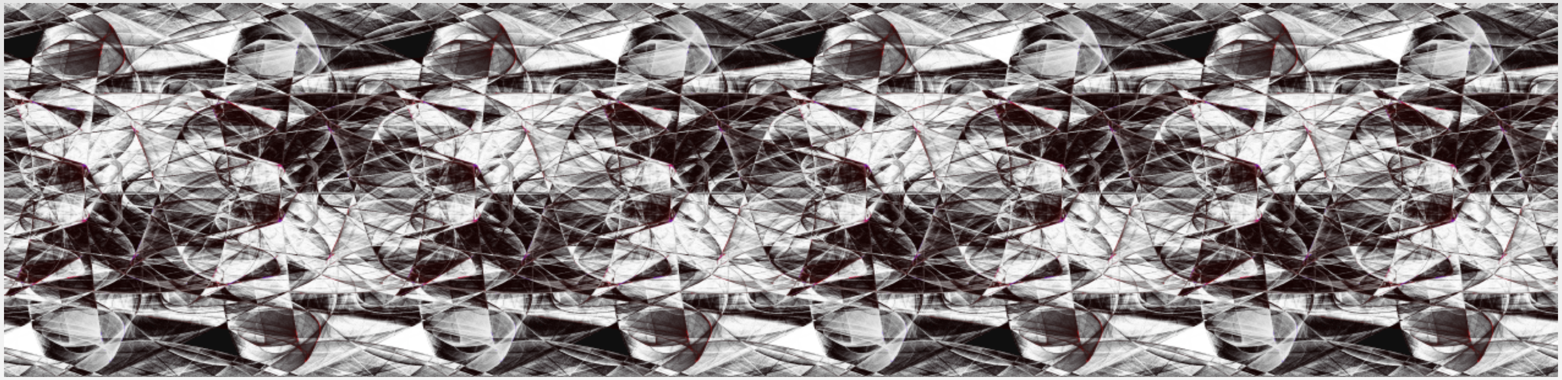
Some of the research reported on here is joint with Peter Ashwin, Exeter

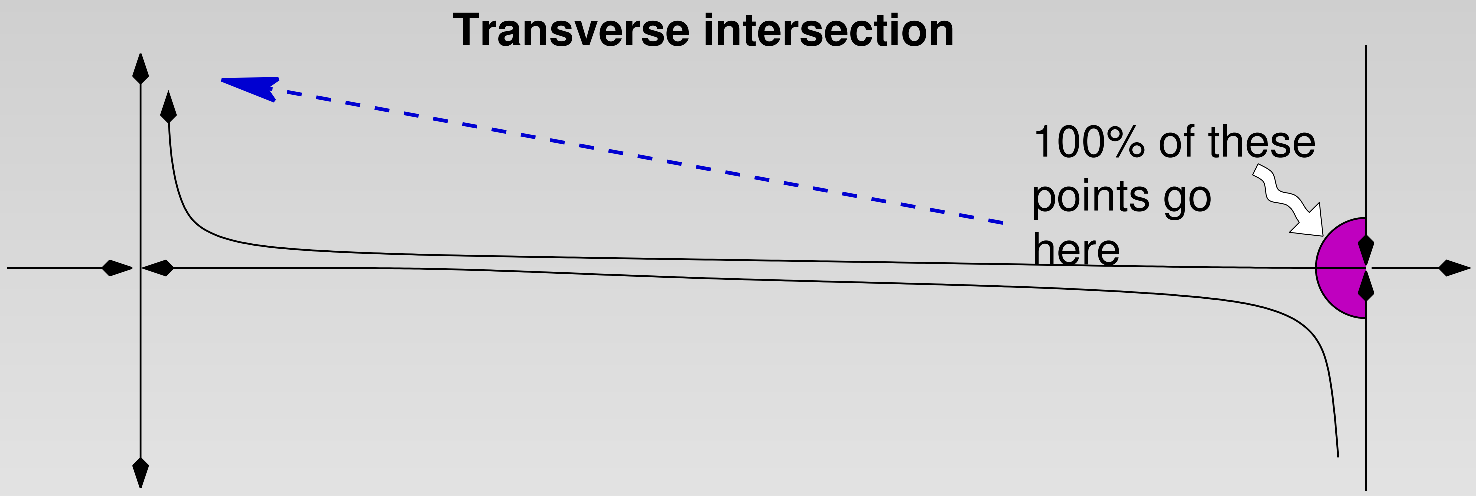


July 17^{th} 2006.

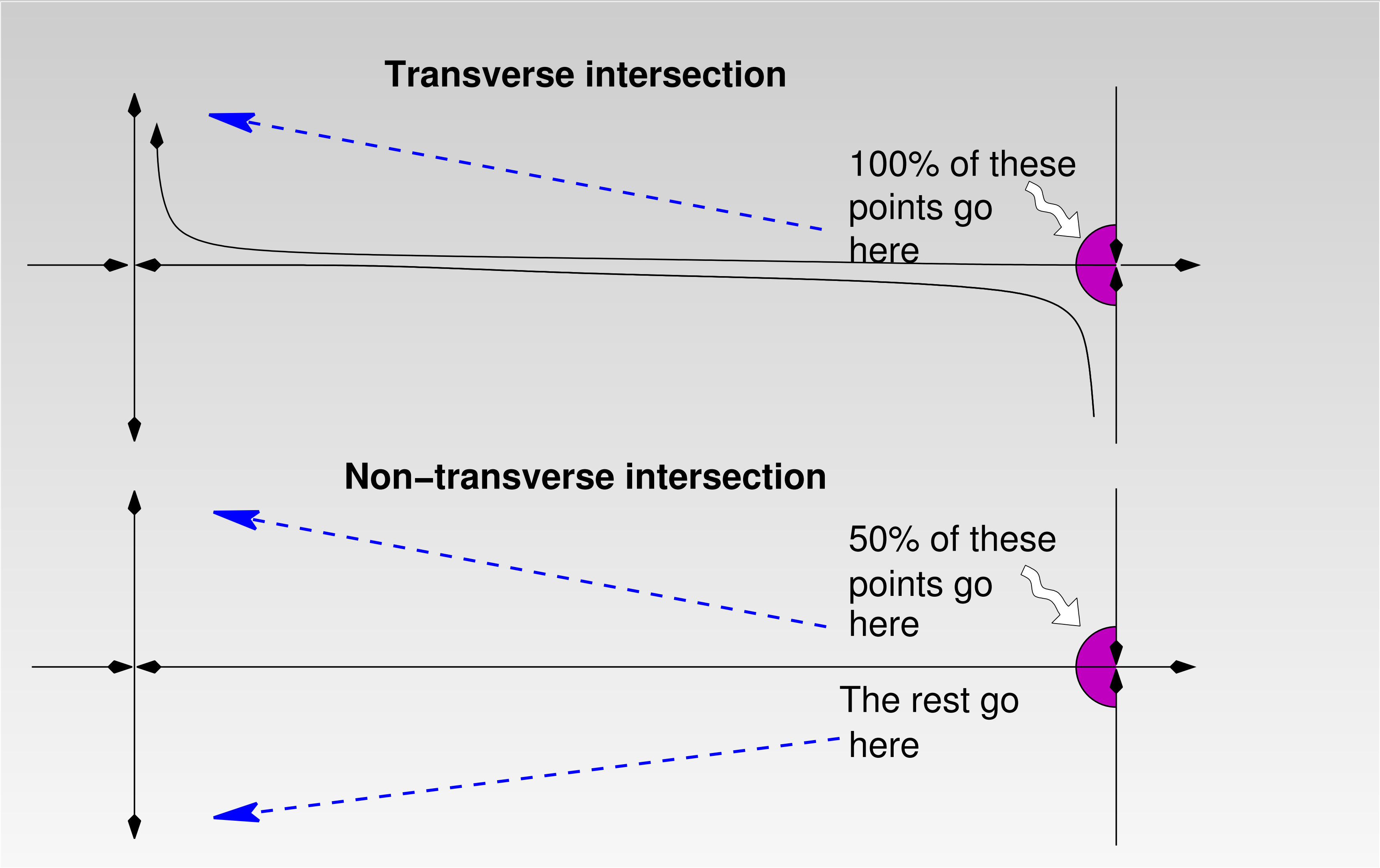
Objects and concepts

- *States/Nodes*: Hyperbolic equilibria, limit cycles, chaotic sets.
- *Invariant manifolds* (Connections): Stable and unstable manifolds of states.
- *Transversality* of stable and unstable manifolds.
- Implications of transversality: “Generic” transition between states.

Transitions



Transitions



Genericity of transversality

Transversality between invariant manifolds of hyperbolic equilibria and limit cycles is a ‘generic’ or ‘typical’ property of vector fields — *unless* there are additional constraints on the vector fields.

For example – this is *the* basic example – if there are subspaces of phase space that are flow invariant for all vector fields in the class, then we can expect to see failure of genericity of transversality of invariant manifolds.

Invariant subspaces

Certain classes of dynamical system naturally have *invariant spaces*. That is, subspaces of phase space that are invariant for the dynamics of all systems in the class.

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- ‘Semilinear’ feedback systems.

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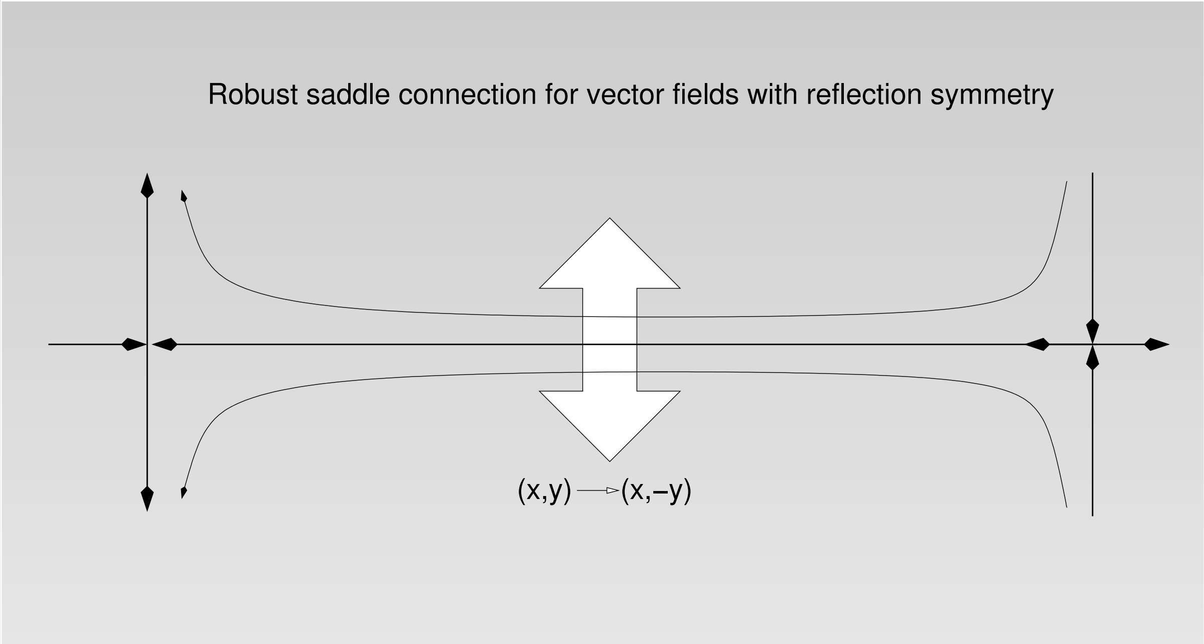
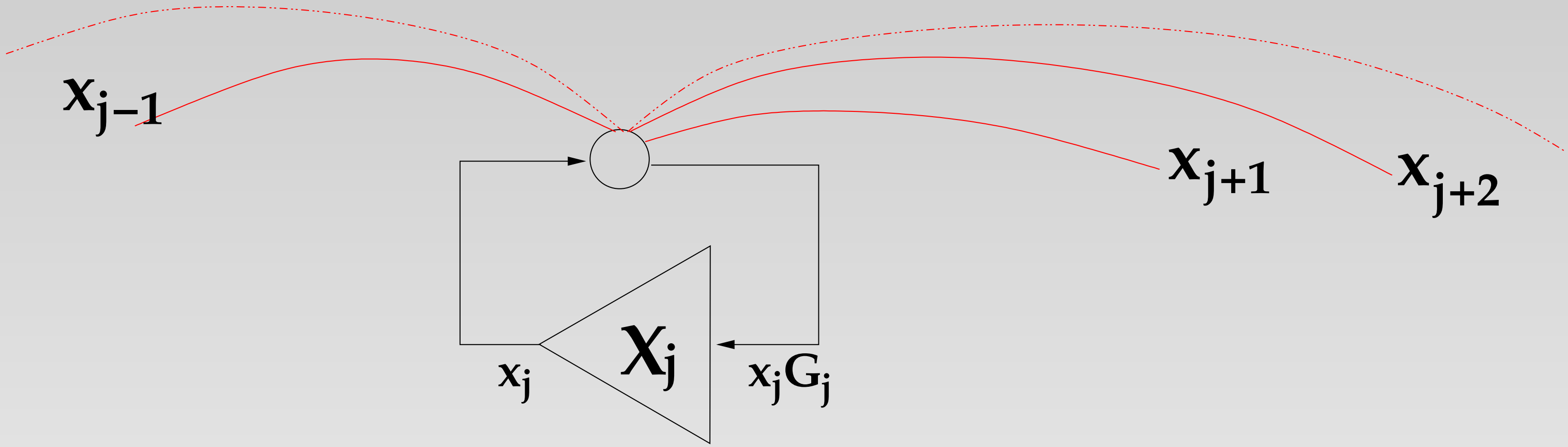
- Symmetric (+ reversible or Hamiltonian) systems.
- Population models based on Lotka-Volterra.
- ‘Semilinear’ feedback systems.
- Coupled cell systems (more later).

Symmetric systems



There is a theory of *equivariant transversality* for G-equivariant vector fields: a concept of 'G-transversal'; G-transversality is generic; intersections will generally be singular – locally diffeomorphic to algebraic varieties. Applications to equivariant dynamics, equivariant bifurcation theory (major), reversible and Hamiltonian (recent).

SLF systems



$$\dot{x}_j = f_j(x_j) + x_j G_j(x_1,..,\hat{x}_j,..,x_n), \quad f_j(0) = 0$$

If there are N cells, then all subspaces $x_{i_1} = \ldots = x_{i_k} = 0, 1 \leq i_1 < \ldots < i_k \leq N$, are flow invariant.

Heteroclinic cycles

Assume given an ODE $\mathbf{x}' = f(\mathbf{x})$ defined on the phase space $\mathbb{R}^n$.

Let $S_0, \ldots, S_N = S_0$ be a set of *nodes* which may be equilibria, limit cycles or more generally (hyperbolic) chaotic sets.

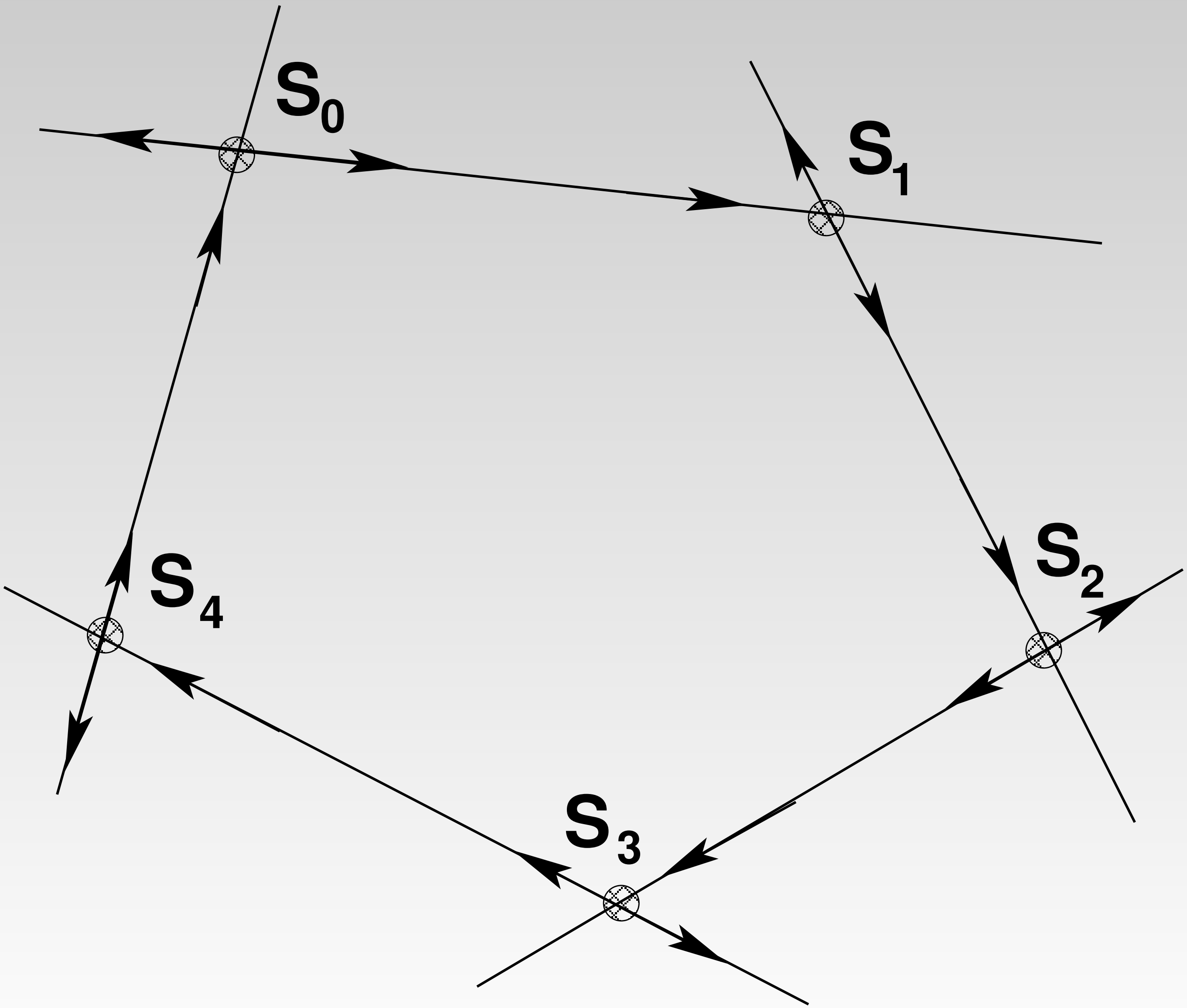
Assumptions
(1) We assume all of the nodes are saddles:

$$\dim(S_i) < \dim(W^u(S_i)) < n.$$

(2) We assume there exist connections between successive nodes:

$$W^u(S_i) \cap W^s(S_{i+1}) \neq \emptyset,\ 0 \leq i < N.$$

Heteroclinic cycles ctd.



Heteroclinic cycles ctd.

If these assumptions hold, we say we have a *heteroclinic cycle* linking the nodes $S_0, \ldots, S_N$.

This is very general. We say the heteroclinic cycle is *simple* if in addition

(3) Each node is an equilibrium.
(4) $\dim(W^u(S_i)) = 1$.

We might then define the 1-dimensional invariant set

$$\Sigma = \cup_i \overline{W^u(S_i) \cap W^s(S_{i+1})}$$

and refer to the *subset* $\Sigma \subset \mathbb{R}^n$ as a heteroclinic cycle. Under certain conditions Σ may be an asymptotically stable attractor.

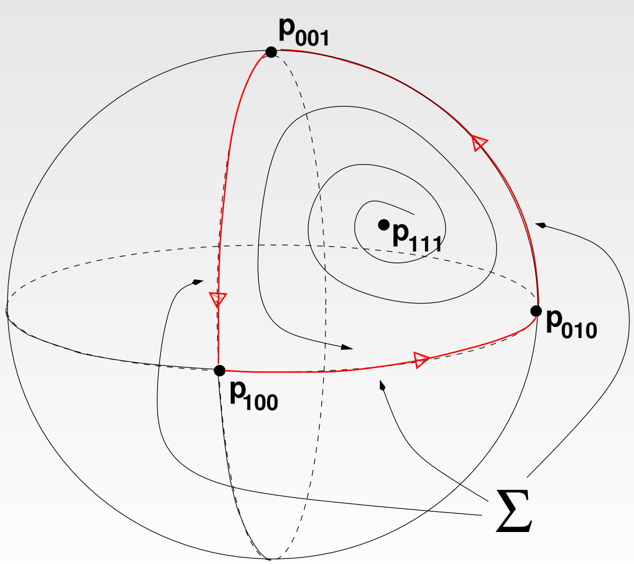
Robust heteroclinic cycles

If Σ is a simple cycle then at least some of the intersections $W^u(S_i) \cap W^s(S_{i+1})$ must be non-transverse. Consequently, without restrictions on the class of ODEs, heteroclinic cycles will never persist under all perturbations of the ODE.

However, if there are subspaces of phase space that are flow invariant for all vector fields in the class, then we can expect to see failure of genericity of transversality of invariant manifolds. This allows for the possibility of robust heteroclinic cycles.

Heteroclinic cycles

In both equivariant dynamics, population models (& **SLF** models), it is possible to have robust cycles of non-transverse saddle connections. First observed by May & Leonard (1975) (population dynamics), later by Dos Reis (1978) (equivariant dynamics on surfaces) and then by Guckenheimer and Holmes (1988) using an equation of Busse & Clever (equivariant bifurcation theory).



Dynamics on flow−invariant attracting sphere.

Symmetry group: $\mathbf{Z}_2^3 \blacksquare \mathbf{Z}_3$

Note the attracting heteroclinic cycle Σ

Heteroclinic cycles: interest

Models for intermittency.

Very much a feature of symmetric and population dynamics.

Of interest in dynamics which have *approximately* invariant subspaces. More precisely: Given a system with lots of invariant subspaces, we might expect to see many different types of heteroclinic cycle. Under small (general) perturbations, the invariant subspaces may disappear and the heteroclinic cycles may bifurcate into periodic orbits. This is analytically quite tractable when we have attracting simple cycles.

Switching.

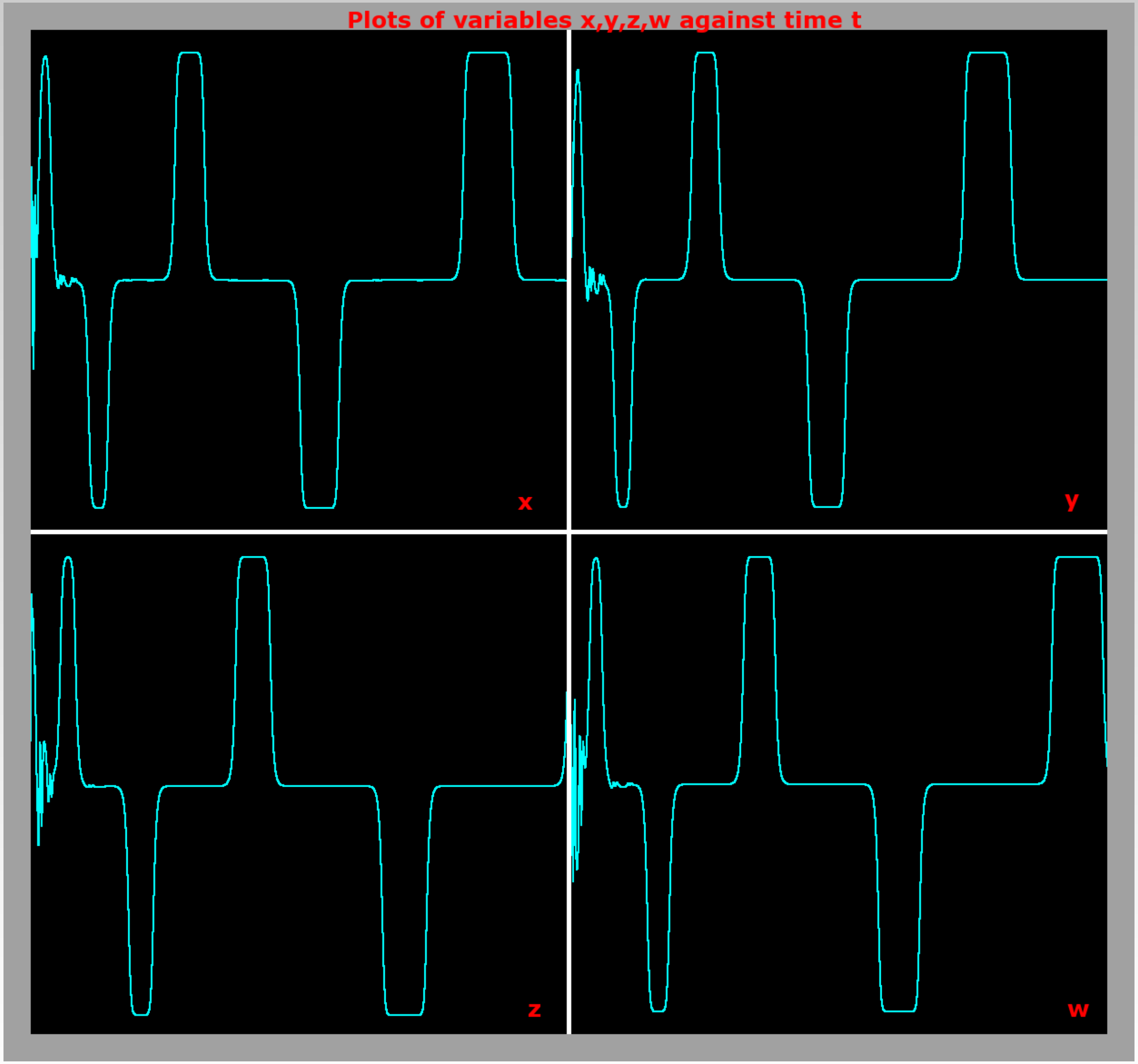
A second example

The (symmetric) system

$$\begin{aligned}
\mathbf{x}' &= \mathbf{x} - \mathbf{x}(\mathbf{x}^2 + a\mathbf{y}^2 + b\mathbf{z}^2 + c\mathbf{w}^2) + d\mathbf{yzw}, \\
\mathbf{y}' &= \mathbf{y} - \mathbf{y}(\mathbf{y}^2 + a\mathbf{z}^2 + b\mathbf{w}^2 + c\mathbf{x}^2) - d\mathbf{zwx}, \\
\mathbf{z}' &= \mathbf{z} - \mathbf{z}(\mathbf{z}^2 + a\mathbf{w}^2 + b\mathbf{x}^2 + c\mathbf{y}^2) + d\mathbf{wxy}, \\
\mathbf{w}' &= \mathbf{w} - \mathbf{w}(\mathbf{w}^2 + a\mathbf{x}^2 + b\mathbf{y}^2 + c\mathbf{z}^2) - d\mathbf{xyz},
\end{aligned}$$

defined on $\mathbb{R}^4$ has equilibria at $(\pm 1, 0, 0, 0), \ldots, (0, 0, 0, \pm 1)$. For appropriate values of $a, \ldots, d$ there is a network of connections between these equilibria. (If $d = 0$, we can get an attracting cycle between $(1, 0, 0, 0), \ldots, (0, 0, 0, 1)$.)

Numerics



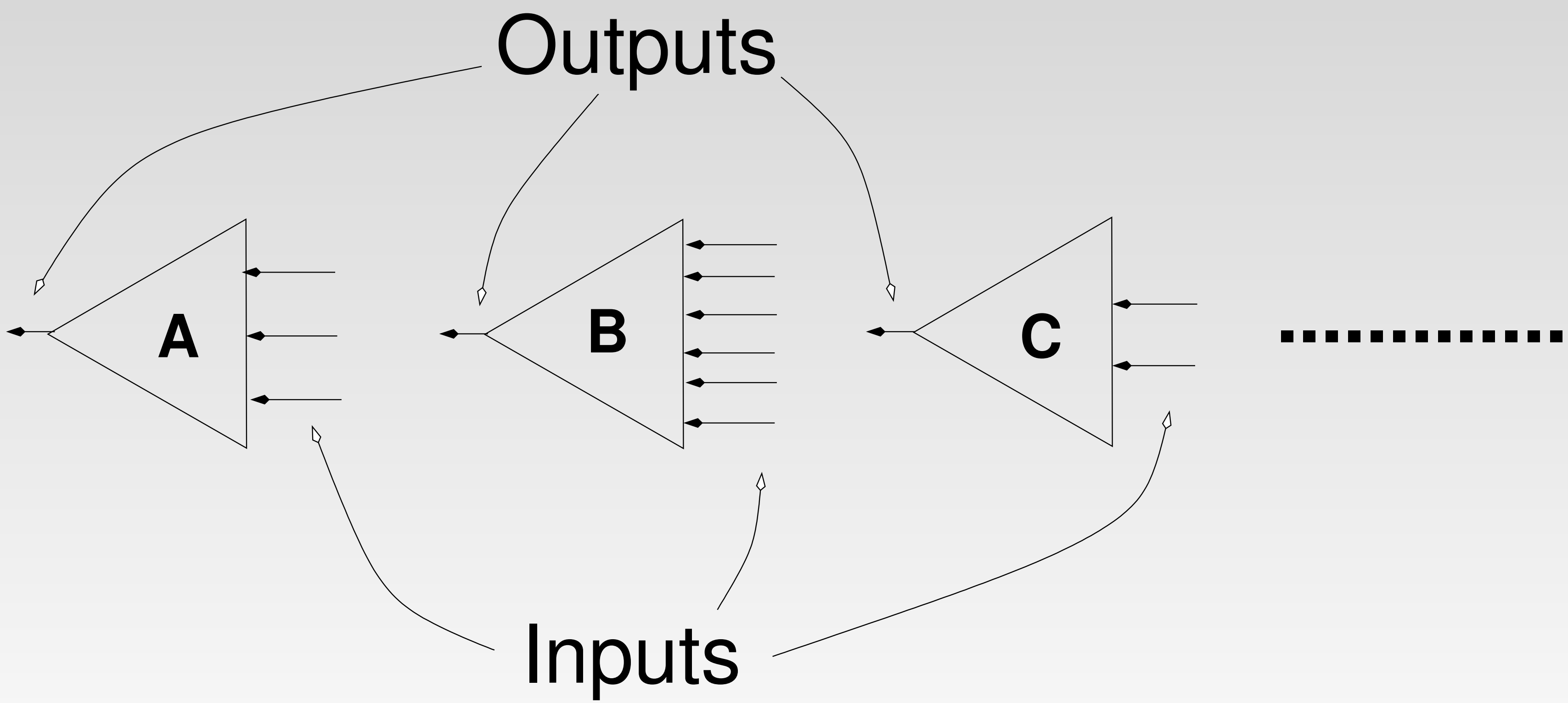
Aim of talk

Our aim in this talk is to show that (simple) heteroclinic cycles are a very common phenomenon in coupled cell systems. As a result we can expect to often see dynamical phenomena like periodic switching between synchronous states in a coupled cell system.

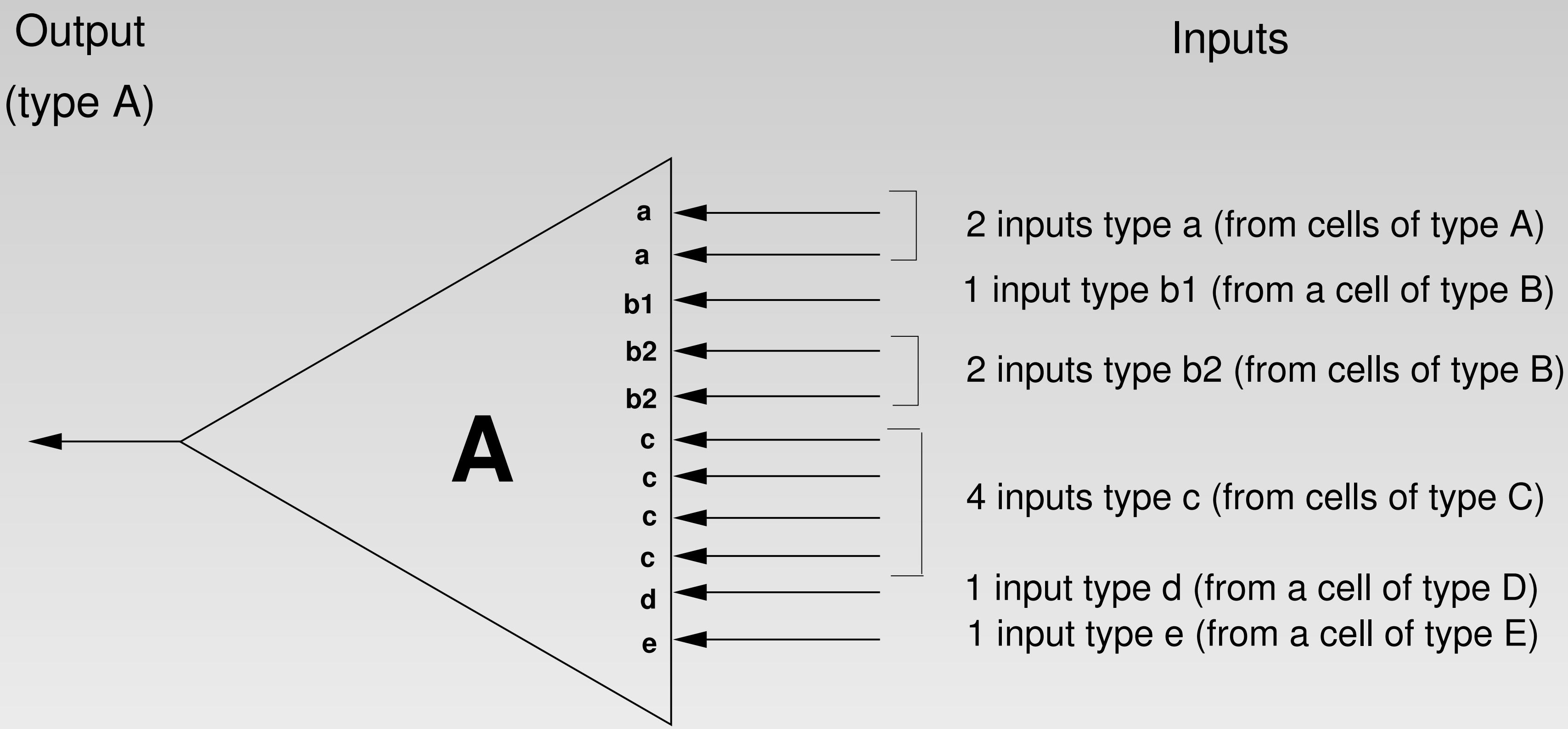
We start by reviewing the concepts of a coupled cell system and synchrony class.

Coupled cell systems: Cell types

We shall be looking at a finite collection of different cell *types*. We write these A, B, C, ... Each cell has a finite number of inputs and an output.



Cells: Inputs



A given cell type may receive inputs from cells of various types. In the figure, a cell of type A receives inputs from cells of types A, B, C, D and E.

Patchcord rules

We interconnect cells using *patchcords*. A type $\mathbf{a}$ patchcord goes from the output of a cell of type $\mathbf{A}$ to the $\mathbf{a}$ input of a cell. If there are type $\mathbf{a1}$, $\mathbf{a2}$,. . . inputs, then we colour code patchcords so as to indicate which type of input the cord should be patched into.

There are no restrictions on the number of outputs we take from a cell.

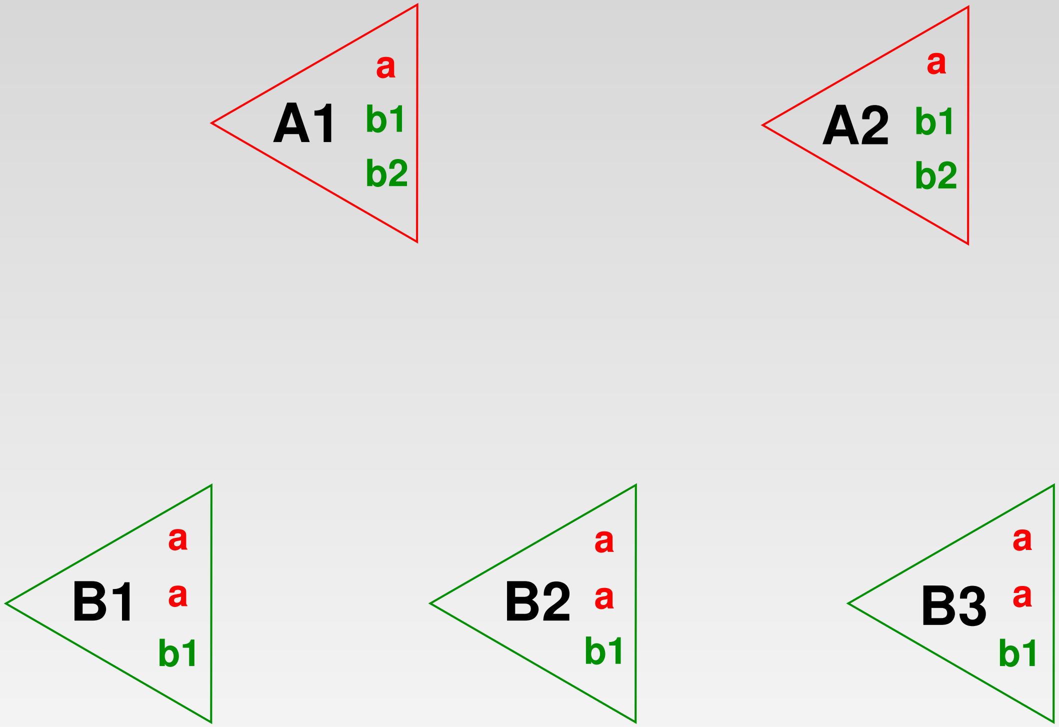
No more than one patchcord is plugged into a given input.

Normally we regard patchcords as ‘dynamically neutral’. However, patchcords could include, for example, a delay line.

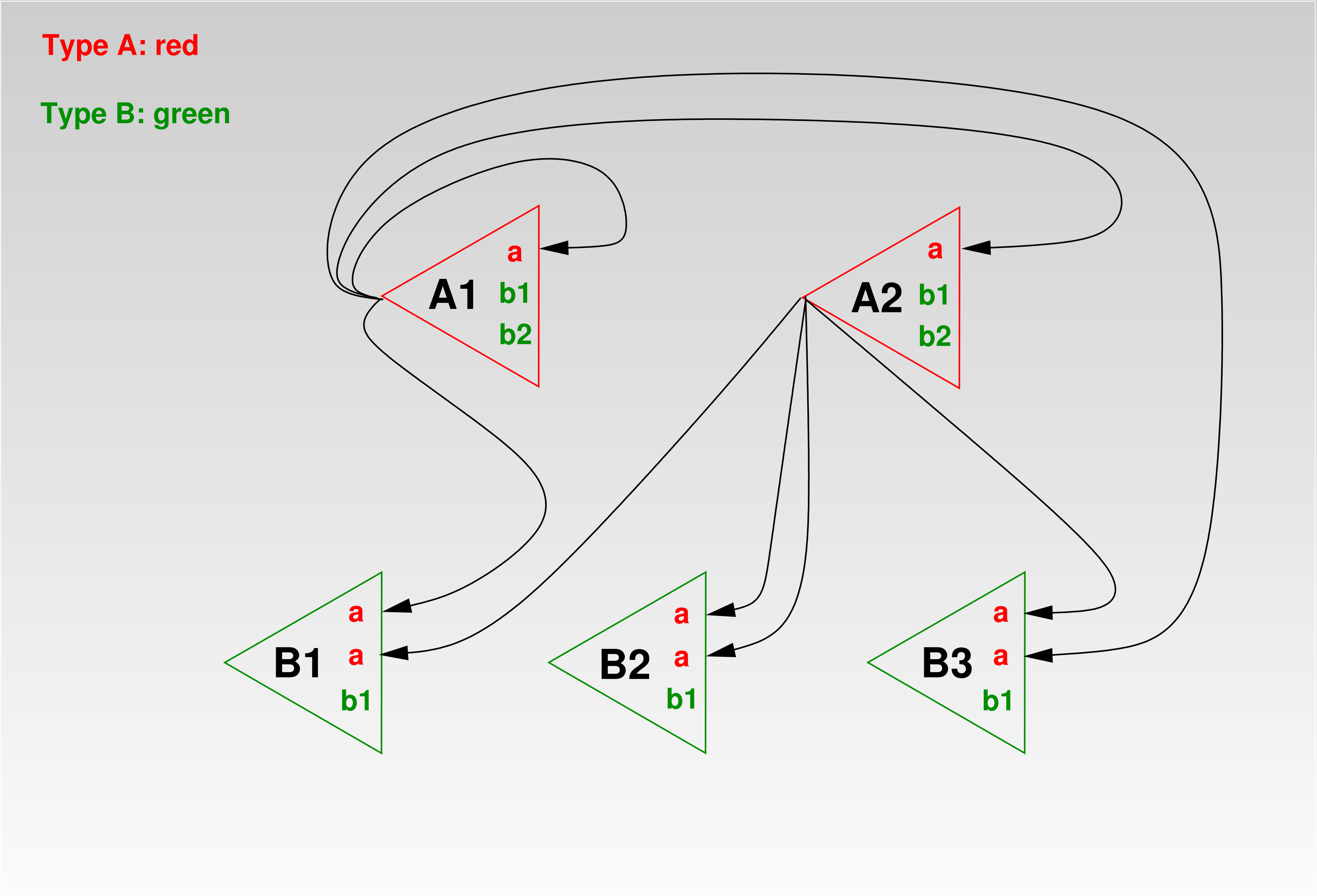
Example

Type A: red

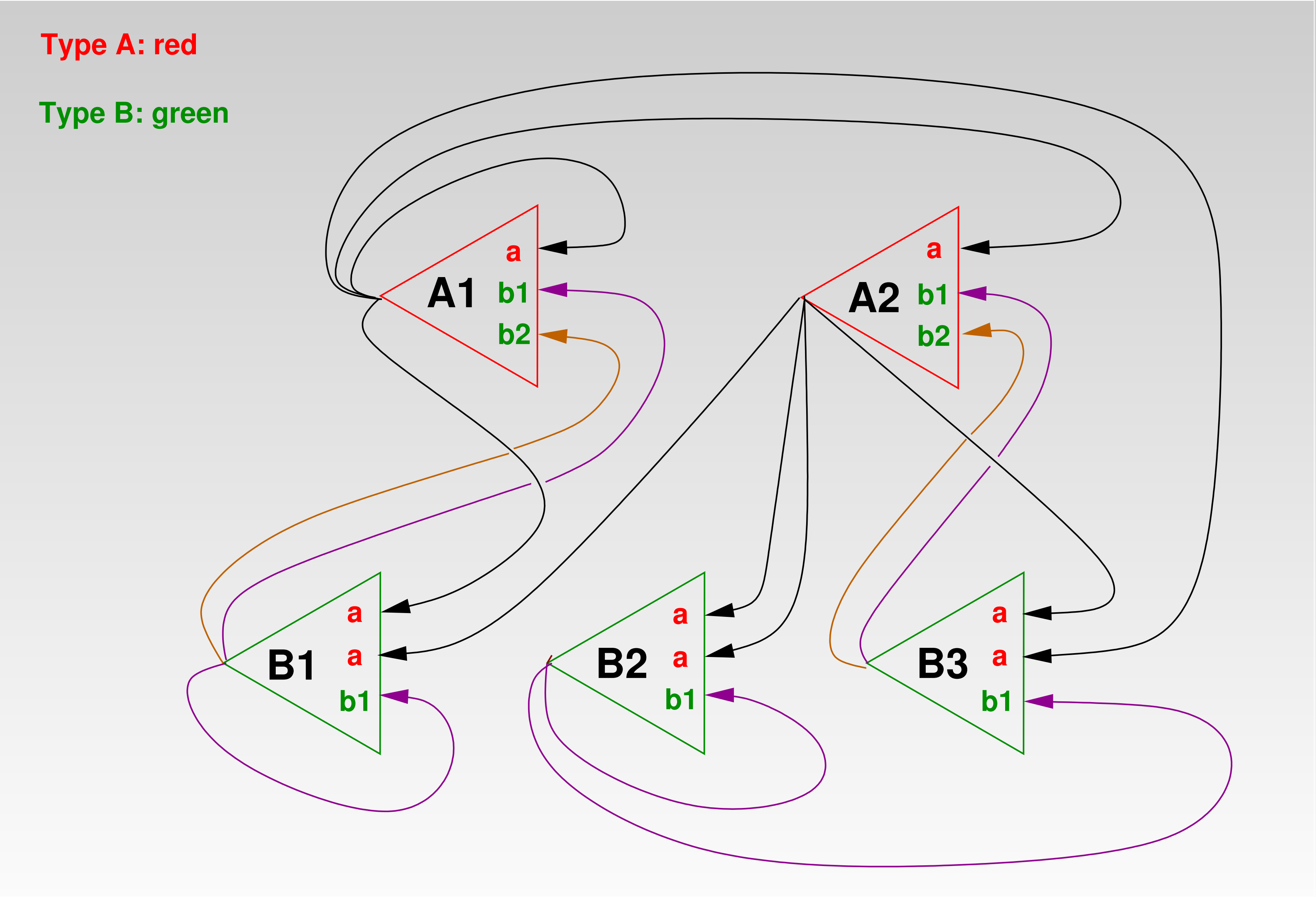
Type B: green



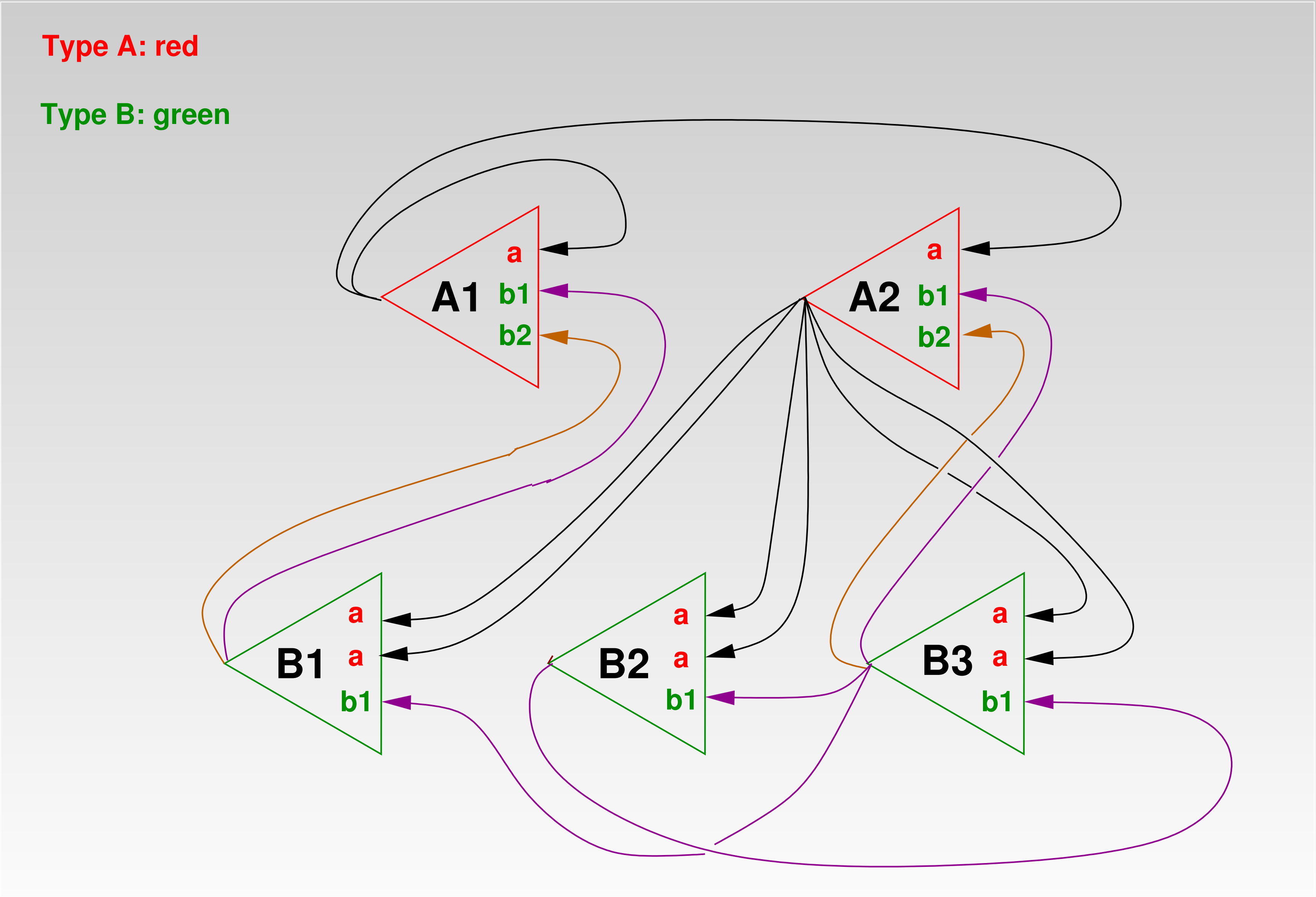
Patching the a inputs.



Patching the b inputs.



Another Patching.



ODE representation

In terms of ODEs we represent the previous coupled cell system by

$$\begin{aligned}
\mathbf{A1}' &= F(\mathbf{A1}; \mathbf{A1}, \mathbf{B1}, \mathbf{B1}), \\
\mathbf{A2}' &= F(\mathbf{A2}; \mathbf{A1}, \mathbf{B3}, \mathbf{B3}), \\
\mathbf{B1}' &= G(\mathbf{B1}; \mathbf{B3}, \mathbf{A2}, \mathbf{A2}), \\
\mathbf{B2}' &= G(\mathbf{B2}; \mathbf{B3}, \mathbf{A2}, \mathbf{A2}), \\
\mathbf{B3}' &= G(\mathbf{B3}; \mathbf{B2}, \mathbf{A2}, \mathbf{A2}).
\end{aligned}$$

The vector field G is symmetric in the $\mathbf{A}$-variables but F is *not* symmetric in the $\mathbf{B}$-variables.

Coupled cell systems

For us, a coupled cell system will consist of

- A finite number of cells (and cell types).

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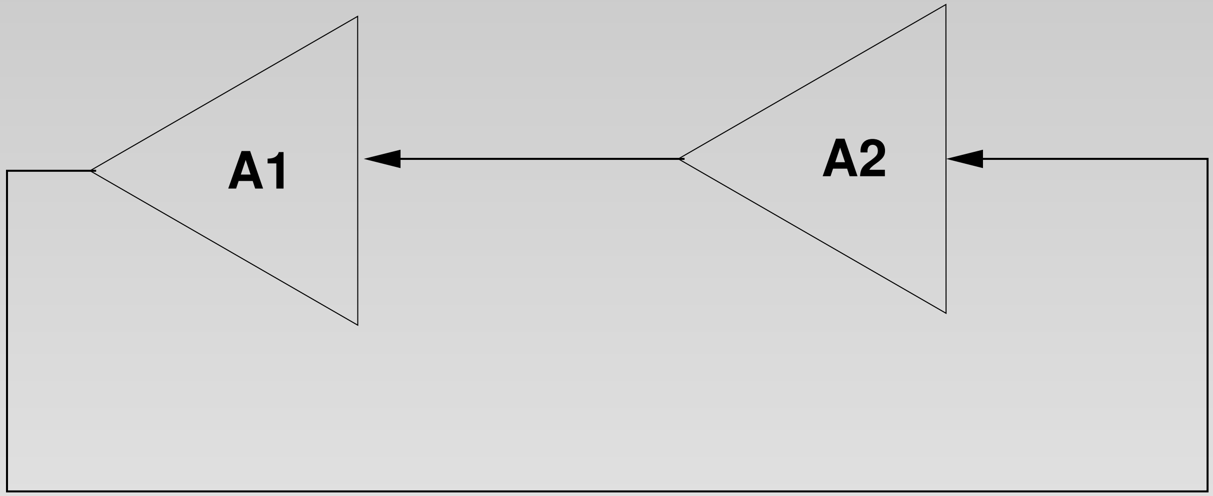
- No inputs will be left unfilled.
- There are no restrictions on the number of outputs from a cell of given type.
- Evolution of cells governed by (say) ODEs — in particular, evolution is *uniquely* determined by initial state.

Invariant subspaces

Given: a coupled cell system. We are interested initially in synchronised solutions of the system. These correspond to certain types of invariant subspace of the phase space.

We illustrate the ideas with some simple examples.

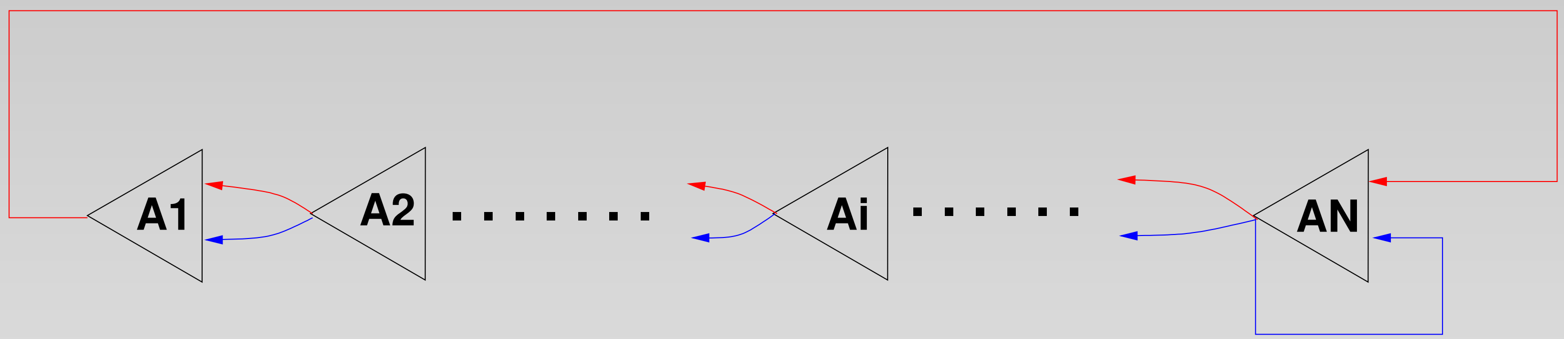
Examples



Two identical cells

The only invariant subspace of synchronous solutions corresponds to both cells being synchronized. We write this $\{\mathbf{A1}, \mathbf{A2}\}$.
(This property is true for *all* coupled cell networks – trivial synchronised state.)

Examples ctd.

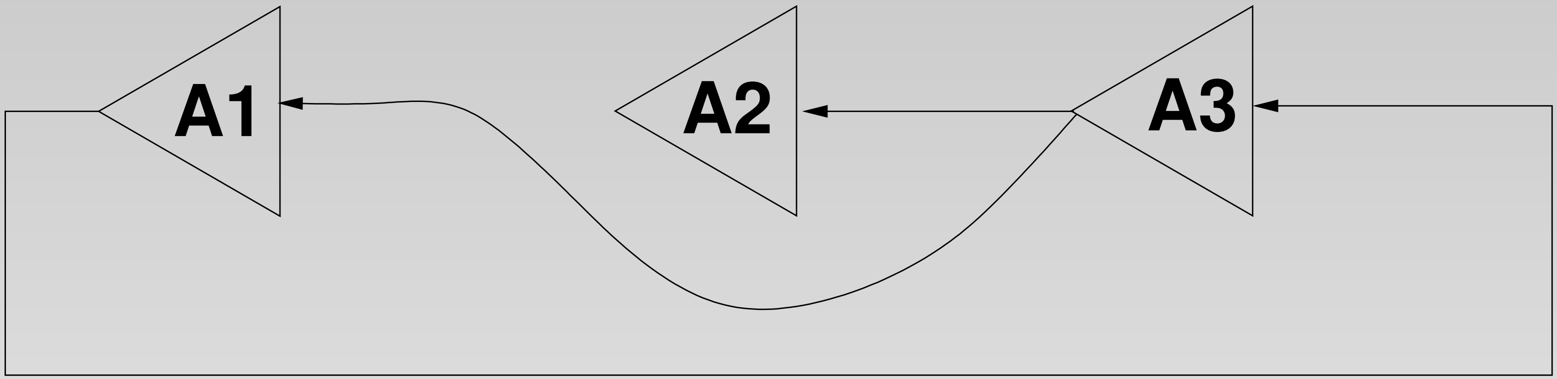


N identical cells
Each cell has two inputs of (different) type.

The only invariant subspace of synchronous solutions is $\{\mathbf{A1}, \ldots, \mathbf{AN}\}$. If $N \geq 2$ is *prime*, we can make do with single input cells.

We call an invariant subspace (synchrony class) which contains no proper invariant subspaces (sub-synchrony class) a *synchrony atom* – or just an *atom*.

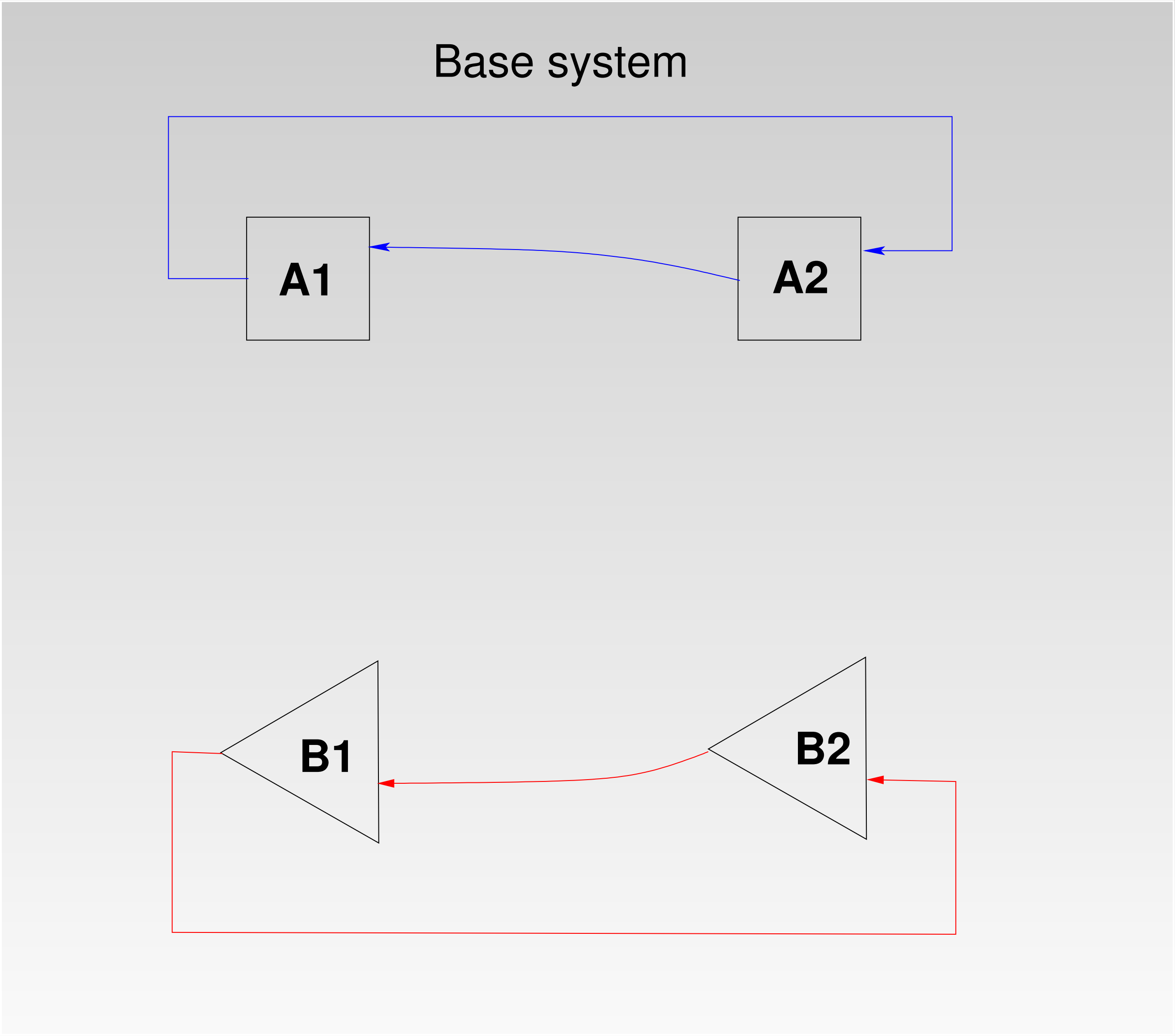
Examples ctd.



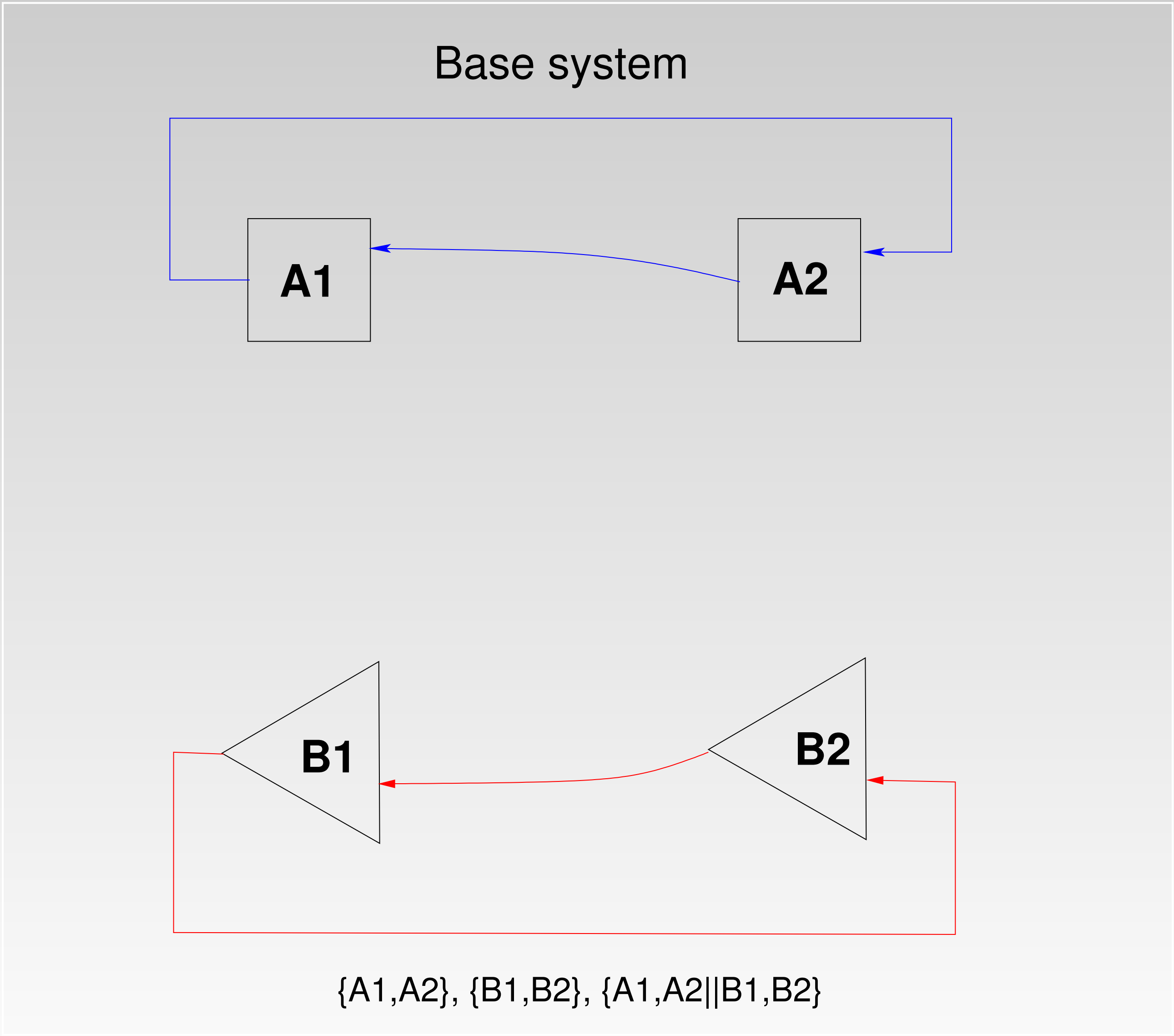
Three identical single input cells

This network has three invariant subspaces of synchronous solutions:
$\{\mathbf{A1}, \mathbf{A2}, \mathbf{A3}\}$ (trivial symchronized state).
$\{\mathbf{A1}, \mathbf{A2}\}$,
$\{\mathbf{A1}, \mathbf{A3}\}$.
Note that $\{\mathbf{A2}, \mathbf{A3}\}$ is *not* an invariant subspace.

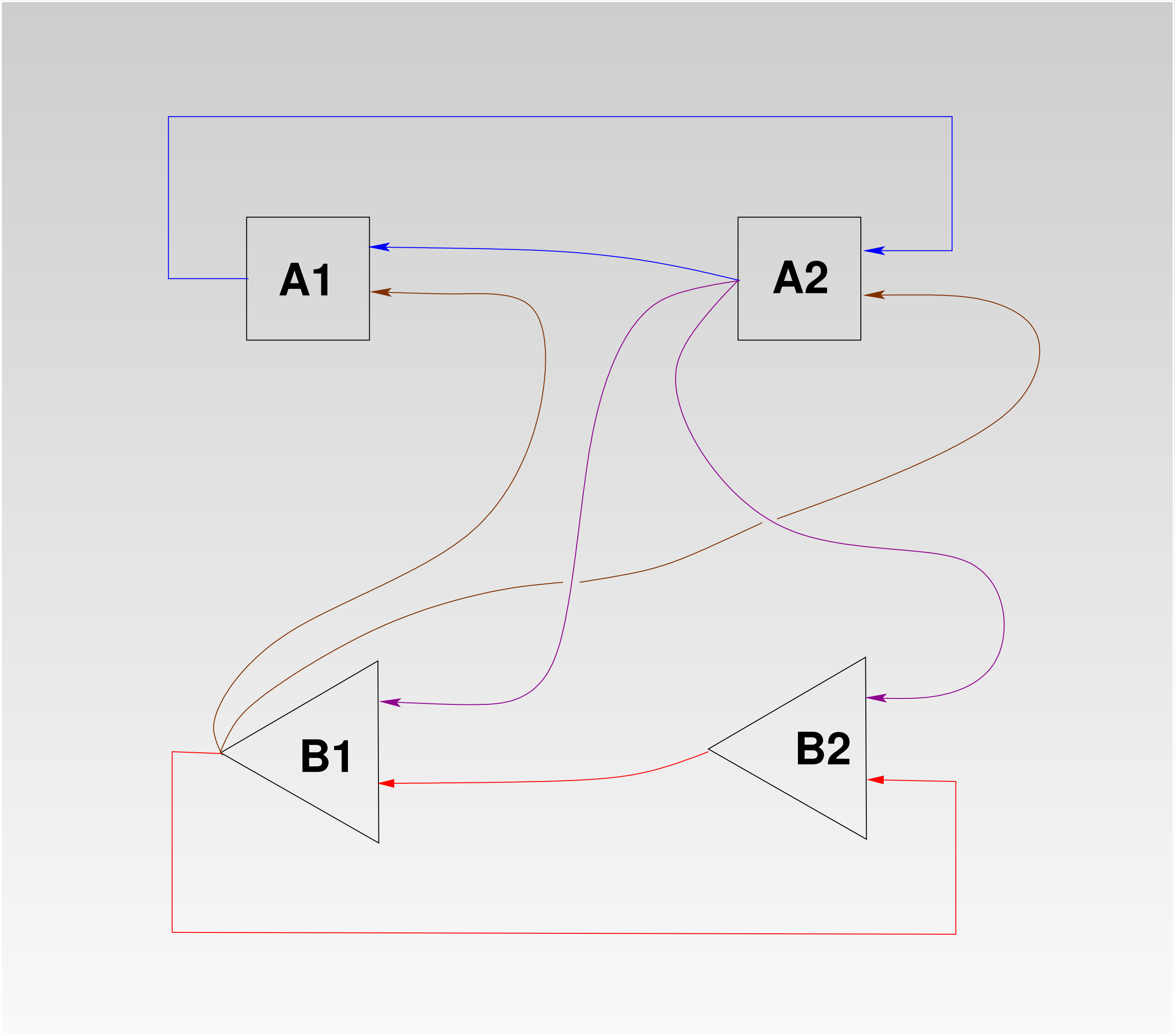
Variations on a 4 cell system



Variations on a 4 cell system



First Variation



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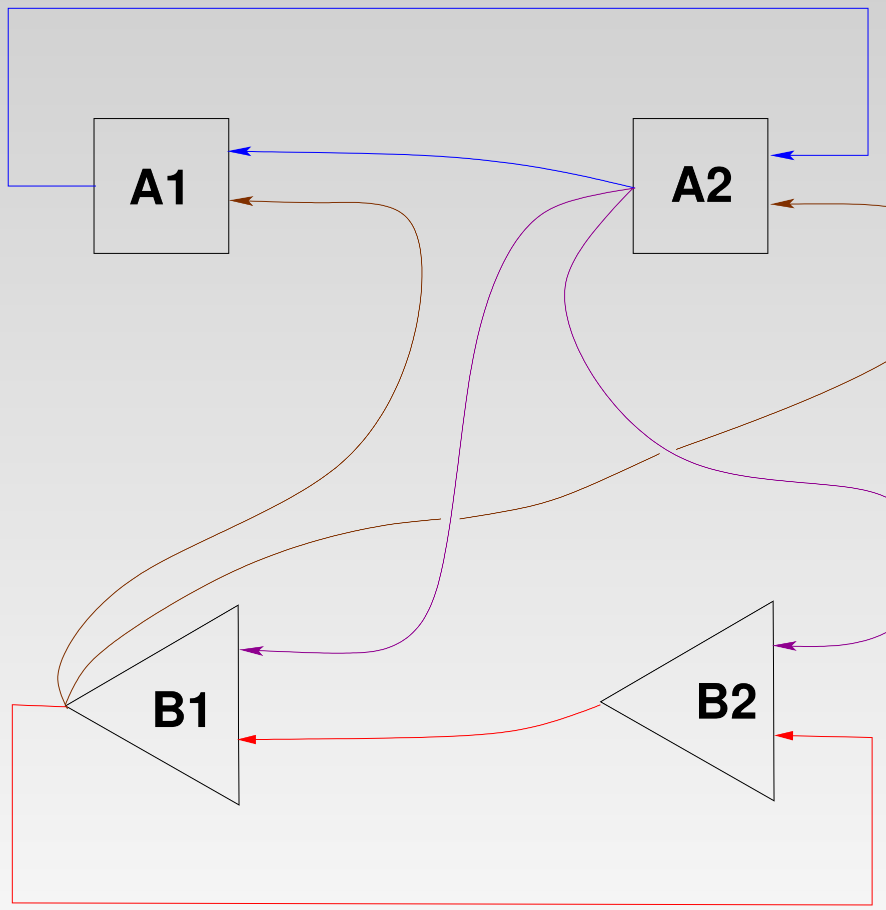
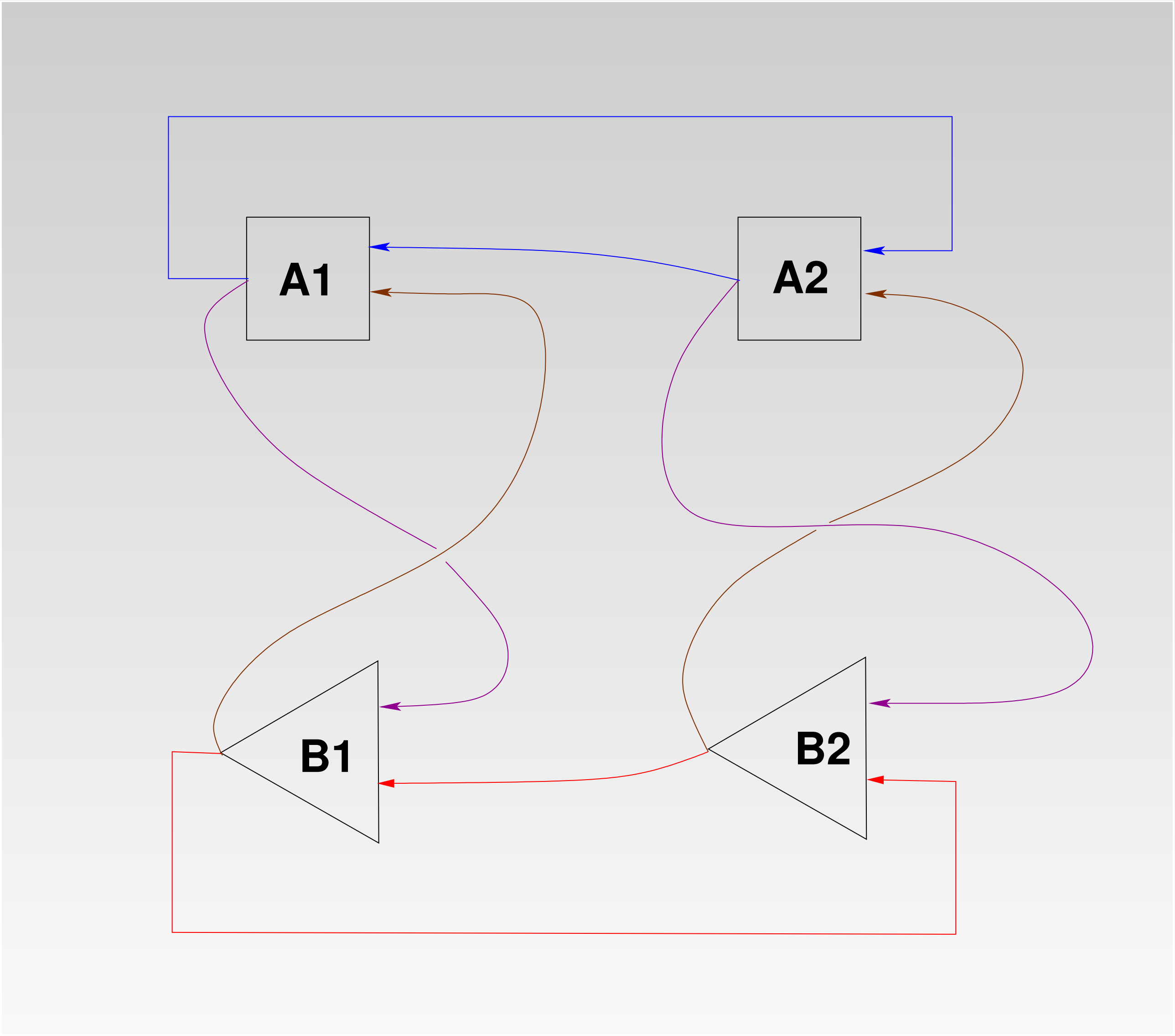


{A1,A2}, {B1,B2}, {A1,A2||B1,B2}

Second Variation



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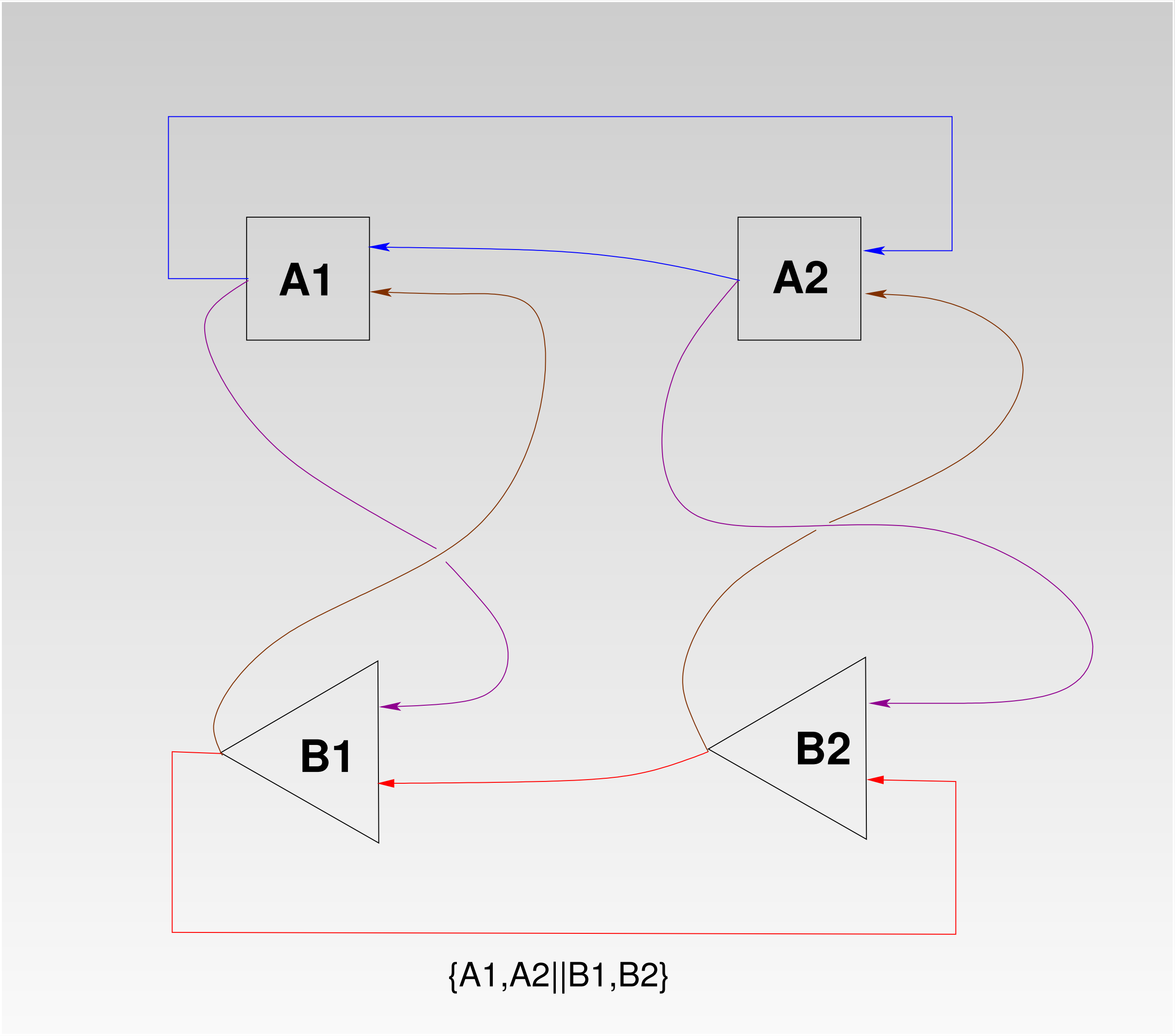
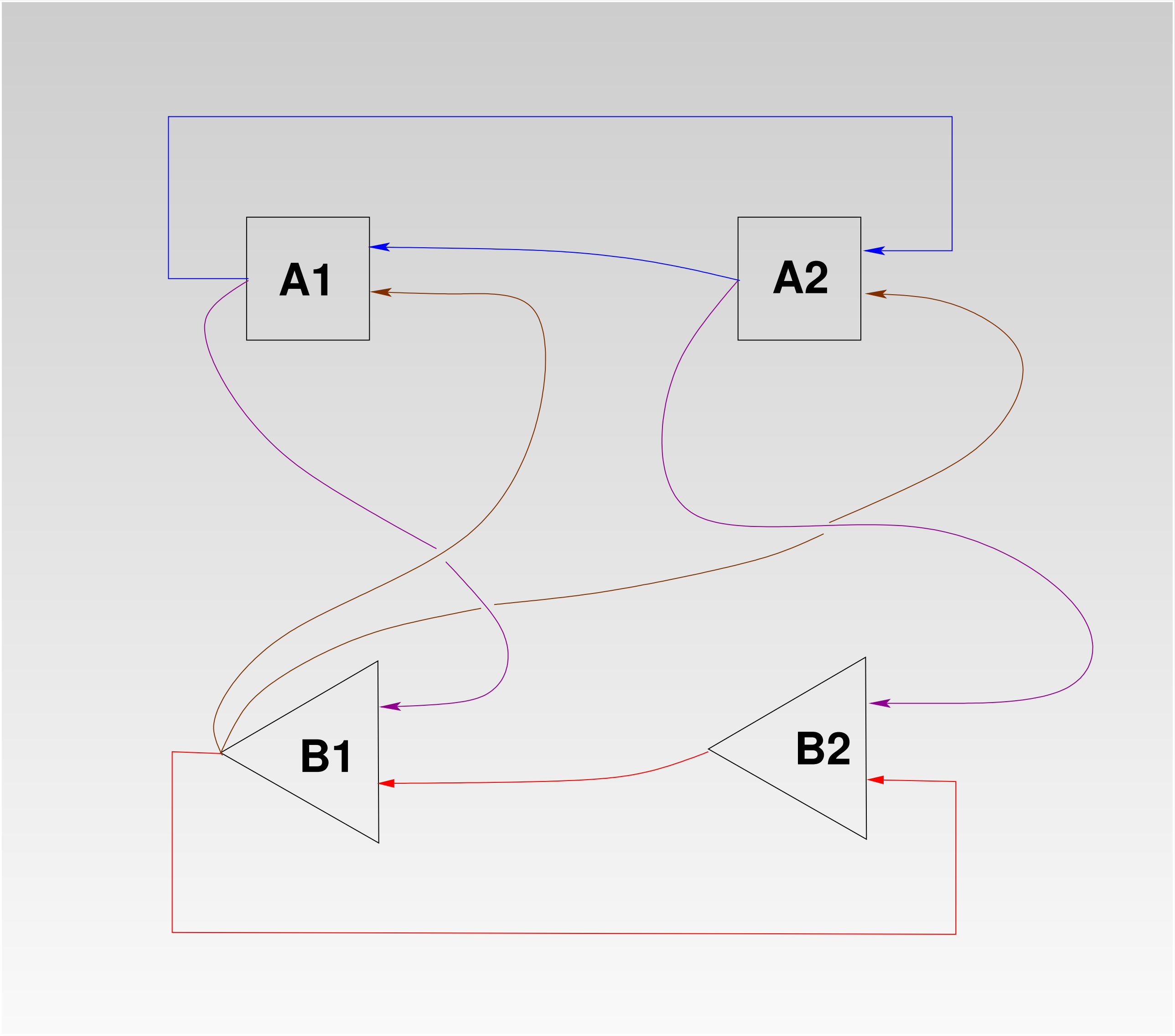


Third Variation



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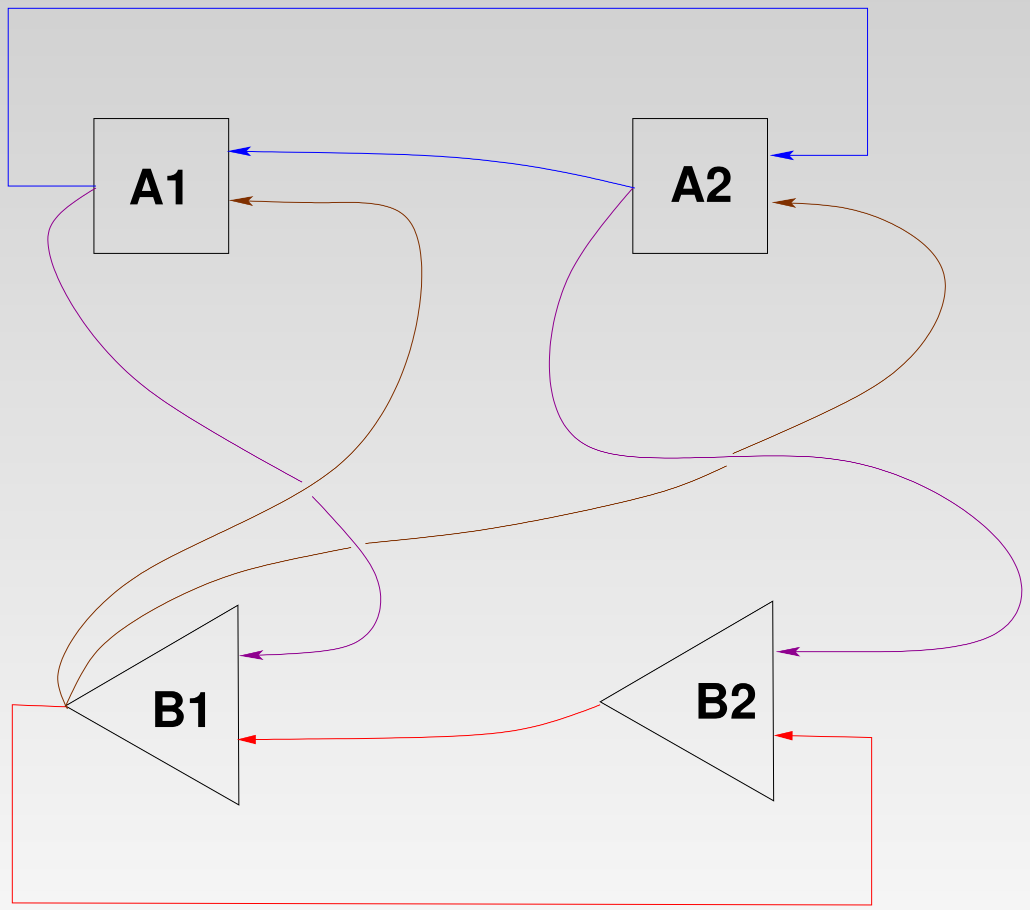


{A1,A2}, {A1,A2||B1,B2}

Some Metatheorems

Associated to a coupled cell system $\mathcal{C}$ are finitely many cell types, each cell type with a prescribed number of inputs, and a set of connections between all pairs of cells. Obviously, this structure can be represented as a graph $\Gamma = \Gamma(\mathcal{C})$ with directed (labelled) edges representing connections and (labelled) nodes representing cells.

The graph $\Gamma(\mathcal{C})$ uniquely determines the set of synchrony classes (invariant subspaces).

Let $\mathcal{U}$ denote the set of all graphs of coupled cell systems.

Synchronous attractors

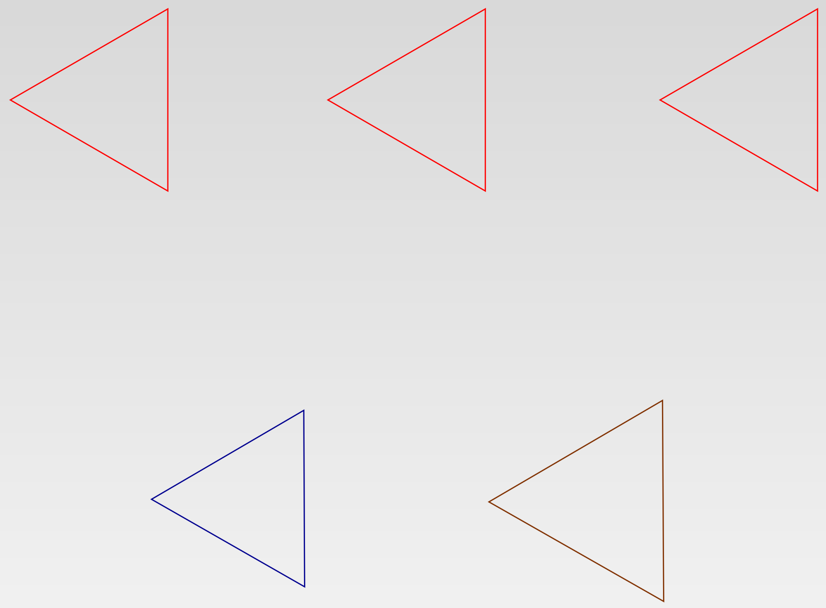
METATHEOREM 1
Suppose $\Gamma \in \mathcal{U}$ and that $\mathcal{S}$ is a synchrony class for Γ. Then there exists a coupled cell system $\mathcal{C}$ such that

- $\mathcal{C}$ has graph Γ.
- There is a hyperbolic attracting equilibrium for $\mathcal{C}$ with synchrony precisely $\mathcal{S}$.

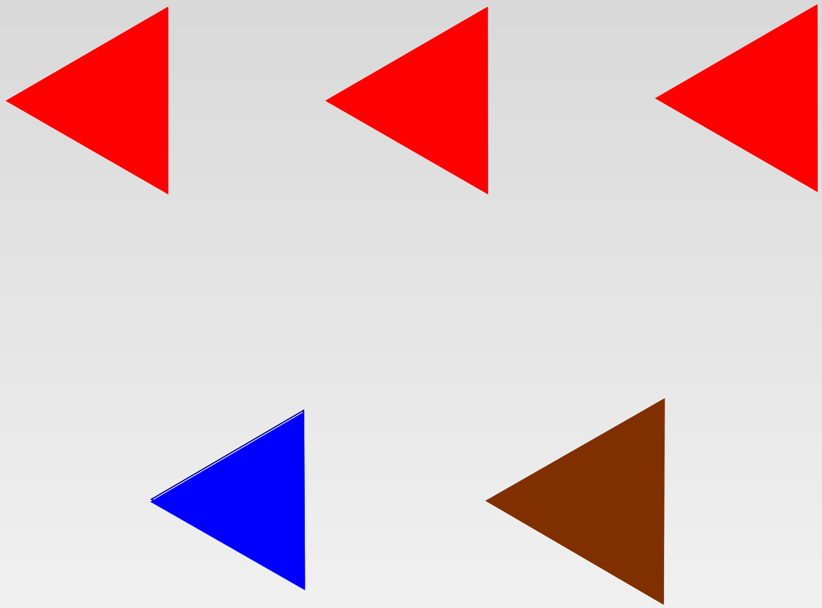
This system can be realized without any restriction on phase space dimensions.

A similar result holds for periodic attractors - in this case phase space dimensions will be at least two for the cells associated to the synchrony class.

Proof...



Proof ctd.



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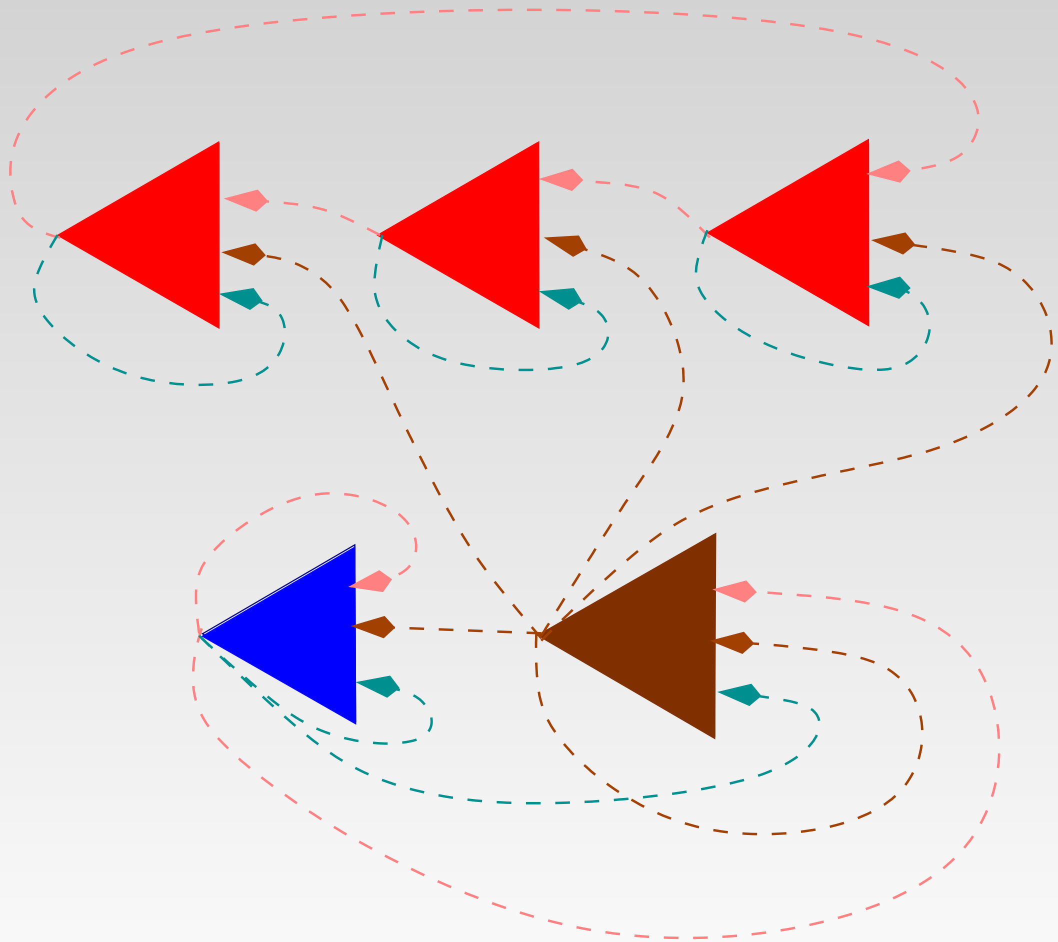

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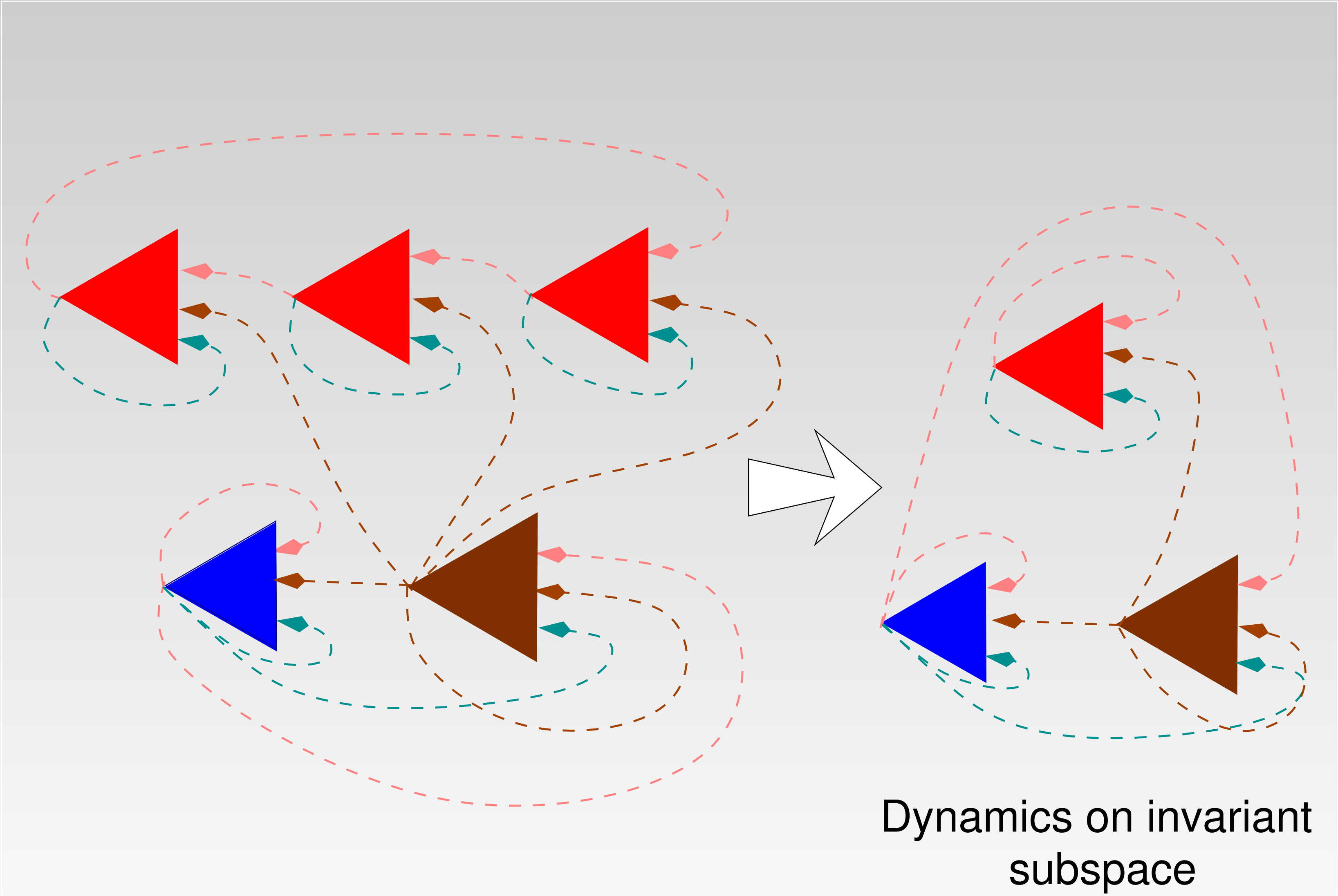


1-cycles

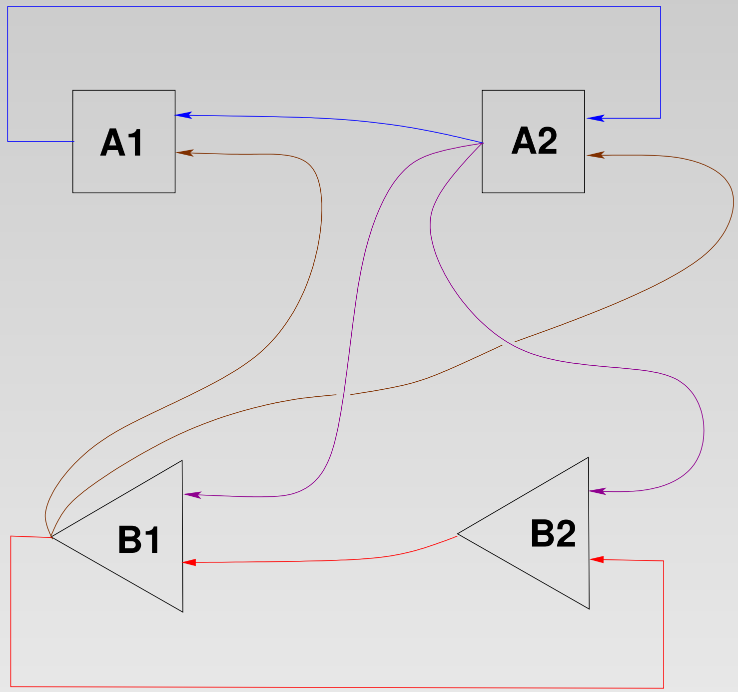
METATHEOREM 2. Let $\Gamma \in \mathcal{U}$ and suppose that $\mathcal{S}_0, \ldots, \mathcal{S}_N = \mathcal{S}_0$ are synchrony atoms. Assume that

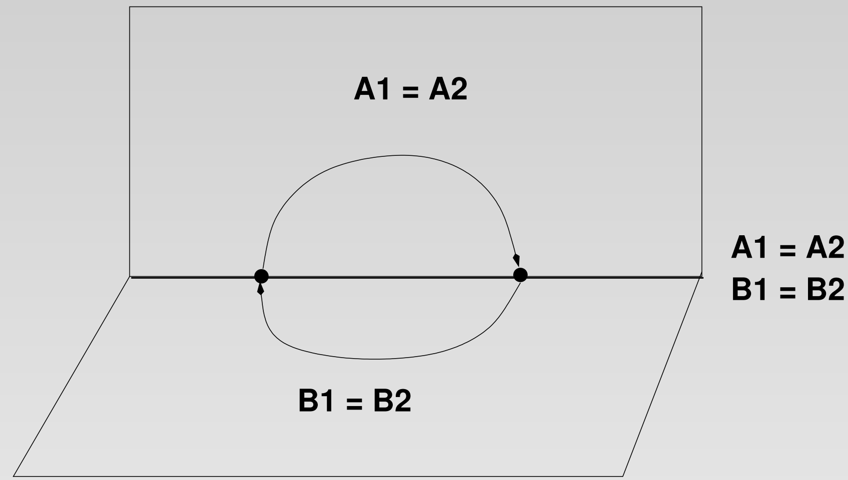
- $\mathcal{S}_i \cap \mathcal{S}_{i+1} = \emptyset,\ 0 \leq i \leq N-1,\ N \geq 2$.
- There exists at least one connection from an $\mathcal{S}_i$-cell to an $\mathcal{S}_{i+1}$-cell, $0 \leq i \leq N-1$.

There exists a coupled cell system $\mathcal{C}$ with graph Γ, which supports an attracting simple cycle with node set $(\mathcal{S}_0, \mathcal{S}_1), \ldots, (\mathcal{S}_{N-1}, \mathcal{S}_0)$. There are no phase space dimension restrictions.

We refer to this type of heteroclinic cycle as an 1-cycle. The connection from $(\mathcal{S}_i, \mathcal{S}_{i+1})$ to $(\mathcal{S}_{i+1}, \mathcal{S}_{i+2})$ will consist of $\mathcal{S}_{i+1}$-synchronized equilibria.

Example: $N = 2$





Here $\mathcal{S}_0 = \{\mathbf{A1}, \mathbf{A2}\}$, and $\mathcal{S}_1 = \{\mathbf{B1}, \mathbf{B2}\}$.

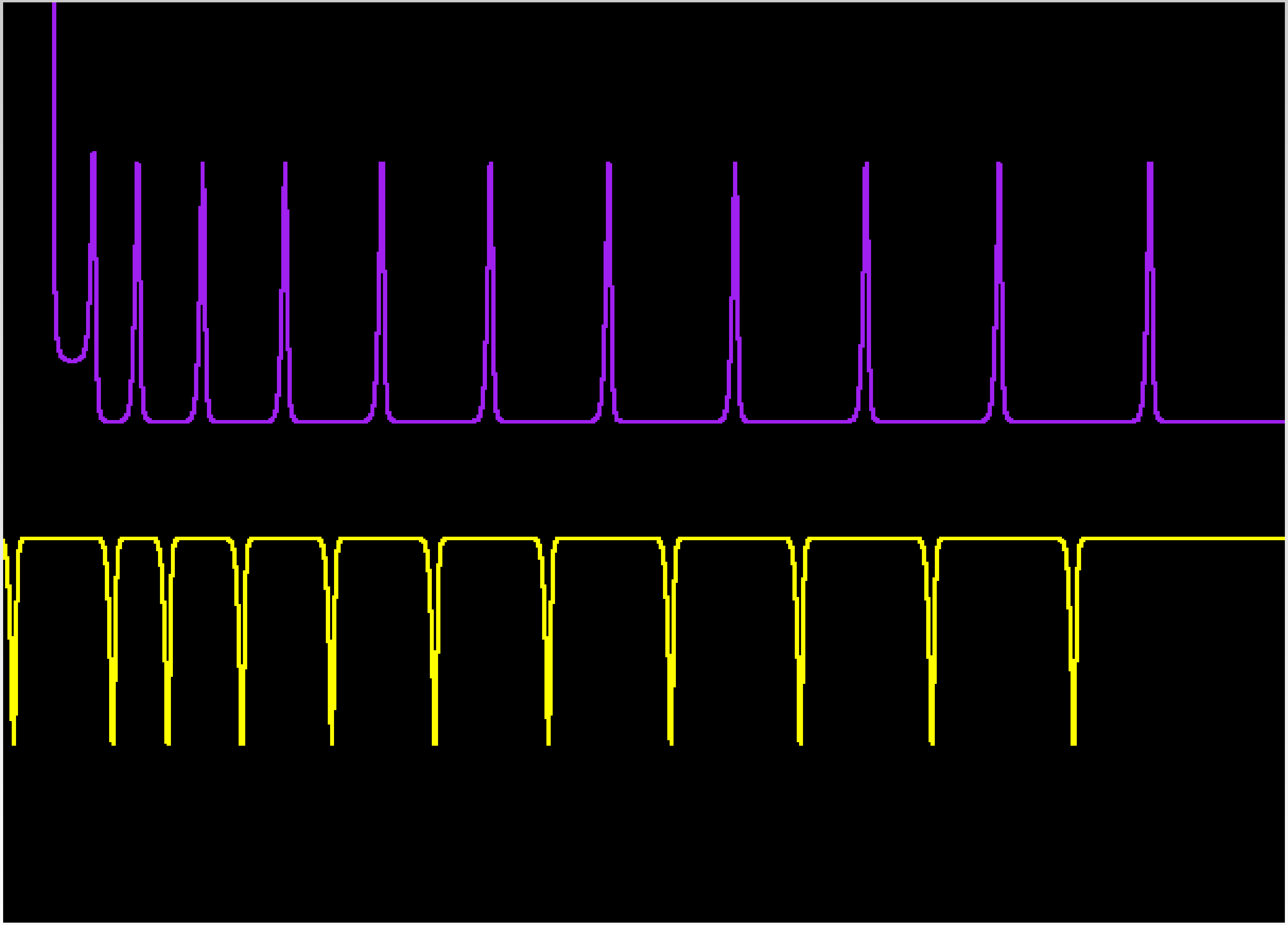
Phase oscillator example

$$g(\theta) = \sin(\theta + a) + r\sin(2\theta).$$

$$\begin{aligned}
\theta_1' &= \alpha g(\theta_1 - \theta_3) + \beta g(\theta_1 - \theta_2), \\
\theta_2' &= \alpha g(\theta_2 - \theta_3) + \beta g(\theta_2 - \theta_1), \\
\theta_3' &= \alpha g(\theta_3 - \theta_2) + \beta g(\theta_3 - \theta_4), \\
\theta_4' &= \alpha g(\theta_4 - \theta_2) + \beta g(\theta_4 - \theta_3).
\end{aligned}$$

$$r = 0.2,\ a = 1.28,\ \alpha = \beta = 1.0$$

Plots of $\theta_1 - \theta_2$, $\theta_3 - \theta_4$



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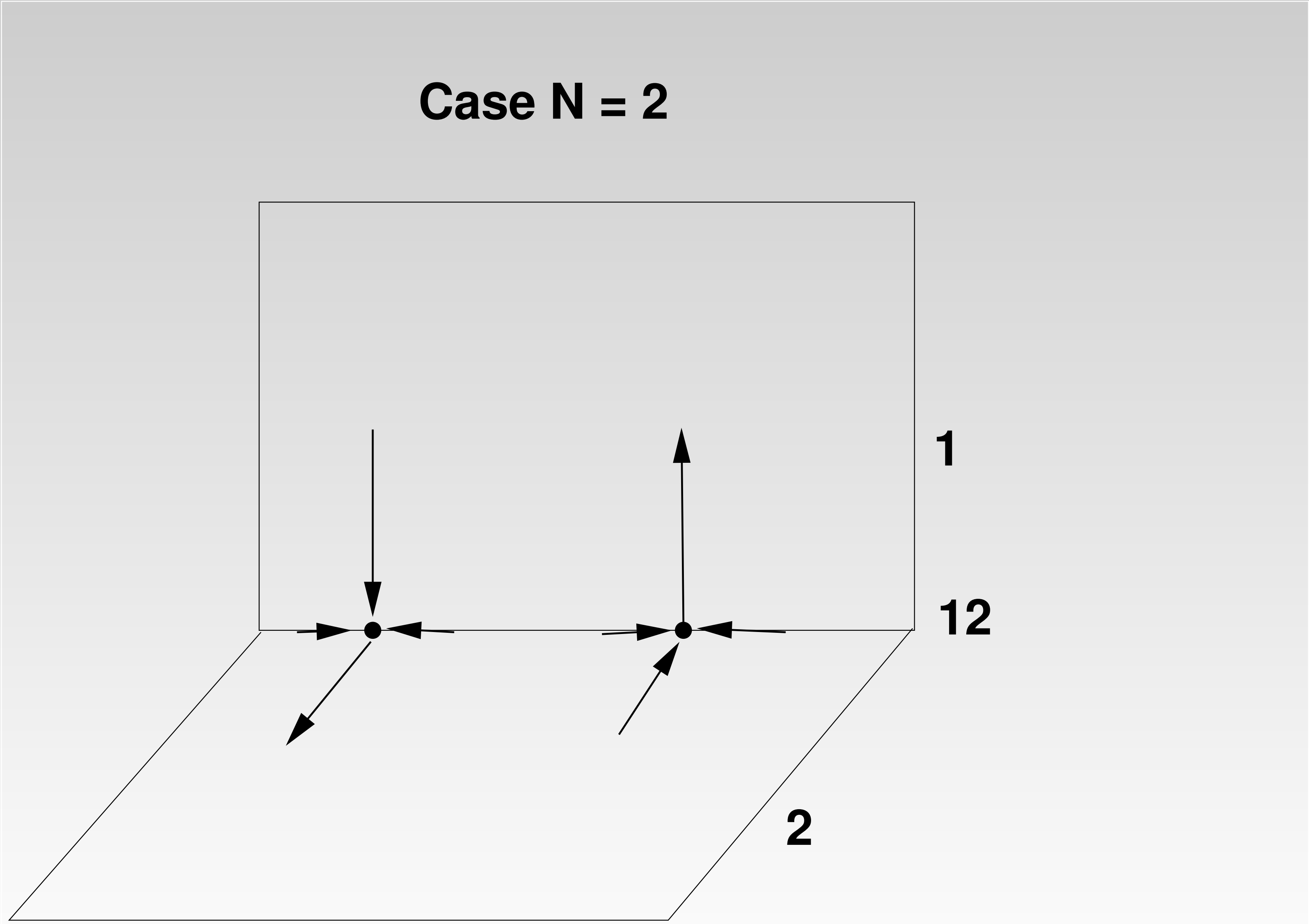

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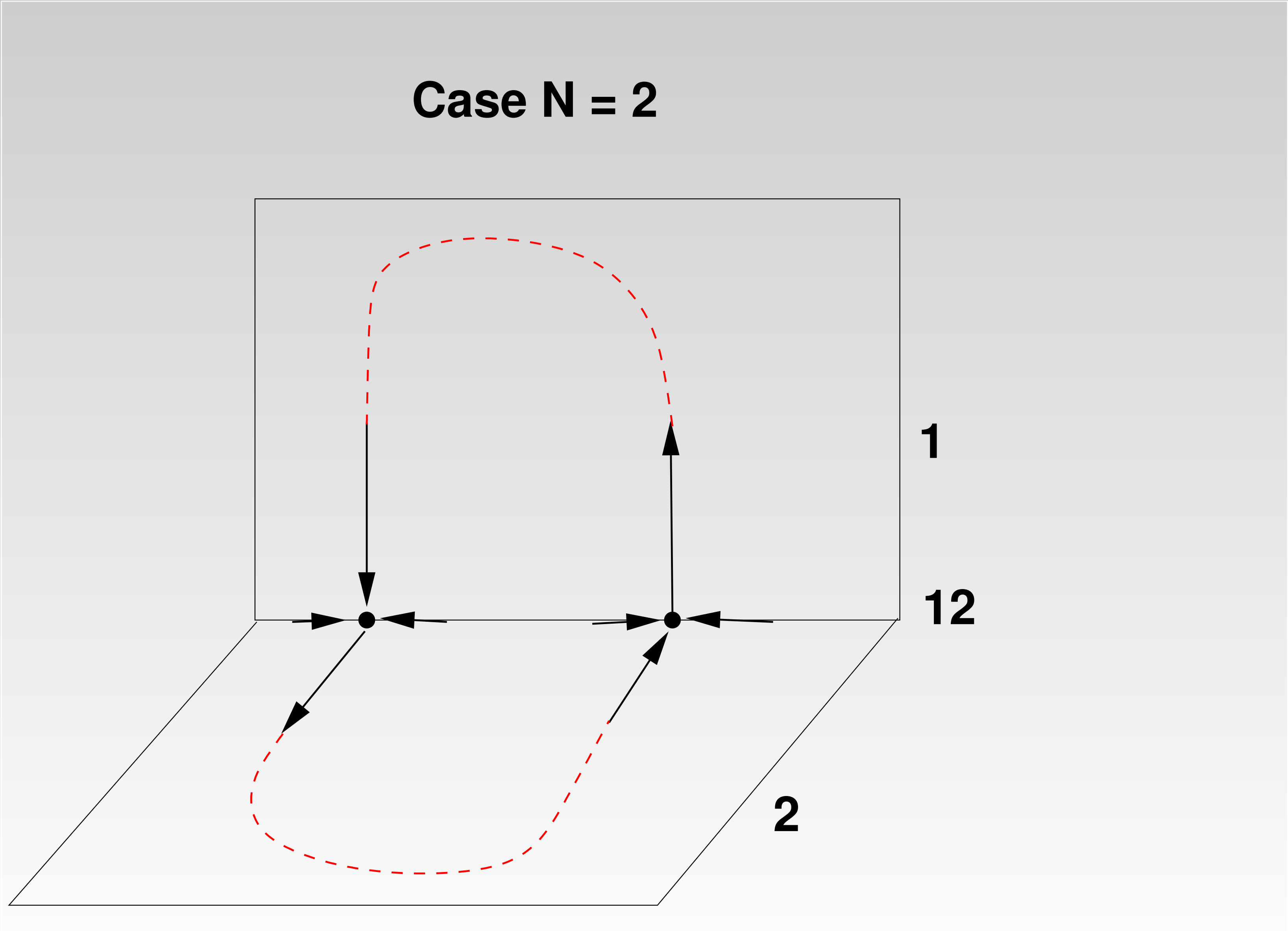
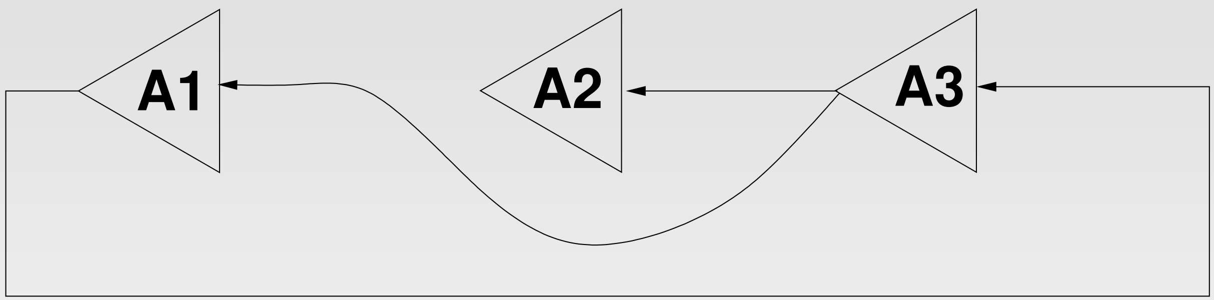


1-cycles ctd.

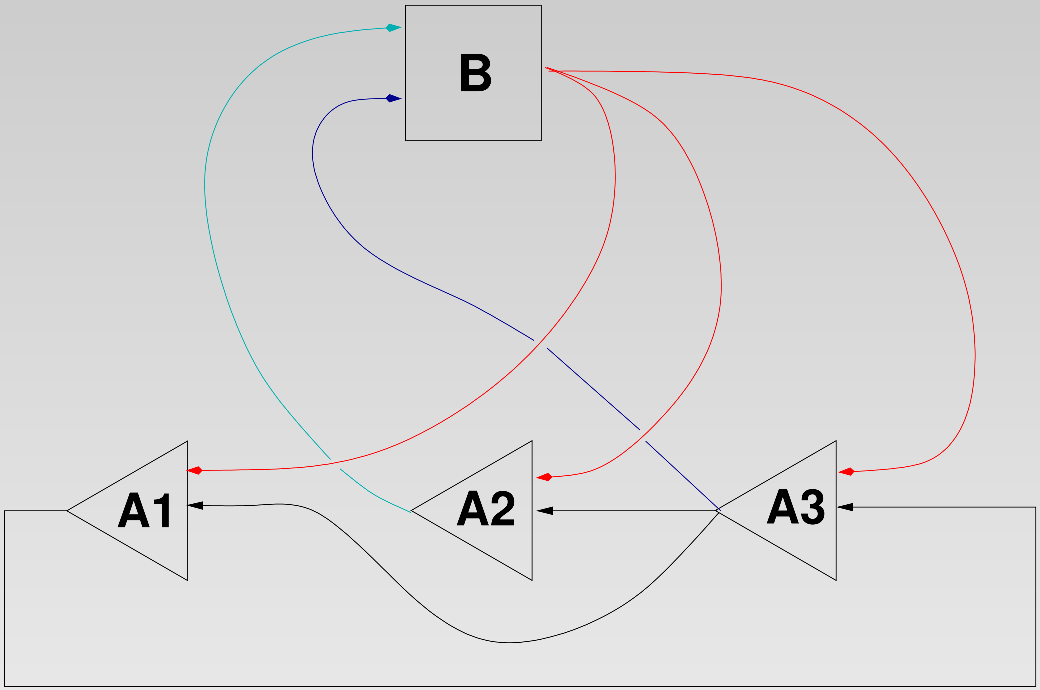
The conditions of Metatheorem 2 are unnecessarily strong and can be significantly weakened. For example, it is not necessary to assume that synchrony classes are disjoint. We illustrate by means of two examples. Recall the earlier example with synchrony classes $\{\mathbf{A1}, \mathbf{A2}\}$, $\{\mathbf{A1}, \mathbf{A3}\}$, $\{\mathbf{A1}, \mathbf{A2}, \mathbf{A3}\}$:



Three identical single input cells

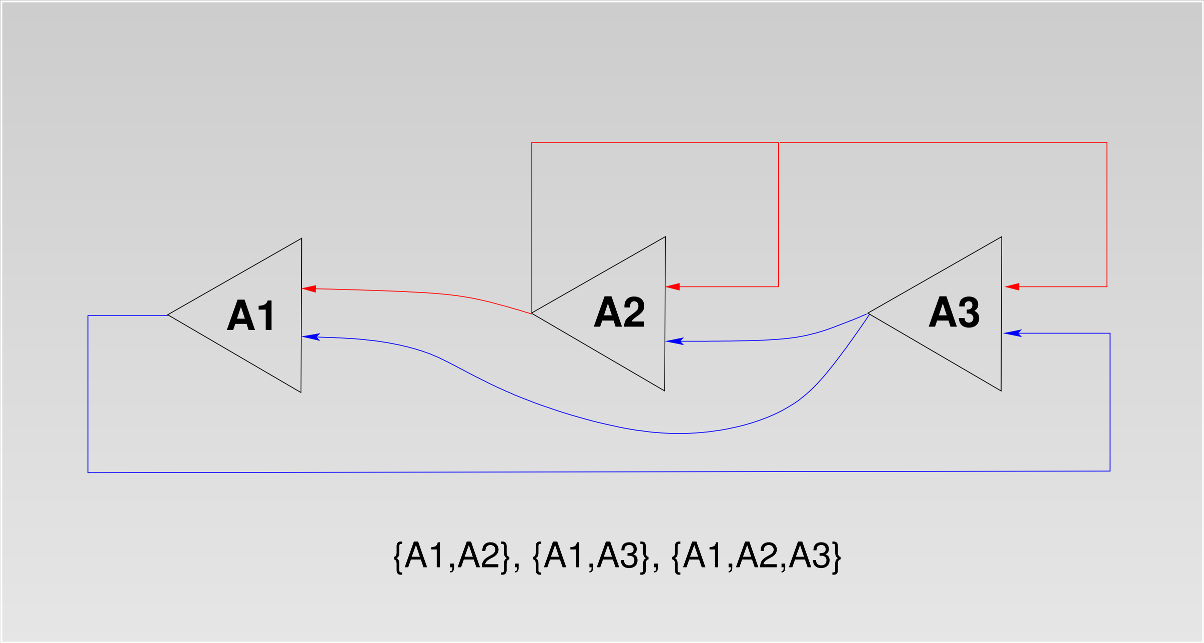
This system does *not* admit a (simple) heteroclinic cycle linking two equilibria in $\{\mathbf{A1}, \mathbf{A2}, \mathbf{A3}\}$.

1-cycles ctd.



It can be shown that this architecture supports a simple attracting heteroclinic cycle such that one connection lies in $\{\mathbf{A1}, \mathbf{A3}\}$, the other in $\{\mathbf{A1}, \mathbf{A2}\}$. The new cell acts as like a 'controller'.

1-cycles ctd.



We briefly indicate why this architecture supports a simple attracting heteroclinic cycle with one connection in $\{\mathbf{A1}, \mathbf{A3}\}$, the other in $\{\mathbf{A1}, \mathbf{A2}\}$, and the phase space dimension equal to one. Three is the minimal number of cells that can support a (robust) heteroclinic cycle in a CCS.

3-cell cycle

The ODEs governing the dynamics on this architecture are

$$\begin{aligned} x' &= f(x; y, z). \\ y' &= f(y; y, z), \\ z' &= f(z; y, x). \end{aligned}$$

Suppose the system has an equilibrium at $\mathbf{a} = (a, a, a)$. Set

$$\frac{\partial f}{\partial x}(\mathbf{a}) = \alpha, \ \frac{\partial f}{\partial y}(\mathbf{a}) = \beta, \ \frac{\partial f}{\partial z}(\mathbf{a}) = \gamma.$$

Jacobian

We find that the Jacobian matrix at $\mathbf{a}$ is

$$J(\mathbf{a}) = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha+\beta & \gamma \\ \gamma & \beta & \alpha \end{bmatrix}$$

A computation verifies that the eigenvalues of $J(\mathbf{a})$ are

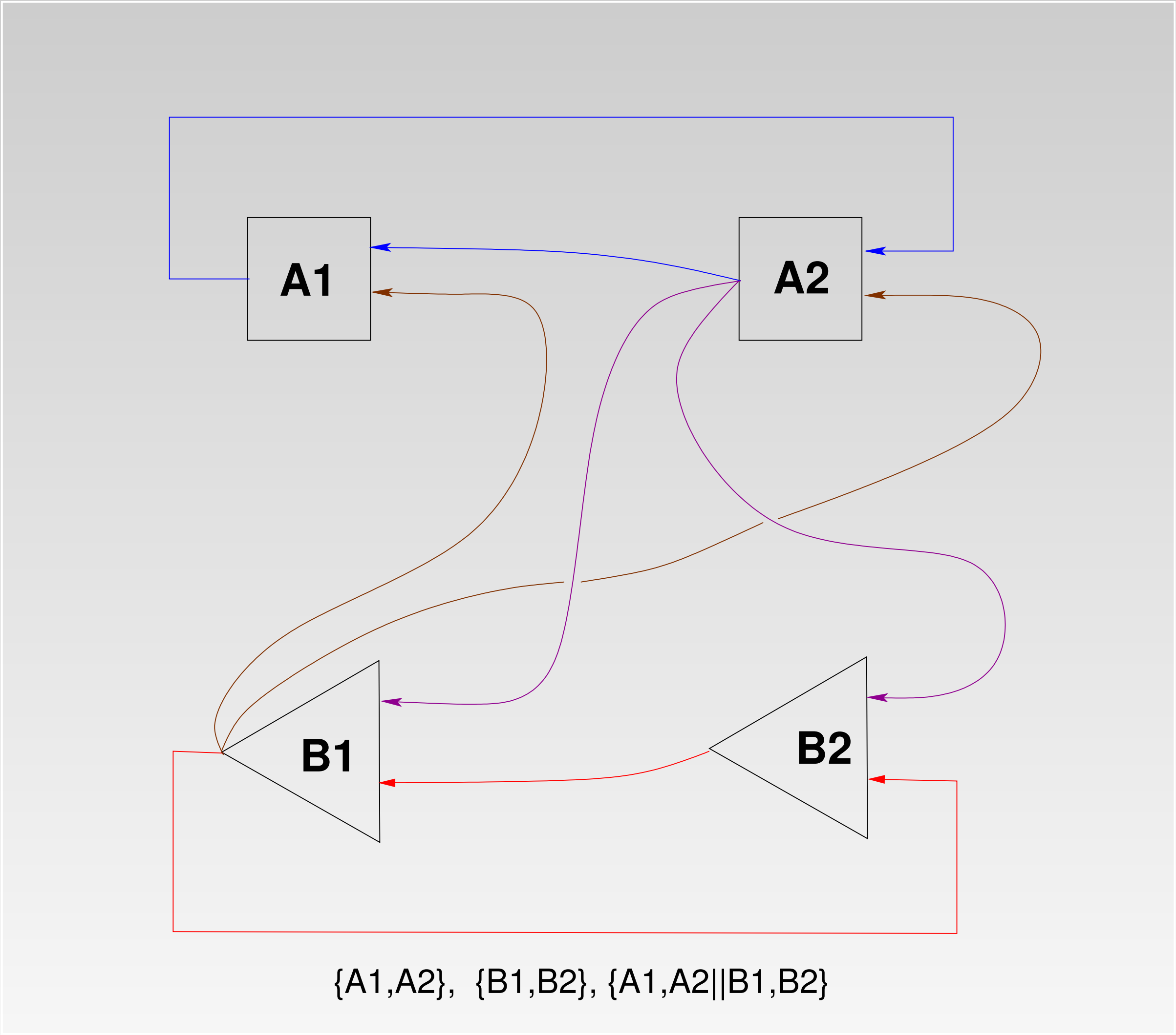
$$\alpha, \alpha-\gamma, \alpha+\beta+\gamma$$

We may choose f so that $\mathbf{a}$ has a 1-dimensional unstable manifold lying in either $\{\mathbf{A1}, \mathbf{A3}\}$ or $\{\mathbf{A1}, \mathbf{A2}\}$. It is then easy to prove the existence of asymptotically stable heteroclinic cycles.

1-cycles ctd.

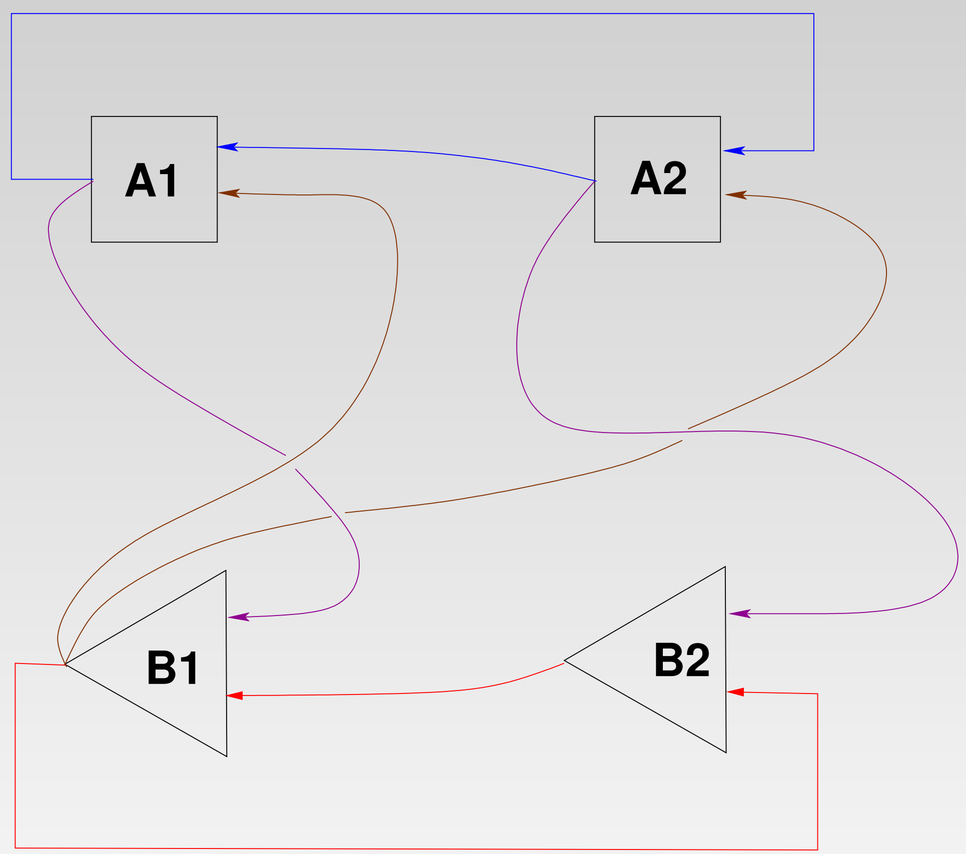
We have to be careful when weaking the condition in Metatheorem 2 that each synchrony class is an atom. We show two examples of nodes for which this condition is weakened. In both cases, the synchrony class is $\{\mathbf{A1}, \mathbf{A2} \| \mathbf{B1}, \mathbf{B2}\}$ (and so the cells synchronize into two blocks). The first example cannot appear as one of the synchrony classes of a node in an 1-cycle; the second can.

First Example



(This can be a component of a node if we drop the word ‘simple’.)

Second Example



{A1,A2}, {A1,A2||B1,B2}

ℓ-cycles

Suppose that we are given a set of N (disjoint) synchrony atoms:

$$\mathbf{S} = \{\mathcal{S}_0, \ldots, \mathcal{S}_N = \mathcal{S}_0\}$$

Fix an integer ℓ, $1 \leq \ell < N$.
A heteroclinic ℓ-cycle with node set $\mathbf{S}$, or ℓ-cycle, consists of a heteroclinic cycle joining hyperbolic equilibria $\mathbf{e}_i$, where

$$\mathbf{e}_i \in \cap_{j=i}^{\ell+i+1} \mathcal{S}_j, \quad 0 \leq i \leq N.$$

We have the usual definition of simple cycle.
We denote ℓ-cycles symbolically by:

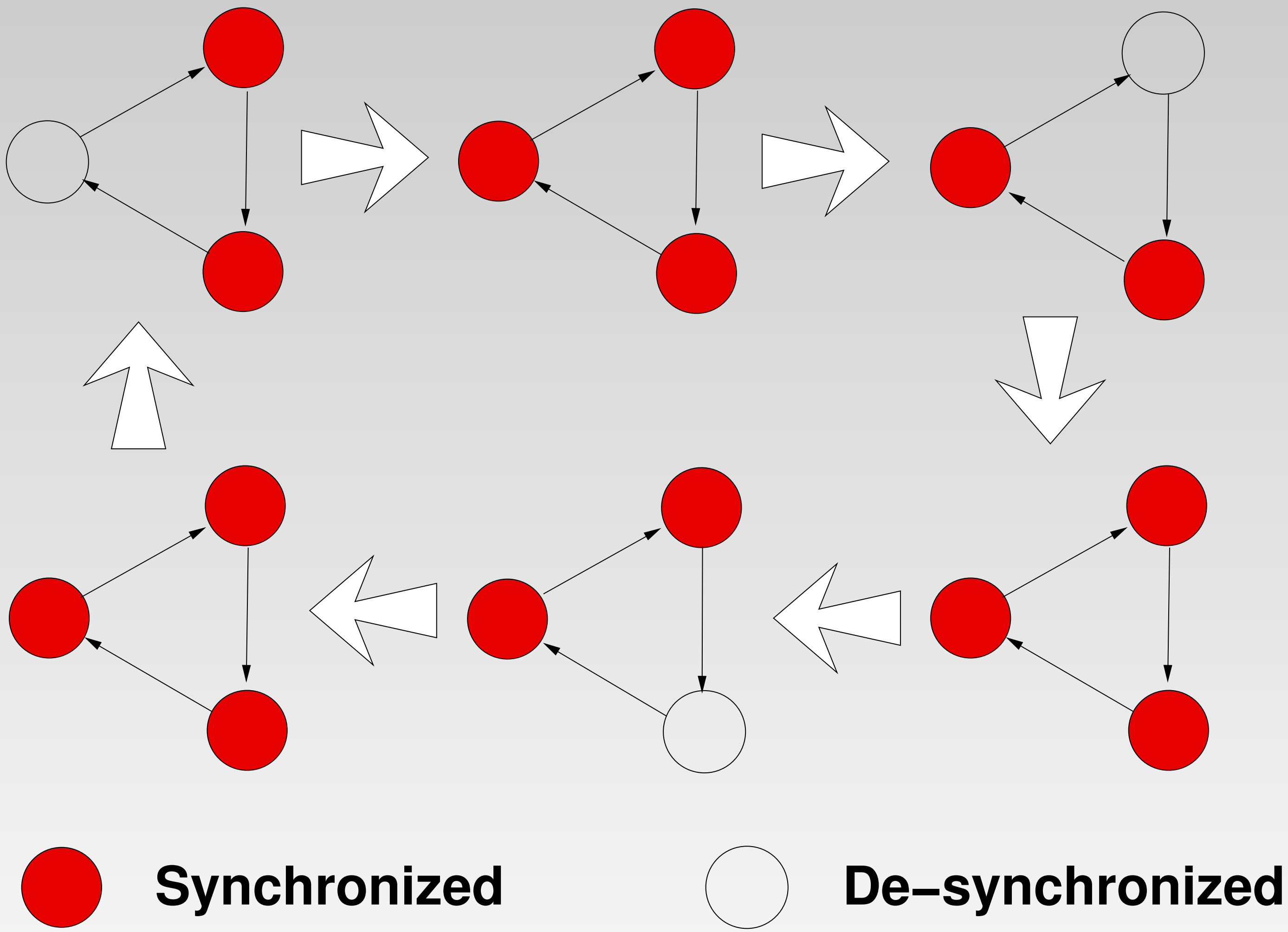
$$\to [\mathcal{S}_0, \ldots, \mathcal{S}_\ell] \to [\mathcal{S}_1, \ldots, \mathcal{S}_{\ell+1}] \to \ldots$$

Example

Given $\Gamma \in \mathcal{U}$ suppose that (a) $N = 3$ and (b) $\mathbf{S} = \{\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2\}$ consists of synchrony atoms for Γ. Assume that there exists at least one connection from an $\mathcal{S}_i$-cell to an $\mathcal{S}_{i+1}$-cell, $0 \leq i \leq 2$. There exists a coupled cell system $\mathcal{C}$, graph Γ, which supports an attracting simple 2-cycle with node set $\mathbf{S}$. There are no phase space dimension restrictions.

In this case, two groups of cells will be synchronized along each connection in the 2-cycle.

2-cycle example



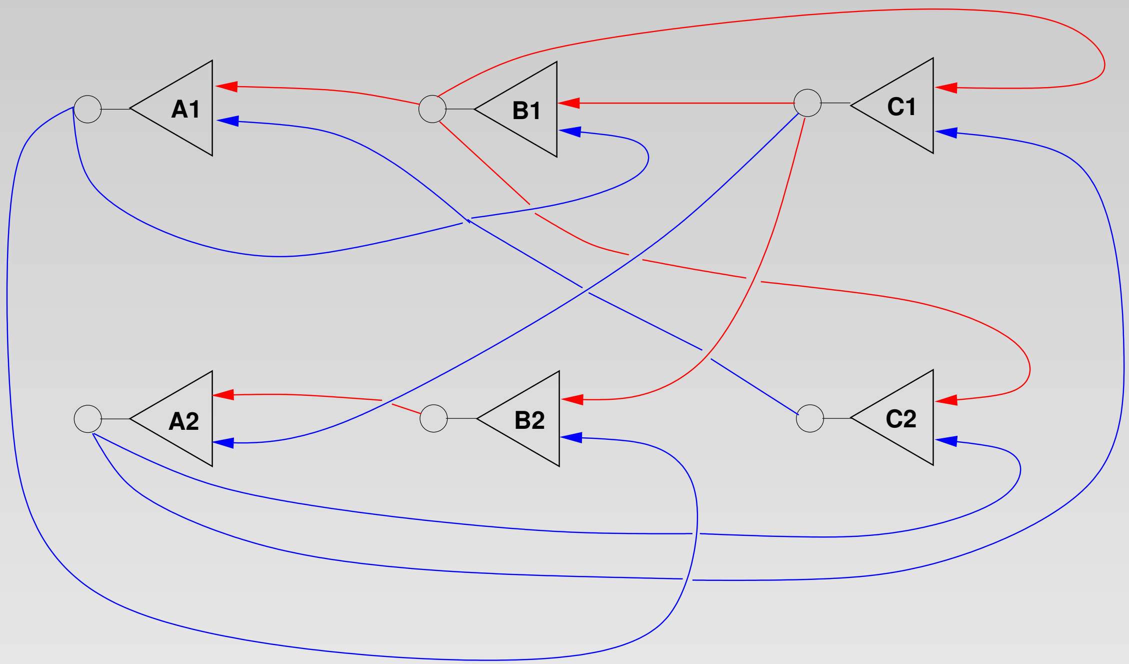
ℓ-cycles

METATHEOREM 3

Let $N \geq 2,\ 1 \leq \ell < N$. Let $\Gamma \in \mathcal{U}$ and suppose that $\mathbf{S}$ is a set of ℓ distinct synchrony atoms. There exists connection data (for example, connections $\mathcal{S}_i \rightarrow \mathcal{S}_{i+j},\ j = 1, \ldots, \ell$) that implies there is a coupled cell system $\mathcal{C}$, graph Γ, which supports an attracting simple ℓ-cycle with node set $\mathbf{S}$. There are no phase space dimension restrictions.

We remark that connection data becomes much stricter if we require that the synchrony classes are identical – that is consist of identical numbers of cells of the same type.

Not so simple, simple cycles



This network admits a simple attracting 1-cycle based on the node set

$$\mathbf{S} = \{\{\mathbf{B1}, \mathbf{B2}\}, \{\mathbf{C1}, \mathbf{C2}\}\}$$

(Connections between two states with synchrony $\{\mathbf{B1}, \mathbf{B2} \| \mathbf{C1}, \mathbf{C2}\}$).

Model equations

Equations – assume 1-dimensional dynamics.

$$
\begin{aligned}
\dot{x}_{A1} &= F(x_{A1}; x_{B1}, x_{C2}), \quad \dot{x}_{A2} = F(x_{A2}; x_{B2}, x_{C1}), \\
\dot{x}_{B1} &= F(x_{B1}; x_{C1}, x_{A1}), \quad \dot{x}_{B2} = F(x_{B2}; x_{C1}, x_{A1}), \\
\dot{x}_{C1} &= F(x_{C1}; x_{B1}, x_{A2}), \quad \dot{x}_{C2} = F(x_{C2}; x_{B1}, x_{A2}).
\end{aligned}
$$

Linearization on $\{B1, B2 \| C1, C2\}$

Computing the jacobian matrix J of the system at $(a_1, a_2; b, b; c, c)$, we find that

$$J(a_1, a_2; b, b; c, c) = \begin{bmatrix} \alpha_1 & 0 & b_1 & 0 & 0 & \bar{b}_1 \\ 0 & \alpha_2 & 0 & b_2 & \bar{b}_2 & 0 \\ c_1 & 0 & \beta & 0 & c_2 & 0 \\ c_1 & 0 & 0 & \beta & c_2 & 0 \\ 0 & e_1 & e_2 & 0 & \gamma & 0 \\ 0 & e_1 & e_2 & 0 & 0 & \gamma \end{bmatrix}$$

This network also supports solutions where the cell pairs $\mathbf{A}_{1/2}$, $\mathbf{B}_{1/2}$, $\mathbf{C}_{1/2}$ are each (approximately) synchronised but one third of a period out of phase with the adjacent pairs.

Non simple cycles

The easiest way to find robust heteroclinic cycles which are not simple is to add a little symmetry. For example, suppose that a candidate node consists of an array of p cells which has $\mathbb{Z}_p$-symmetry. For example, if $p = 3$ we might take

$$\begin{aligned} \mathbf{x_1}' &= f(\mathbf{x_1}; \mathbf{x_2}, \mathbf{x_3}), \\ \mathbf{x_2}' &= f(\mathbf{x_2}; \mathbf{x_3}, \mathbf{x_1}), \\ \mathbf{x_3}' &= f(\mathbf{x_3}; \mathbf{x_1}, \mathbf{x_2}). \end{aligned}$$

If p is *odd*, the node can never occur in a simple 1-cycle. The reason is that the unstable eigenspace associated to asynchronous solutions is always even dimensional (representation theory of $\mathbb{Z}_p$).

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Some conclusions

- Coupled cell systems support many different types of heteroclinic cycle – including attracting simple cycles.
- Finding whether a heteroclinic cycle exists in a given network can be a difficult problem.
- Even for small networks, we are a long way from obtaining any reasonable sort of classification.
- There are many interesting and significant questions relating to synchrony breaking and the appearance of periodic or chaotic phenomena near vanishing cycles.